Rotation-Rotation Mechanism moving with Respect to a Plane

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Abstract: *The relative motion of a robotic arm formed by homogeneous bars of different lengths and masses, hinged to each other is investigated. The first bar of the mechanism is articulated on a platform, considered for three cases: placed on the surface of the Earth, so it is fixed; located on Earth, but in rotation with respect to it and in the final case, located on a space station, moving around the Earth. For all the analyzed cases the motion equations are determined using the Lagrangian formalism.*

Key Words: *Lagrange equations, relative motion, inertial and non-inertial reference frame*

1. INTRODUCTION

In this paper the relative motion of a robotic arm formed by bars of different lengths and masses, hinged to each other is investigated. The first bar of the mechanism is articulated on a platform, considered for three cases: placed on Earth, so it is fixed; located on Earth, but in rotation with respect to it and located on a space station [1], moving around the Earth.

The first of these three cases corresponds to the motion of the mechanism with respect to a fixed reference frame and the other two with respect to the mobile reference systems. For all the analyzed cases the motion equations are determined using the Lagrangian formalism [2, 3], applied in its traditional form, valid with respect to an inertial reference system, conventionally considered as fixed. However, in some cases, a generalized form of the formalism valid in relation to a non-inertial reference frame [6], [7], [8] will also be applied. The numerical calculations were performed using a Matlab program.

2. CONFIGURATION OF THE MECHANISM

A robotic arm consisting of two homogeneous bars is studied (Fig. 1). The first bar, *OA*, is hinged in point *O* on the fixed element while the second bar, *AB*, is articulated in point *A* on the first bar. The bars have the length l_1 and l_2, respectively, and the masses m_1, m_2, respectively. The bars move in the *Oxy* plane.

The configuration of the mechanism is defined by the angles φ_1 and φ_2, formed by the two bars with respect to the *Ox* axis. The bar *OA* is driven by the torque motor with the moment M_1 and the bar *AB* is driven by the bar *OA* via the torque motor with the moment M_2. The variations of moments M_1 and M_2 are determined so that the motion of the mechanism takes place according to the equations of motion:

$$\varphi_1(t) = A_1 \frac{\omega_0}{2\pi}\left(t - \frac{1}{\omega_0}\sin(\omega_0 t)\right), \tag{1}$$

$$\varphi_2(t) = A_2 \frac{\omega_0}{2\pi}\left(t - \frac{1}{\omega_0}\sin(\omega_0 t)\right). \tag{2}$$

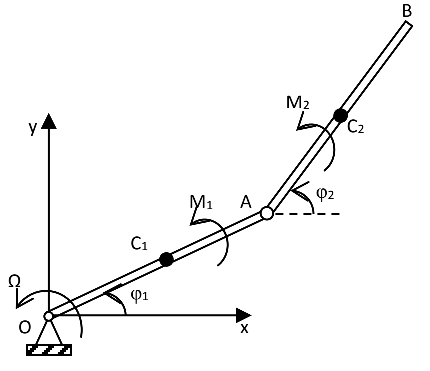

Fig. 1 Rotation around the Oz axis

2.1 The Mechanism is Articulated on a Fixed Platform

In this case, the Lagrangian formalism in the traditional form

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial E}{\partial \dot{\varphi}_1}\right) - \dfrac{\partial E}{\partial \varphi_1} = Q_1 \\ \dfrac{d}{dt}\left(\dfrac{\partial E}{\partial \dot{\varphi}_2}\right) - \dfrac{\partial E}{\partial \varphi_2} = Q_2 \end{cases}, \tag{3}$$

is applied, where the kinetic energy of the system is

$$E = \tfrac{1}{2} J_O \omega_1^2 + \tfrac{1}{2} m_2 v_{C2}^2 + \tfrac{1}{2} J_{C2}\omega_2^2. \tag{4}$$

The generalized forces Q_1 and Q_2 from the right side of the Lagrange equations are determined using the virtual work

$$\delta L = M_1\delta\varphi_1 + M_2(\delta\varphi_2 - \delta\varphi_1) = (M_1 - M_2)\delta\varphi_1 + M_2\delta\varphi_2. \tag{5}$$

Thus, the differential equations of motion are obtained:

$$\begin{cases} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2\cos(\varphi_2 - \varphi_1) - A_{12}\dot{\varphi}_2^2\sin(\varphi_2 - \varphi_1) = M_1 - M_2 \\ A_{12}\ddot{\varphi}_1\cos(\varphi_2 - \varphi_1) + A_{22}\ddot{\varphi}_{12} + A_{12}\dot{\varphi}_1^2\sin(\varphi_2 - \varphi_1) = M_2 \end{cases}, \tag{6}$$

where the following notations were made:

$$A_{11} = \left(\frac{m_1}{3} + m_2\right) l_1^2, \tag{7}$$

$$A_{22} = \frac{m_2}{3} l_2^2, \tag{8}$$

$$A_{12} = \frac{1}{2} m_2 l_1 l_2, \tag{9}$$

2.2 The Mechanism is Hinged on a Rotating Platform

In this case the platform is located on the Earth, but in rotation with respect to it, about a vertical axis, with the constant angular velocity Ω (Fig. 2).

The motion of the rods that make up the robotic arm takes place in the rotation plane of the platform.

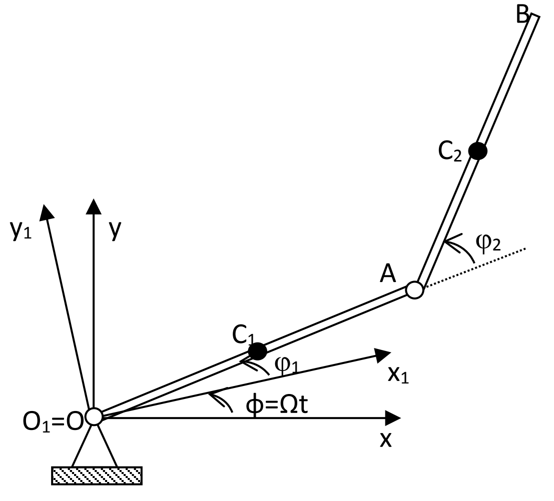

Fig. 2 Mechanism on a mobile platform on the ground

2.2.1 Lagrange Equations with Respect to a Fixed Reference Frame

The equations of motion are determined using the system (3), where the kinetic energy of the system, in this case, is

$$E = \frac{1}{2} J_O (\Omega + \dot{\varphi}_1)^2 + \frac{1}{2} m_2 v_{C2}^2 + \frac{1}{2} J_{C2} (\Omega + \dot{\varphi}_2)^2. \tag{10}$$

The generalized forces from the right side of the Lagrange equations will maintain their previous form, thus, the differential equations of motion will be:

$$\begin{cases} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2 \cos(\varphi_2 - \varphi_1) - A_{12}(\dot{\varphi}_2 + \Omega)^2 \sin(\varphi_2 - \varphi_1) = M_1 - M_2 \\ A_{12}\ddot{\varphi}_1 \cos(\varphi_2 - \varphi_1) + A_{22}\ddot{\varphi}_{12} + A_{12}(\dot{\varphi}_1 + \Omega)^2 \sin(\varphi_2 - \varphi_1) = M_2 \end{cases}. \tag{11}$$

2.2.2 Lagrange Equations with Respect to a Mobile Reference Frame

In order to obtain the generalized transport force, the following is calculated:

$$E_c = E_c^{OA} + E_c^{AB}, \tag{12}$$

$$E_c^{OA} = \frac{1}{2} J_o \Omega^2 = \frac{1}{2} \frac{m_1 l_1^2}{3} \Omega^2, \tag{13}$$

$$E_c^{AB} = \frac{1}{2}J_o^{AB}\Omega^2 = \frac{1}{2}\left[\frac{m_2 l_2^2}{12} + m_2(x_{1C2}^2 + y_{1C2}^2)\right]\Omega^2. \tag{14}$$

For the robotic arm formed by the two homogeneous bars, the expressions

$$Q_{kt}^{\omega} = \frac{\partial E_c}{\partial q_k}, (k = 1, 2), \tag{15}$$

depend on the generalized coordinates of the system, φ_1 and φ_2:

$$\begin{cases} Q_{1t}^{\omega} = \dfrac{\partial E_c}{\partial \varphi_1} = \dfrac{1}{2}\Omega^2 m_2 l_1 l_2 \sin(\varphi_2 - \varphi_1) \\ Q_{2t}^{\omega} = \dfrac{\partial E_c}{\partial \varphi_2} = -\dfrac{1}{2}\Omega^2 m_2 l_1 l_2 \sin(\varphi_2 - \varphi_1) \end{cases}. \tag{16}$$

From Fig. 2 it can be noted that the bar *OA* is in rotation, therefore the contribution of this body to the generalized Coriolis force

$$Q_{kc} = -2\overline{\omega_0} \cdot \overline{\overline{P_{o1}}} \cdot \left(\overline{\omega_r} \times \frac{\partial \overline{\omega_r}}{\partial \dot{q}_k}\right), (k = 1, 2) \tag{17}$$

is zero, because the two variables of the cross product are parallel vectors.

For the bar *AB*, which is a rigid body in an arbitrary relative motion, formula [5]

$$Q_{kc} = -2\overline{\omega_0} \cdot m \cdot \left(\overline{v_{rc}} \times \frac{\partial \overline{v_{rc}}}{\partial \dot{q}_k}\right) - 2\overline{\omega_0} \cdot \overline{\overline{P_{o_1}}} \cdot \left(\overline{\omega_r} \times \frac{\partial \overline{\omega_r}}{\partial \dot{q}_k}\right), (k = 1, 2), \tag{18}$$

can be applied and, taking into account that the second cross product is zero, because the vectors are parallel, the following expression for the contribution of this bar to the generalized Coriolis force is obtained:

$$Q_{kc}^{AB} = -2\overline{\Omega} m_2 \cdot \left(\overline{v_{rC_2}} \times \frac{\partial \overline{v_{rC_2}}}{\partial \dot{q}_k}\right), (k = 1, 2). \tag{19}$$

This way, the Lagrange equations with respect to a non-inertial reference system will be:

$$\begin{cases} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2\cos(\varphi_2 - \varphi_1) - A_{12}(\dot{\varphi}_2)^2\sin(\varphi_2 - \varphi_1) = \\ \quad = M_1 - M_2 + A_{12}\Omega^2\sin(\varphi_2 - \varphi_1) + 2A_{12}\Omega\dot{\varphi}_2\sin(\varphi_2 - \varphi_1) \\ A_{12}\ddot{\varphi}_1\cos(\varphi_2 - \varphi_1) + A_{22}\ddot{\varphi}_2 + A_{12}(\dot{\varphi}_1)^2\sin(\varphi_2 - \varphi_1) = \\ \quad = M_2 - A_{12}\Omega^2\sin(\varphi_2 - \varphi_1) - 2A_{12}\Omega\dot{\varphi}_1\sin(\varphi_2 - \varphi_1) \end{cases}. \tag{20}$$

It can be noticed that the systems (11) and (20) are equivalent, which proves that the two study methods lead to identical results.

2.3 The Mechanism is Located on a Space Station

It is further considered that the mechanism previously presented is articulated on a platform located on a space station orbiting at an arbitrary altitude with the constant angular velocity Ω (Fig. 3).

The motion of the mechanism takes place in the plane of the space station orbit.

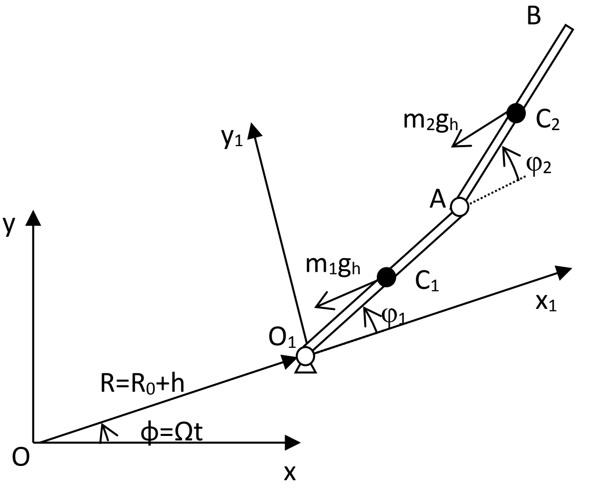

Fig. 3 Mechanism on a mobile platform

2.3.1 Lagrange Equations with Respect to a Fixed Reference Frame

The kinetic energy of the system illustrated in Fig. 3 is

$$E=\frac{1}{2}m_1v_{c1}^2+\frac{1}{2}J_{c1}(\dot{\varphi}_1+\Omega)^2+\frac{1}{2}m_2v_{c2}^2+\frac{1}{2}J_{c2}(\dot{\varphi}_2+\Omega)^2. \tag{21}$$

The angular velocity Ω is

$$\Omega=\sqrt{\frac{\mu}{R^3}}, \tag{22}$$

where R is the radius of the circular orbit of the space station, while $\mu=3{,}986004418\cdot10^{14}\,\mathrm{m}^3/s^2$ and denotes the standard gravitational parameter of the Earth.

If the system is acted on by conservative and nonconservative forces, then the generalized forces will be of the form [3, 4]

$$\begin{cases}Q_1=\dfrac{\delta_1L^*}{\delta\varphi_1}+\dfrac{\partial U}{\partial\varphi_1}\\ Q_2=\dfrac{\delta_2L^*}{\delta\varphi_1}+\dfrac{\partial U}{\partial\varphi_2}\end{cases}, \tag{23}$$

where δ_kL^* is the virtual work produced by the non-conservative forces.

For conservative systems,

$$\begin{cases}Q_1=\dfrac{\partial U}{\partial\varphi_1}\\ Q_2=\dfrac{\partial U}{\partial\varphi_2}\end{cases}, \tag{24}$$

where U is the function of force corresponding to the conservative forces that act upon the system.

Thus, for the studied mechanism, the terms corresponding to the non-conservative forces are

$$\begin{cases} \dfrac{\delta_1 L^*}{\delta\varphi_1} = M_1 - M_2 \\ \dfrac{\delta_2 L^*}{\delta\varphi_1} = M_2 \end{cases}, \tag{25}$$

and the ones corresponding to the conservative forces are of the form (24).

If the gradient of the gravitational field is neglected, the function of force U will have the form:

$$u = -m_1 g_h x_{1C_1} - m_2 g_h x_{1C_2} + C = -D_1 \cos\varphi_1 - D_2 \cos\varphi_2 + C, \tag{26}$$

where

$$\begin{cases} D_1 = \left(\dfrac{m_1}{2} + m_2\right) g_h l_1 \\ D_2 = m_2 g_h l_2 \end{cases}. \tag{27}$$

Replacing all the terms obtained above, the following equations of motion are obtained:

$$\begin{cases} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2\cos(\varphi_2 - \varphi_1) - A_{12}(\dot{\varphi}_2 + \Omega)^2\sin(\varphi_2 - \varphi_1) + B_1\Omega^2\sin\varphi_1 - \\ \quad D_1\sin\varphi_1 = M_1 - M_2 \\ A_{12}\ddot{\varphi}_1\cos(\varphi_2 - \varphi_1) + A_{22}\ddot{\varphi}_2 + A_{12}(\dot{\varphi}_1 + \Omega)^2\sin(\varphi_2 - \varphi_1) + B_2\Omega^2\sin\varphi_2 -' \\ \quad D_2\sin\varphi_2 = M_2 \end{cases} \tag{28}$$

where

$$B_1 = \frac{1}{2}(m_1 + 2m_2) R l_1, \tag{29}$$

$$B_1 = \frac{1}{2} m_2 R l_2. \tag{30}$$

Remarking that the last two terms in the left members of the above system cancel each other out, the system takes the form (11) found when the platform is placed on the ground and it is in a rotation motion with respect to it.

2.3.2 Lagrange Equations with Respect to the Mobile Reference Frame

In this section, the equations of motion of the robotic arm hinged on the platform located on the space station are determined, using the Lagrangian formalism with respect to the mobile reference system $O_1x_1y_1z_1$, linked to the platform (Fig. 3).

This reference system has a constant angular velocityΩ, and the acceleration of the origin $\bar{a}_{O1}$ is:

$$\bar{a}_{O_1} = -\Omega^2 R \bar{\iota_1}. \tag{31}$$

In order to calculate the relative linear momentum $\bar{H}_r$[7], the relative velocity of the second center of mass is used and the velocity of the first center of mass is calculated [5], [9].

The following is obtained:

$$\begin{cases} Q_{1t}^{a} = -\left(\frac{m_1}{2} + m_2\right) l_1 R\Omega^2 \sin\varphi_1 = -B_1\Omega^2\sin\varphi_1 \\ Q_{2t}^{a} = -m_2 R\frac{l_2}{2}\Omega^2\sin\varphi_2 = -B_2\Omega^2\sin\varphi_2 \end{cases}. \tag{32}$$

In order to determine the final form of the Lagrange equations when the platform is located on the space station, terms (32) are added to (20) and it follows that

$$\begin{cases} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2\cos(\varphi_2-\varphi_1) - A_{12}(\dot{\varphi}_2)^2\sin(\varphi_2-\varphi_1) = M_1 - M_2 + D_1\sin\varphi_1 + \\ \quad + A_{12}\Omega^2\sin(\varphi_2-\varphi_1) + 2A_{12}\Omega\dot{\varphi}_2\sin(\varphi_2-\varphi_1) - B_1\Omega^2\sin\varphi_1 \\ A_{12}\ddot{\varphi}_1\cos(\varphi_2-\varphi_1) + A_{22}\ddot{\varphi}_2 + A_{12}(\dot{\varphi}_1)^2\sin(\varphi_2-\varphi_1) = M_2 + D_2\sin\varphi_2 - \\ \quad - A_{12}\Omega^2\sin(\varphi_2-\varphi_1) - 2A_{12}\Omega\dot{\varphi}_1\sin(\varphi_2-\varphi_1) - B_2\Omega^2\sin\varphi_2 \end{cases}. \tag{33}$$

It is noted that systems (28) and (33) are equivalent, which again proves that the two methods of study lead to identical results.

3. NUMERICAL APPLICATIONS

Several sets of numeric values were considered for the system parameters: A_1, A_2, l_1, l_2, m_1, m_2. For each set, the variation curves of the angles of rotation, angular velocities and angular accelerations of the two bars, as well as the variation curves of the motor moments, for various values of the angular velocity Ω, were determined (Fig. 4 - Fig. 7). The calculations were performed using a program developed in Matlab.

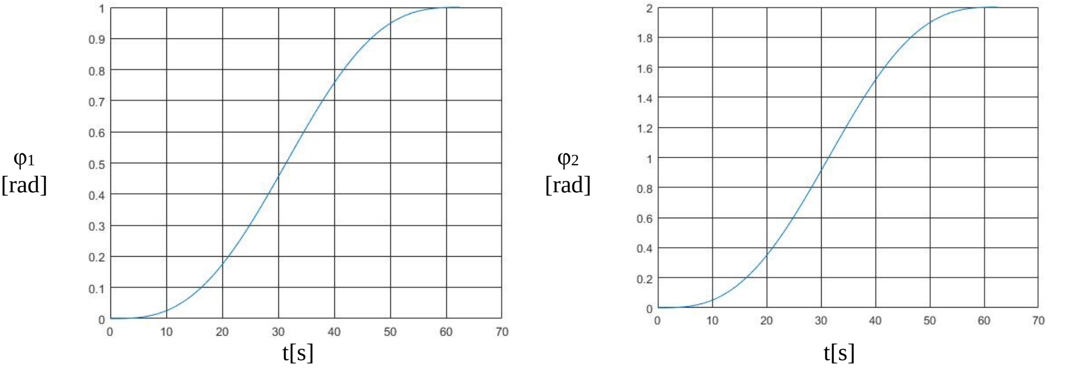

Fig. 4 The law of motion for $\boldsymbol{A_1 = 1\ rad, A_2 = 2\ rad}$

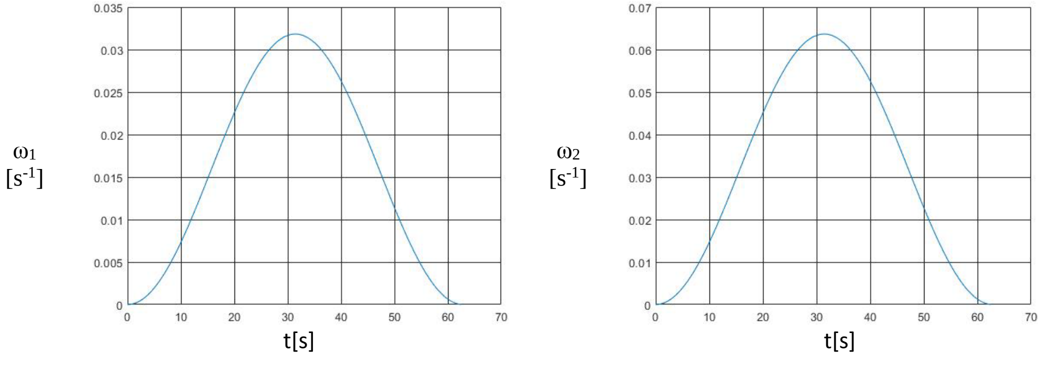

Fig. 5 The angular velocities of the components of the mechanism for $\boldsymbol{A_1 = 1\ rad, A_2 = 2\ rad}$

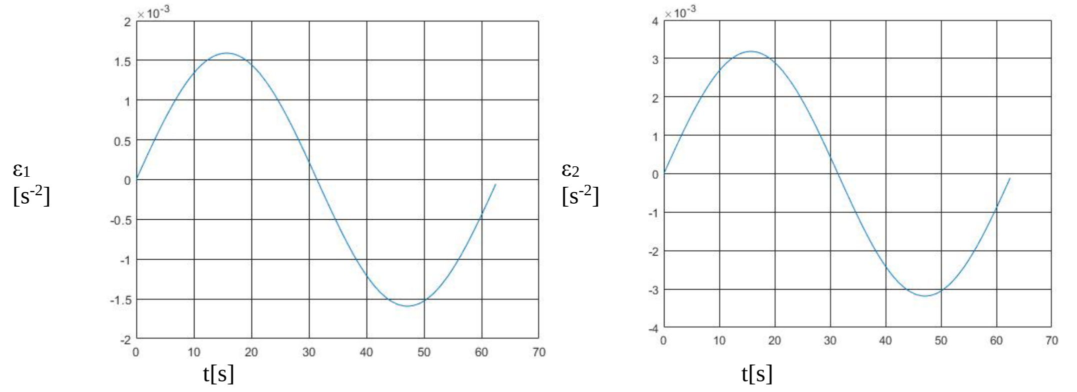

Fig. 6 The angular accelerations of the components of the mechanism for $A_1 = \mathbf{1}\ \boldsymbol{rad}$, $\boldsymbol{A_2 = 2\ rad}$

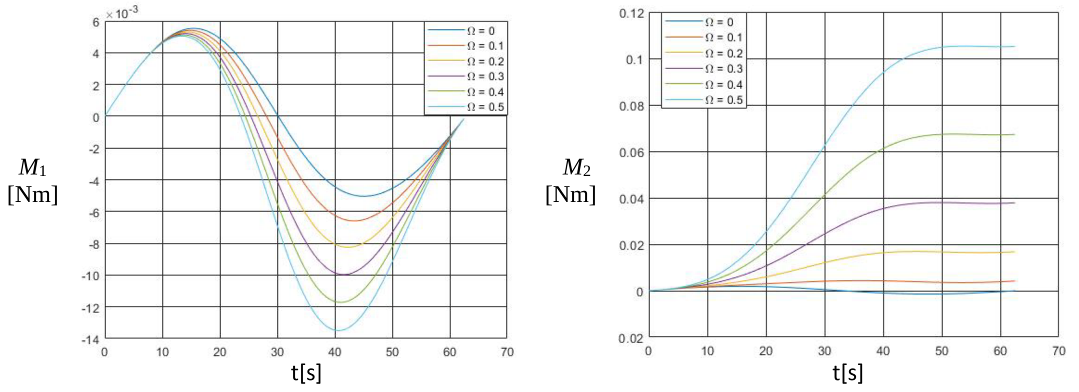

Fig. 7 Motor moments for $\boldsymbol{A_1 = 1\ rad, A_2 = 2\ rad}$, $\boldsymbol{l_1 = l_2 = 1\ m}$, $\boldsymbol{m_1 = m_2 = 1\ kg}$

4. CONCLUSIONS

For all the analyzed cases, the motion equations are determined using the Lagrangian formalism, applied in its traditional form, valid with respect to an inertial reference system, conventionally considered as fixed. A generalized form of the formalism valid with respect to a non-inertial reference system has been also applied.

It was noted that the two versions of the Lagrangian formalism have led to the same results. It has been shown that the motion equations obtained for a mechanism, i.e. a robotic arm, located on a platform situated on a space station orbiting at an arbitrary altitude with a constant angular velocity Ω, are equivalent to those obtained when the platform is located on the Earth, in a rotation motion with respect to it, if the gravitational gradient is neglected.

The numerical studies have shown that the values of the motor moments increase with the values of the amplitudes A_1 and A_2. It also follows that the values of the motor moments generally increase with the platform's angular velocity.

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