# Supersonic and transonic Mach probe for calibration control in the Trisonic Wind Tunnel

Alexandru Marius PANAIT*

*Corresponding author
INCAS – National Institute for Aerospace Research "Elie Carafoli", Flow Physics Department, Trisonic Wind Tunnel Unit,
B-dul Iuliu Maniu 220, Bucharest 061126, Romania,
panait.marius@incas.ro

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***Abstract:*** *A supersonic and high speed transonic Pitot Prandtl is described as it can be implemented in the Trisonic Wind Tunnel for calibration and verification of Mach number precision. A new calculation method for arbitrary precision Mach numbers is proposed and explained. The probe is specially designed for the Trisonic wind tunnel and would greatly simplify obtaining a precise Mach calibration in the critical high transonic and low supersonic regimes, where typically wind tunnels exhibit poor performance. The supersonic Pitot Prandtl combined probe is well known in the aerospace industry, however the proposed probe is a derivative of the standard configuration, combining a stout cone-cylinder probe with a supersonic Pitot static port which allows this configuration to validate the Mach number by three methods: conical flow method – using the pressure ports on a cone generatrix, the Schlieren-optical method of shock wave angle photogrammetry and the Rayleigh supersonic Pitot equation, while having an aerodynamic blockage similar to that of a scaled rocket model commonly used in testing. The proposed probe uses an existing cone-cylinder probe forebody and support, adding only an afterbody with a support for a static port.*

***Key Words:*** *Supersonic Pitot Prandtl, transonic Mach number measurement, Mach calibration and validation in transonic regimes, calculation method, numeric method*

## 1. INTRODUCTION

The supersonic Pitot Prandtl tube appeared as a necessary adaptation to circumvent the limitations of the method of speed measurement for gas flows in the high-speed regimes where Bernoulli's equations do no longer apply. The classic theory of compressible subsonic flows works for flow speeds up to the establishment of the first shock wave when the isentropic principle is violated. For supersonic speeds, any type of body that exhibits a sharp edge, point or slope causes a complex of expansion and simple waves that culminate in a shock wave. The shock wave is a discontinuity point in the flow where energy is lost to thermal processes as flow parameters jump from the free flow to the post-shock values. This regime where shock waves appear is governed by a different set of laws but is by no means sudden and uniform but partial and complicated, where subsonic flow co-exists with transient phenomena of flow separation and re-attachment etc. and, in consequence, as the flow approaches the speed of sound or Mach 1, we find ourselves in a transitional regime where everything is hard to compute or measure due to complexities in the flow field. This regime, called transonic, marks a wide interval where the compressibility term for the dynamic pressure correction increases with the fourth power of the Mach number as described below (1), (2)

$$p_t = p_s + p_d(1+\eta) \tag{1}$$

where $p_t$ is the total pressure, $p_s$ the static pressure and $p_d$ the dynamic pressure as per the incompressible gas theory (2)

$$p_d = \frac{1}{2}\rho V^2 \tag{2}$$

with 1+η the correction factor for gas compressibility, experimentally found to be expressed as (3):

$$1+\eta = \left(1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^6}{1600} + \cdots\right) \tag{3}$$

This shows the importance of Mach number calculation and usage even in subsonic flows for every speed above approximately 50 m/s as discovered by NACA in the late 40's, when the first studies on flow compressibility in wind tunnels were made [1], [4], [5].

## 2. PRACTICAL MACH NUMBER CALCULATION IN WIND TUNNELS

A more practical way to measure speed in wind tunnels is not to rely on the definition for the dynamic pressure at all but to realize that, although complicated, (2) and (3) the dynamic pressure is always equal to the difference between the total pressure and static pressure in a flow with no hydrostatic pressure gradient (such as in most wind tunnels), and we can rewrite the Mach number equations for the subsonic and supersonic regimes to use the total pressure and the static pressure, readily available in wind tunnels by means of simple Pitot tubes in the settling chamber and static pressure ports in various locations along the experimental chamber and Laval nozzle.

To fully characterize the flow speed in all regimes, a probe capable of dealing with the onset of shock waves and a proper Mach calculation method have long been established, since before the 50's [2]. For the purely subsonic yet compressible flow in wind tunnels we find it most practical to express the Mach number as (4) with Bernoulli's equation while for high transonic/supersonic flows the Rayleigh equation is valid (5).

$$M_{subsonic} = \sqrt{5\left[\left(\frac{p_t}{p_s}\right)^{\frac{2}{7}} - 1\right]} \tag{4}$$

$$M_{supersonic} = 0{,}88128485\sqrt{\frac{p_t}{p_s}\left(1 - \frac{1}{7M^2}\right)^{\frac{5}{2}}} \tag{5}$$

We immediately notice the recursive nature of the equation (5) for the supersonic Mach number and conclude that we must find a good starting value to establish a process of simple iteration. Indeed, it is shown that the most numerically simple method, the simple iteration having the subsonic values described by (4) as a first term, quickly converges to a Mach value as the derivative of (5) is sub-unitary for any flow conditions described by $p_t$ and $p_s$. Some clarifications are necessary.

As we have shown above, the relations are only valid in the assumption of isentropic flow, but as in supersonic we have shock waves we need to assume that the flow before and immediately after the shock wave is isentropic, and we use the total pressure measured behind the shock wave for $p_t$., assumed to be normal to the flow, and same for the static pressures that will be measured behind the shock and expansion wave complex on the probe body tip.

However, the method makes use of the observation that for these calculations to be valid, a good calibration must already be in effect, i.e. a relation that links static pressure on wall taps to pressures in the experimental chamber pressures.

Moreover, the method fails when we try to apply it for supersonic Mach values because with an empty wind tunnel /experimental chamber, there is no simple correspondence to the shock waves in the wind tunnel (caused aft of the model support mechanism by the latter's parallelogram mechanism) and those formed on a model or supersonic probe, imposed by the geometry of these objects.

That is why in supersonic we must find a relation that works only with pressures ahead of the shock wave to predict speeds after the shock wave without needing much information on where the shock wave is situated apart from the obvious jump in static pressures that can be observed on the wall taps.

In practice, only speeds after a shock wave matter because the shock wave is always produced by the object that is immersed in the supersonic flow but we can easily measure only pressures and calculate speeds with the isentropic assumption before the shock wave. The so-called Rayleigh relations link the parameters across the shock wave, by describing the evolution for pressure, stagnation pressure, velocity and temperature of the fluid before and after the shock wave (assimilated to the choking point in the flow).

## 3. PROPOSED MACH CALIBRATION METHOD

Therefore, choosing a right set of pressure taps is paramount, both when using the probe and when measuring with the empty experimental chamber, to avoid those where the shock wave affects the normal static pressure distribution.

That is the reason why I propose this method of supersonic Mach calibration using a specially modified cone-cylinder probe (Figure 1) that uses several static pressures taps on its cylindrical body to best measure the reference static pressure: immediately behind the shock wave in supersonic, and at a practically observed distance of 4 to 10 probe diameters from the cone to cylinder junction for the subsonic regime. Also, to avoid the interference effects in subsonic regimes as well as in transonic ones, the last pressure taps on the probe are also to be eliminated from calculations (ignored).

Data is valid only if we have no shock waves that are reflected on the probe from the wind tunnel walls.

If that is the case, the pressure distribution one expects from a classical cone-cylinder probe (Figure 2, [3]) would not be met and the assumption that the pressure taps on the cylindrical section approximate static pressures in the experimental chamber's centerline is no longer valid.

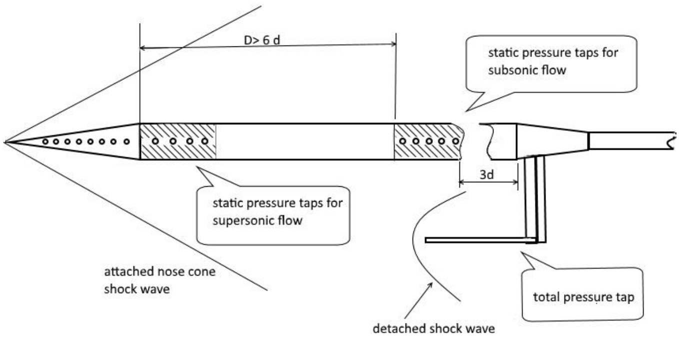



Figure 1. The modified cone-cylinder probe is transformed in a supersonic Pitot tube with expanded capabilities in transonic and subsonic regimes if appropriate selection of static pressure taps is made

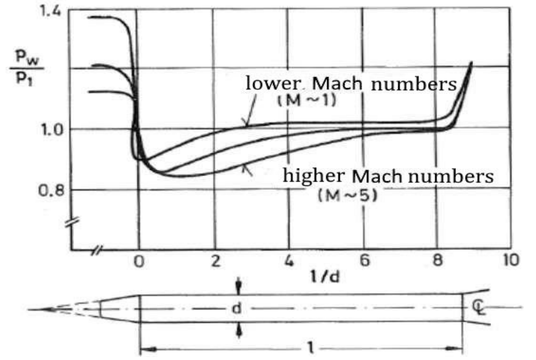



Figure 2. The theoretical pressure distribution for a cone-cylinder probe is taken as reference for the validation of the experimental conditions of the supersonic Pitot tube measurements [3]

As seen in Figure 2, the pressure measured by the taps on the cylindrical section approximate the static pressure distribution for distances of 3 to basically the end of the cylinder section. As shown in the diagram given in [3] the cone-cylinder probe continues with a frustum that causes static pressures to jump again but that is not the case with our probe that in its initial configuration had no afterbody at all but ended in a sharp edge and dropped to the diameter of the sting (support). With the proposed modification we expect a drop in static pressures that is smooth and due to area ruling, further aft from the last pressure taps on the cylindrical section. However, there will be some pressure taps that will be ignored in the calculation of the Mach number – those affected by interference as seen in Figure 2. If other anomalous static pressure distributions appear, the cause must be a reflected shock wave and the theory for Mach calculation with the described method no longer stands, as it is based on the “two isentropic regions” flow approximation- the flow behind and ahead of the shock wave are considered to be two separate isentropic flows linked together by a parameter (pressure, temperature, entropy etc.) jump at the shock wave described by a set of known equations similar to (5) that only describes pressures and Mach. In Figure 1 we marked two areas of pressure reference that are to be used for static pressures behind the shock wave and in subsonic regimes. The values measured with these taps are subjected to some basic statistic work. The mean is extracted and the standard deviation computed from groups of up to 8 taps in the chosen regions. If outliers are present -with the 3-sigma criterion, we conclude that some shock wave reflection must have occurred and the data is no longer valid. Choosing the static pressure taps to work statistics on is made by observing the characteristics of flows on slender cone-cylinder bodies. Some taps are eliminated from the process by virtue of their unfavorable positioning, close to the probe ends (tip and support). The number of these pressure taps that are not useful for static pressure measurements in speed measurement can be reduced by adapting the geometry using a profiled, area-ruled afterbody as well as replacing the cone forebody of the probe with an elliptic or parabolic one in the regimes these are optimal for (high transonic). The effect of these modifications to the standard cone-cylinder probe is a better approximation of the centerline pressure distribution in the experimental chamber from the pressure taps in the cylindrical section coming from reducing the influences of the fore and afterbodies on the pressure values in the static measurement section.

Finding the exact beginning of the sections to measure static pressures from is more complicated in the supersonic region (Figure 3). The flow in Figure 3 can be considered isentropic before the shock wave and again behind it. In fact, the conical flow is even considered frictionless and irrotational around the cone – valid assumptions for a symmetric flow on a symmetric cone that is sufficiently polished and carefully built. This leads to the conic flow to have only two components, along the generatrix and perpendicular to it. The Taylor-Maccall equations (6) then describe the conical flow by giving the shock wave angle relation to the flow speed and have their results tabulated as ready-to-use guides. It is a good control parameter for the calculated Mach values using the pressure method described above when Schlieren is available.

If that is not the case, the values of the probe pressure distribution on the conical section are also linked to the well-known solutions to the Taylor-Maccall equations for conical flows and as such can be directly compared with the theoretical values and thus the flow velocity can be found in diagrams like Figure 4 [5], [6].

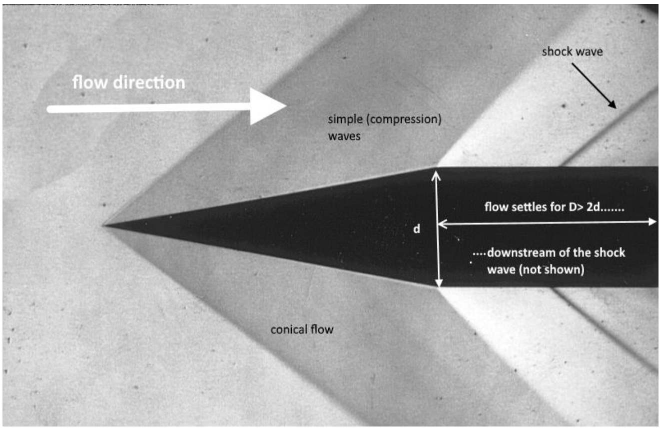



Figure 3. A cone-cylinder probe head in supersonic flow. Note the system of the simple waves and the thin shock wave that forms approximately at 2d from the cone-cylinder junction

The simplified conical flow equations are:

$$\begin{cases} \frac{\gamma-1}{2}\left[1-V_r^{\,2}-\left(\frac{dV_r}{d\theta}\right)^2\right]\left[2V_r+ctg\theta\frac{dV_r}{d\theta}+\frac{d^2V_r}{d\theta^2}\right]-\frac{dV_r}{d\theta}\left[V_r\frac{dV_r}{d\theta}+\frac{dV_r}{d\theta}\frac{d^2V_r}{d^2\theta}\right]=0 \\ V_\theta=\frac{dV_r}{d\theta} \end{cases} \tag{6}$$

where $V_r$ is the flow speed along the radius, $V_\theta$ the speed perpendicular to it, with θ the simple wave angle.

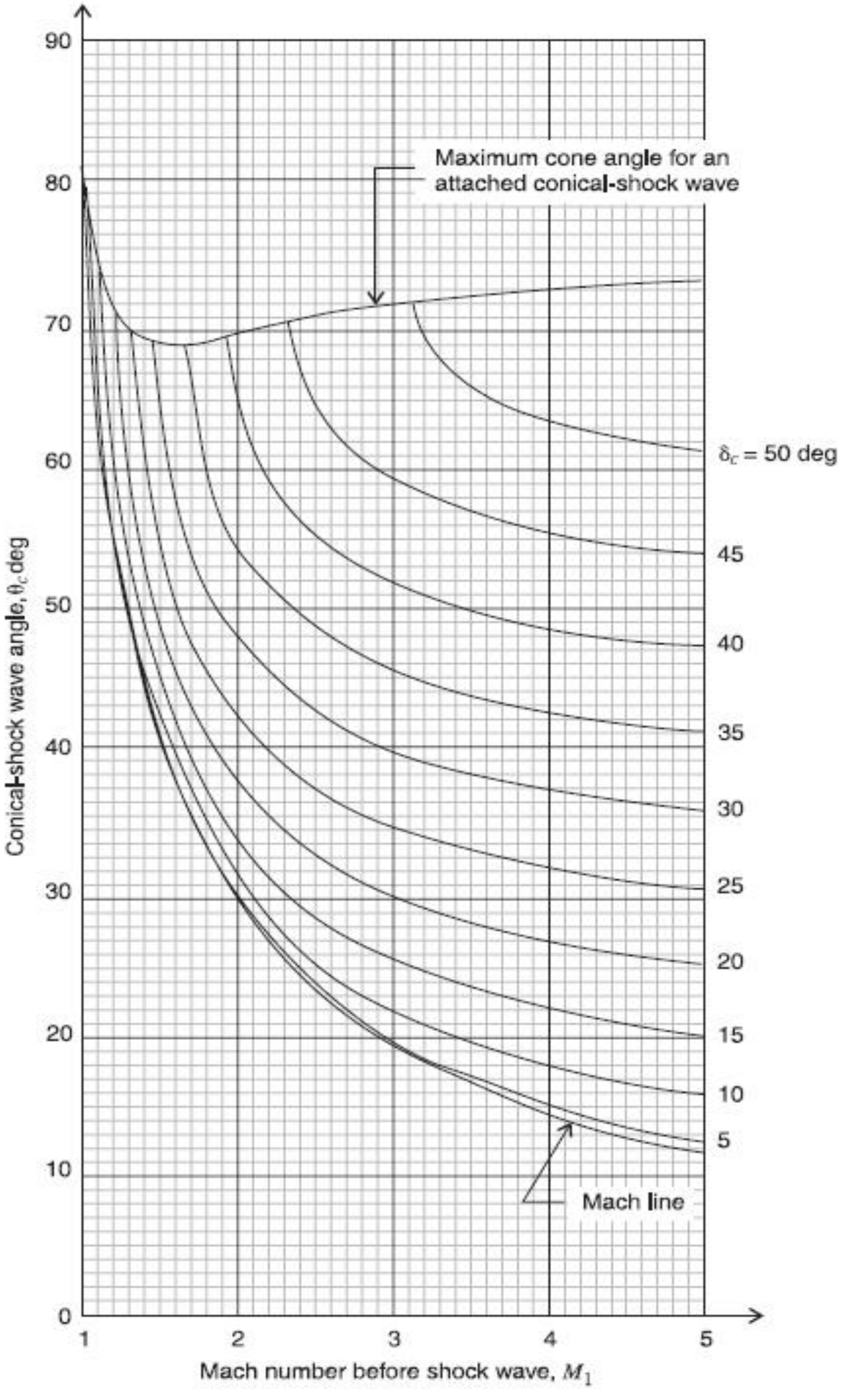
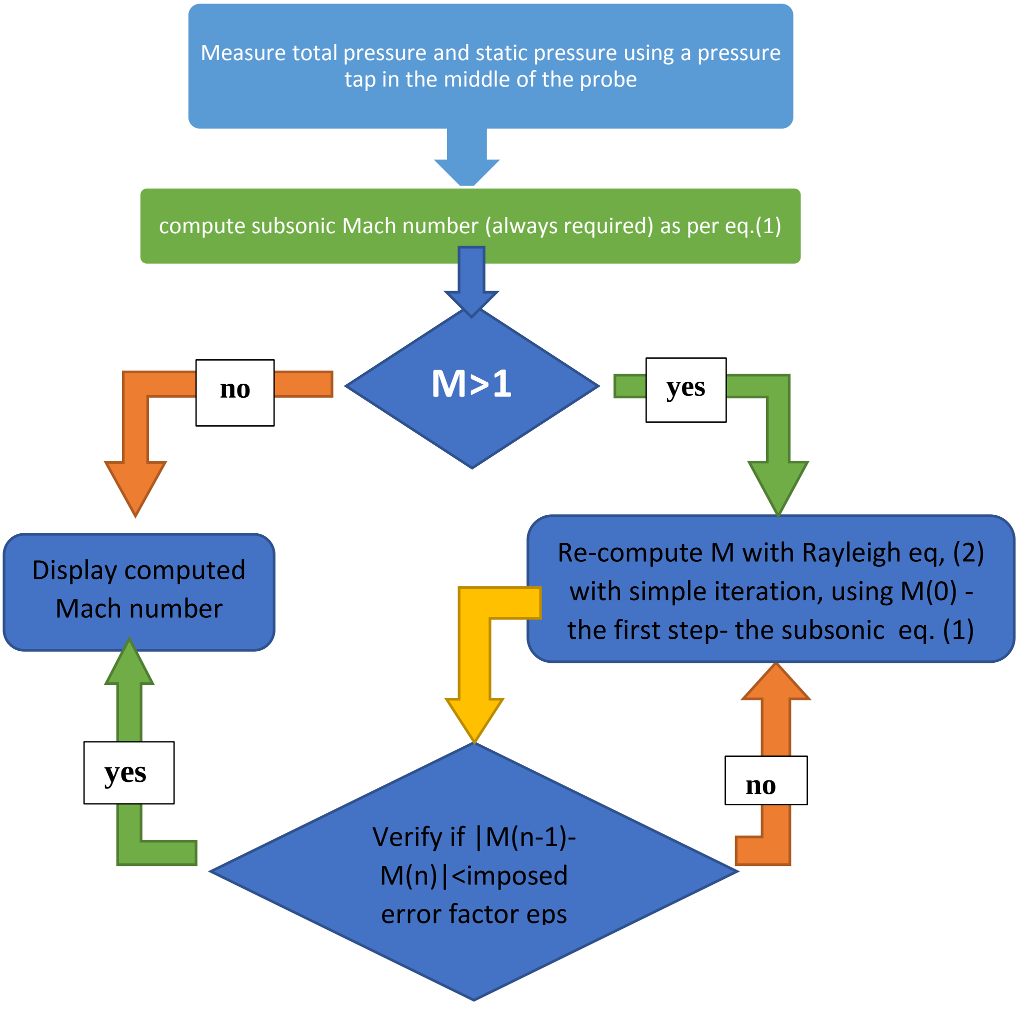



Figure 4. Finding the Mach number by Schlieren photogrammetry is made possible by charts like these [5], [6]

We solve the equation numerically to get $V_r$ and an angle θ for which $V_\theta$ =0, that would give the speeds behind the shock wave having that angle. The method is solving numerically for a

number of angles until the second equation in (6) equals zero. Iterations typically use a Runge-Kutta fourth order method and usually the convergence is quick. The results are tabled and given, for example, in specialty literature such as [2] to be directly used to determine flow velocities from Schlieren pictures of conical probes or conical forebodies.
For our intents the conical flow is to be used for probe calibration and verification of the correctness of results obtained with the pressure method described above as we need to confirm the static pressure taps were chosen correctly for the particular flow regime investigated.
We adapt the Mach-meter velocity calculation procedure for using our modified probe as a subsonic/transonic and supersonic Pitot, choosing at first pressure taps in the middle of the cylindrical region as reference so as to minimize errors and then changing the reference static pressure group as shown in Figure 1 as required by the computed flow speeds (sub/supersonic).
The algorithm for determining the Mach number using the pressure method using the modified probe is described by the logic diagram below:



## 4. RAYLEIGH OR FANNO?

Another valid simplification for flow conditions in a blowdown wind tunnel is the so-called Fanno flow- an adiabatic, unidimensional and friction-enabled model. Its equations describe how a flow entering a sufficiently long and rough conduit is choked by friction with the conduit walls [4]. It is not generally desired to reach Fanno flow in supersonic blow -down wind tunnels, as it is a wasteful dissipative regime when the flow energy is lost by friction with rough walls.

As a consequence, the Laval nozzle walls and the brief divergent section right at its exit are extremely smooth and care is taken to eliminate needless friction with the wind tunnel walls. In reality there is some friction but the upstream energy of the flow is maintained as high as necessary to push the flow directly into the Rayleigh regime. Fanno flow occurs for brief moments before tunnel start-up and at the blowdown sequence end, when the pressure regulating valve closes and the flow is no longer maintained by upstream energy. The last motions of the air column in the wind tunnel, now closed at the supply end, are those of an oscillating tube and the flow is best described by the Fanno model. In operation, we prefer the Rayleigh model.

## 5. CONCLUSIONS

The calibration procedures would benefit from the implementation of the proposed special probe configuration as it facilitates the quick velocity conformity checking with a high degree of accuracy and with no important modifications to the tunnel baseline configuration- unlike the central probe that needs installation of a specially designed support system and special operating regimes of the wind tunnel so as not to damage it from vibration. Still it is considered that a well-designed and equipped central probe is necessary for a correct and complete characterization of the flow quality after each major repair, modification and revision or whenever the tunnel contours or wall quality was in some way altered. To better assess flow quality in the wind tunnel, the cone tip can be equipped with more pressure taps along several generatrixes and the values of the pressure on the taps at the same distance from the tip can be compared. If significant differences appear between these pressure taps then there is some degree of asymmetry in the flow- angularity or rotational components present. Also, to improve the analysis, the total pressure probe can be substituted with a five-hole probe to check for angularity in the probe body wake as well. This supplemental information ensures that the probe body did not have too much influence on the total pressure readout.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