# A new approach in the numerical simulation for the blood flow in large vessels

Titus PETRILA[1], Balazs ALBERT*[,2]

*Corresponding author
[1]Aerospace Consulting
B-dul Iuliu Maniu 220, Bucharest 061126, Romania
*[,2]Babes-Bolyai University, Cluj-Napoca
Mihail Kogalniceanu 1, 400084, Cluj-Napoca, Romania
balazsalb@gmail.com

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***Abstract:*** *In this paper we are proposing a new approach in the numerical simulation of the blood flow in large vessels. The initial conditions are set to be compatible with the non-Newtonian model used. Numerical experiments in stenosed artery and in artery with aneurysm (using COMSOL 3.3), are presented.*

***Key Words:*** *non-Newtonian model, blood flow, Cross type rheological model.*

## 1. INTRODUCTION

The present paper continues the recent researches of the authors [2], by elaborating some appropriate initial conditions for the simulation of the blood flow in arteries with stenosis or aneurysm, initial conditions which describe more correctly the blood flow than the others used in previous papers and which observed the Newtonian behavior.

For blood we accept a non-Newtonian rheological behavior with variable viscosity under the conditions of an unsteady (pulsatile) flow regime connected with the rhythmic pumping of the blood by the heart. As the same time we admit the incompressibility and homogeneity of the blood while its flow is laminar and the exterior body forces are neglected. The vessel wall is considered to be viscoelastic, the approach closest to the reality.

The proposed mathematical model has been numerically tested in the case of the blood flow in large arteries with stenosis and aneurysm taking into consideration viscoelasticity of the limiting walls. We will present some final numerical results.

## 2. MATHEMATICAL MODEL

Accepting the axial-symmetric behavior of the blood flow in the considered vessel, the axis of symmetry being *Oz*, the flow domain $\Omega$ in cylindrical coordinates $(r,\theta,z)$ at any moment *t* is defined by

$$\Omega(t) \equiv \{(r,\theta,z)/r < R+\eta(z,t),\ \ \theta \in [0,2\pi), z \in (0,L)\},$$

where *R* and *L* are, respectively, the (initial, at rest) radius and the length of the envisaged vessel tube while $\eta(z,t)$ is the classical deformation (displacement) at the considered moment of the vessel wall.

In the half meridian plane $\theta = const$, if $u$ and $v$ are the components of the blood velocity on the directions $r$ and $z$ respectively while $p$ is the pressure (assessed versus a reference pressure $p_{ref}$), then, in the absence of the exterior forces, the continuity equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial z}=0\,. \tag{1}$$

The corresponding motion equations result from the general Cauchy equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=div\mathbf{T}\,, \tag{2}$$

where we accept for the stress tensor $\mathbf{T}$ the following representation (rheological model for blood)

$$\mathbf{T}=-[p+\lambda(\frac{\partial \mathcal{K}}{\partial \dot{\gamma}}\ddot{\gamma}+\frac{\alpha}{\mu_p}\mathcal{K}^2)]\mathbf{I}+2(\mu_s+\mu_{RBC})\mathbf{D}, \tag{3}$$

Here $\mu_{RBC}$ is given by the non-Newtonian Cross type rheological model [3]

$$\mu_{RBC}=\frac{\mu_0^*}{1+(k\dot{\gamma})^{1-n}}\equiv\mu_e+\lambda\mathcal{K}(\dot{\gamma})\,, \tag{4}$$

where $\mu_e$ and $\mu_0^*$ are viscosity coefficients of the blood, $k$ is a time constant and $n$ is the index for a shear thinning behavior.

We have also used another Cross type rheological model elaborated by M. Ohta el al. [4], namely

$$\mu=\mu_\infty+\frac{(\mu_0-\mu_\infty)}{1+\left(\frac{\dot{\gamma}}{\dot{\gamma}_C}\right)^p}, \tag{5}$$

where $\mu_\infty$ is the dynamic viscosity at very high shear rate $\mu_0$ is the value for the null shear rate $\dot{\gamma}$ is the shear rate, $\dot{\gamma}_C$ is a constant and $p$ a constant exponent.

The evolution equations are joined to some boundary conditions which express either the viscoelastic behavior of the wall or the existence of a pressure gradient along $Oz$ axis (according to the heart beats and implicitly to the rhythmic blood pushing into the vessel). Specifically

$$\frac{\partial v}{\partial r}=0 \text{ and } u=0 \text{ at } r=0$$

and no slip condition at $r=R$. The boundary conditions at "edges" $z=0$ and $z=L$ of the vessel agree with a physiological pulse velocity given by a periodic time-varying function.

## 3. IMPROVED INITIAL CONDITIONS

In the previous researches we have used two type of initial conditions at the inlet boundary (z = 0) of the arterial segment (according to the former Newtonian behavior):

1. a parabolic profile for the inlet velocity, described by the following function

$$v_{in} = \left(1 - \frac{r^2}{R^2}\right) \cdot (1{,}1 - \cos(2\pi t)) \tag{6}$$

2. an oscillatory physiological velocity profile, described by

$$v_{in} = F(t)\left(1 - \left(\frac{r}{R}\right)^2\right) \tag{7}$$

where

$$F(t) = \frac{a_0}{2} + \sum_{k=1}^{7} (a_k \cos(2\pi kt) + b_k \sin(2\pi kt)). \tag{8}$$

The second velocity profile is similar to the one used in [5], see figure 1.

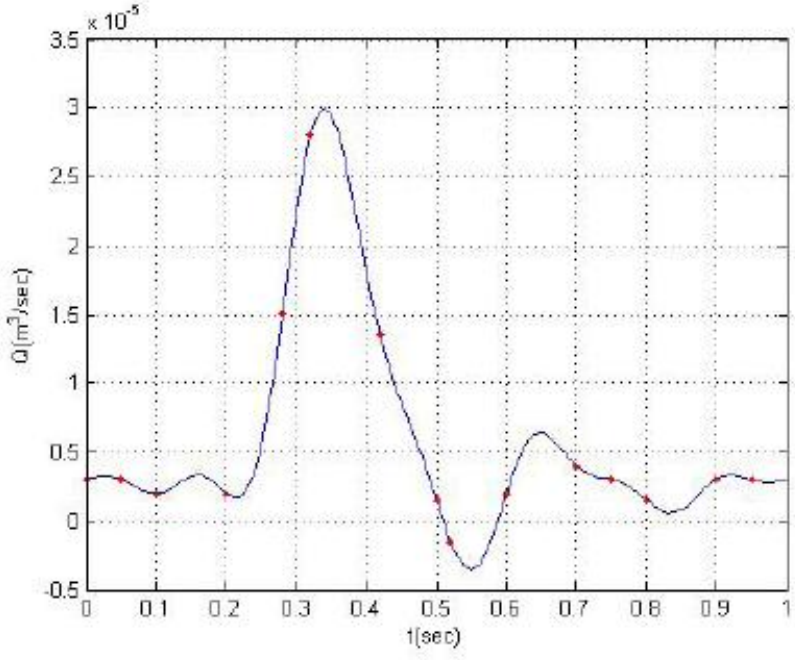


Figure 1: Inlet velocity profile

To obtain more realistic initial conditions - which are compatible with the used non-Newtonian model - at the inlet boundary of the arterial segment with stenosis or aneurysm we will make the following approach.

We have lengthened “theoretically” the envisaged arterial segments just like in figure 2a and 2b.

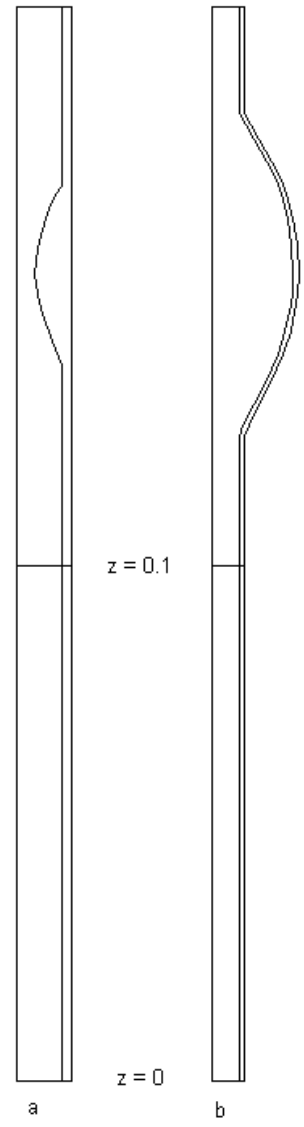


Figure 2: a) lengthened artery with stenosis; b) lengthened artery with aneurysm

At the inlet boundary ($z = 0$) of the whole arterial segment we have first imposed a parabolic profile for the velocity, as in the Newtonian case. Due to the evolution of the blood flow this artificial initial condition has been modified at the beginning of the arterial segment with stenonsis or aneurysm.

The inlet velocity has not a parabolic profile anymore, it has now a realistic profile and the further simulations in the vicinity of the stenosis and the aneurysm are then made using these modified new initial conditions.

The modified velocity profiles are presented in figure 3 (in the case of stenosed artery) and 4 (in the case of artery with aneurysm). The length of the whole arterial tube is $L = 0.2m$, the velocity profiles are presented at $z = 0m$ and $z = 0.1m$.

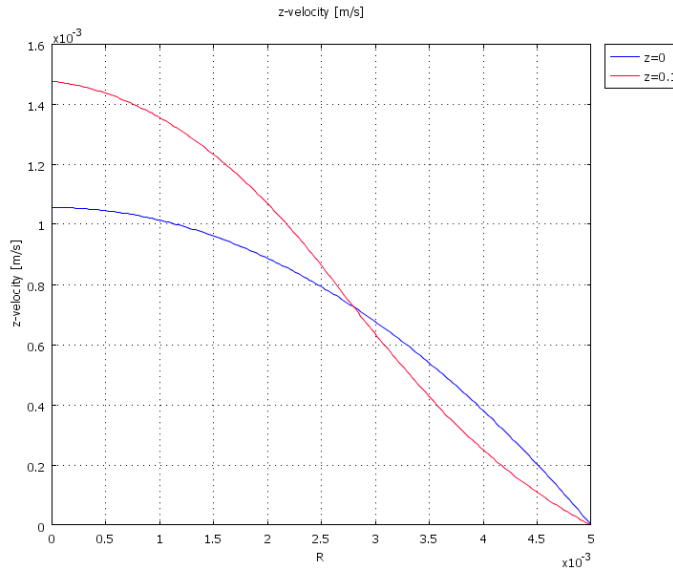


Figure 3: Velocity profiles at $z = 0$ and $z = 0.1$ in the case of stenosed artery

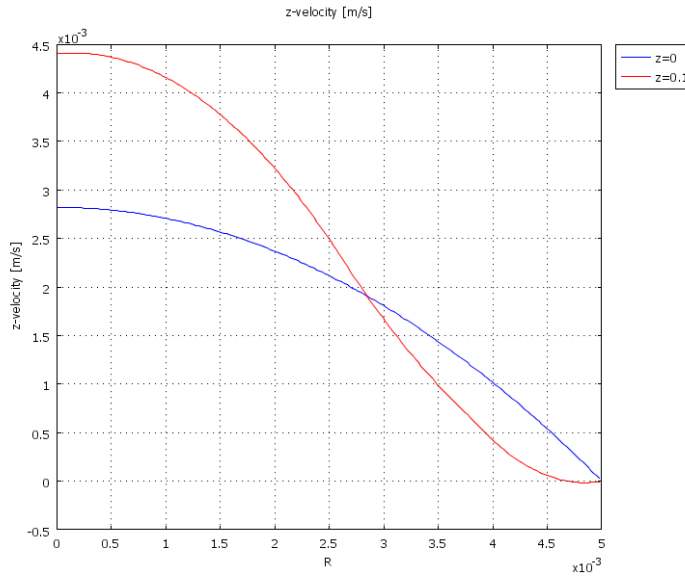


Figure 4: Velocity profiles at $z = 0$ and $z = 0.1$ in the case of artery with aneurysm

## 4. NUMERICAL SIMULATIONS AND CONCLUSION

Numerical simulations have been performed, using COMSOL Multiphysics 3.3, a powerful modelling package based on the Finite Element Method, to investigate the performance of the proposed mathematical model. Let us consider an artery "segment" of radius $R = 0.005m$, length $L = 0.2m$, the thickness of the limiting wall is $0.001m$. The mass density of the blood has been fixed at $\rho = 1060 kg/m^3$.

At the points of the vessel axis of symmetry $r = 0$ we have imposed the axially symmetry requirements while on the vessel walls - no slip conditions. In order to avoid the

transient effect of the initial conditions, the time integration interval is $t \in [0, 10]$ and the results are presented only for the last 5 periods, $t \in [5, 10]$.

We made the numerical simulations both for an arterial segment with a stenosis and for an arterial segment presenting an aneurysm. For all cases, we calculated the wall shear stress $wss = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)$ at particular points (see figure 5), this quantity is responsible for possible ruptures of the vessel wall. The evolution of the *WSS* for $t \in [5, 10]$ is given in figures 6 and 7.

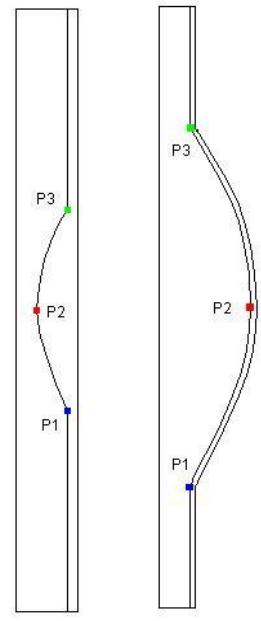


Figure 5: stenosed artery with 3 particular points; artery with aneurysm with 3 particular points

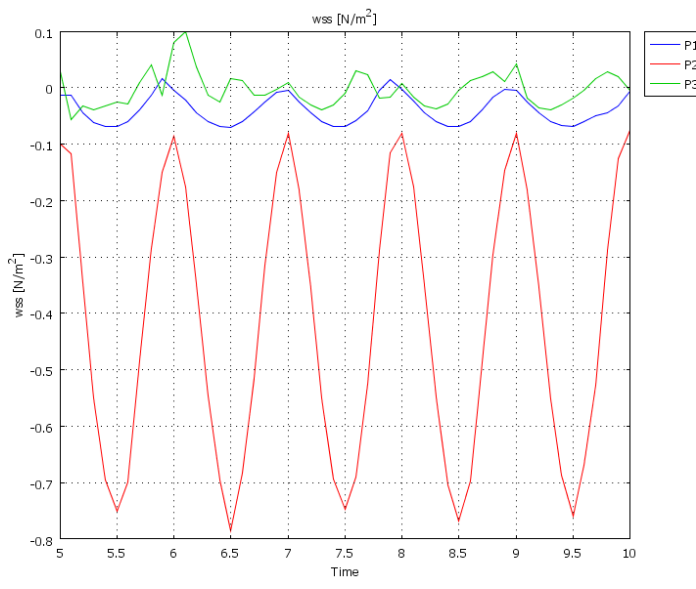


Figure 6: WSS at point P1, P2 and P3 (case of stenosed artery)

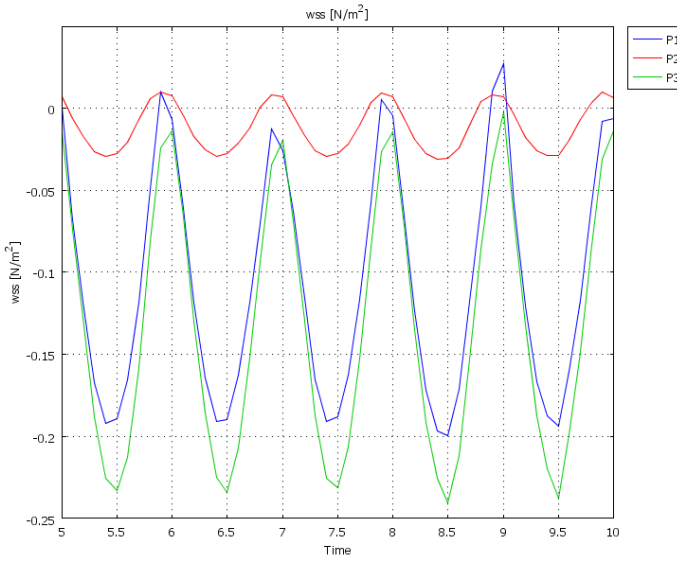


Figure 7: WSS at point P1, P2 and P3 (case of artery with aneurysm)

On figure 6 can be clearly seen that the values for the WSS are much higher in the middle of the stenosis than in the zone right after the stenosis. On figure 7 we can observe that the values for the WSS are much lower in the middle of the aneurysm.

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