Analytical Estimation of Time Dilation of a Satellite in Elliptical Orbit

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Abstract: *This article describes the analytical estimation of time dilation along the elliptical orbit of a satellite due to effects of relativity where the functioning has been time-dependent. The satellites in circular orbit are calibrated as moving frame of reference at constant velocity in constant non inertial frame of reference, namely Global Positioning System satellites where effects of both relativistic and gravitational time dilations can be seen. This article deals with the variation of the time dilation along the elliptical orbit whose speed of reference frame varies with respect to time and also varying the non-inertial frame of reference. The relativistic effects on the time dilation, is caused by the Earth rotation, due to the orbit eccentricity. The estimated values have a scope in the field of space mechanics. As a result the plot has been intended along the orbit comparing to circular orbit in both special and general relativity perspective.*

Key Words: *Time dilation, Elliptic orbit, Relativity, Satellite path, Global Positioning System*

1. INTRODUCTION

Newtonian methods are applicable to any moving frame of reference at low speed whose motion is relative to observing frame of reference, but for frame moving at high speed nearly at speed of light, Einstein's Theory of Relativity has been applicable. Newton's method has been widely used in **e**ngineering field because of its simplicity comparing to Einstein's Theory. Newtonian method is simple but inaccurate. None of the fundamental experiments on which the restricted principle of relativity is based requires for their explanation that the classical concept of absolute time be modified [1]. The GPS satellite system is one of the applications of the Theory of relativity. Doppler effect in canal rays involved velocities of the moving particles high enough to show that the expected effect has been actually a positive effect of Einstein's theory [2]. In Doppler effect, time flow has been affected by the general theory as well as by the special theory of relativity. Lorentz factor has been used to evaluate time dilation in special relativity [3]. Because of the special relativity the GPS satellites which are orbiting at 3888 m/s slow down the time by 7 μs per day. Also, at 20000 km altitude from the surface of the earth they cause time to speed up by 45μs per day. Thus, the total dilated time per day has been about 38 μs in total. Consequently, the clock slowed down by that 38 μs per day. In nonlinear accelerating objects, for example a satellite orbiting at elliptical orbit, the time dilated also varies nonlinearly with respect to instantaneous velocity and distance from the planet. One interesting consequence of the square light clock geometry is the change of orientation of each of the mirrors [4]. Fig. 1 shows the current methodology adopted in the article.

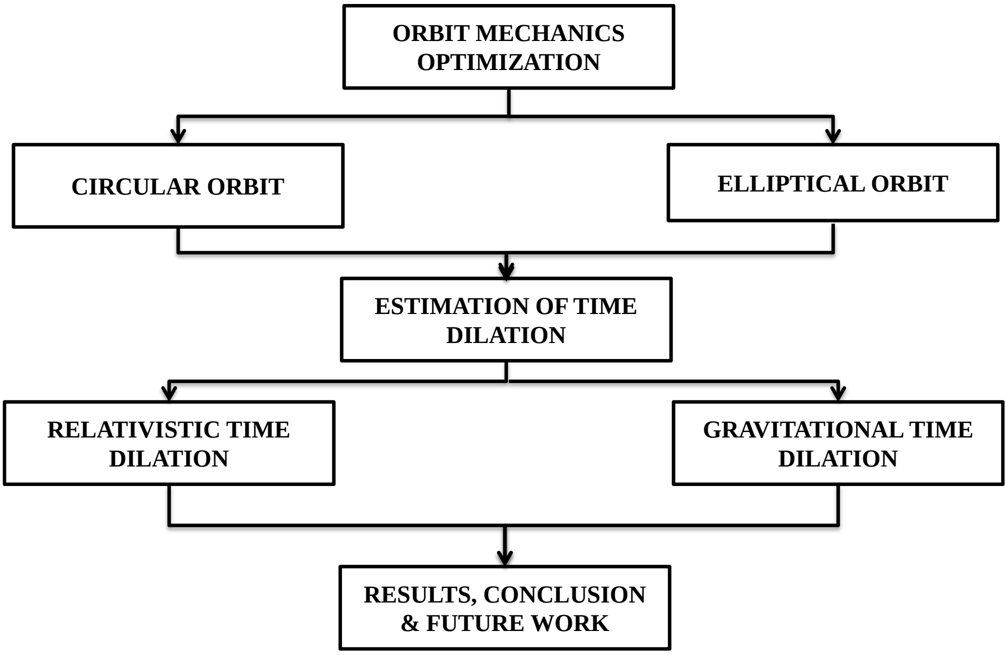

Fig. 1 - Methodology adopted the in current article

2. ESTIMATION OF VELOCITY AND POSITION

The postulates of the special theory of relativity are, [5]

1. The laws of physics have the same form in all inertial reference frames.
2. The speed of light c is a constant independent of the motion of the source.

of free fall. This can be stated as:

3. The universality: stated as over a small region of space and time, it has been impossible to distinguish between a gravitational field due to the mass, and a fictitious gravitational field due to acceleration.

The nomenclatures used are as follows : T - orbital time period, M - mean anomaly, E - eccentric anomaly, t' - time in a moving frame of reference, t - time in an inertial frame of reference, v - velocity of moving frame of reference, c - speed of light, G - Gravitational constant (6.647×10^{-11} $m^3 kg^{-1} s^{-2}$), m – center mass being approached by object and R - distance between source of gravity to observing object.

Raymond [6] proposed that definitive statements regarding detectability can only be made by solving the mission specific comprehensive inverse problem through realistic mock data generation. Apart from basic algebra and geometry, the only assumption needed for this derivation has been the constancy of the speed of light [7].

3. CALCULATION OF THE CIRCULAR ORBIT

Inappropriate velocity of that object with respect to its distance from focus of orbit has been the reason for an object that orbits around a planet or for a star in an elliptical orbit. The velocity that has been required to keep the object in circular orbit has been determined by the equilibrium between the centripetal force exerted on the center mass by the orbit object and the gravitational force exerted on orbiting object by the center mass.

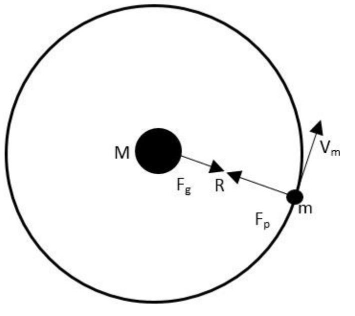

Fig. 2 - Circular orbit

The gravitational force exerted on the satellite by its force has been counteracted by the centripetal force exerted on the center mass by the satellite's velocity when the satellite orbits around a center mass as illustrated in Fig. 2. Therefore, equating the gravitational force with centripetal force which is equal

$$\frac{mMG}{R^2} = \frac{mV^2}{R} \tag{1}$$

Orbital velocity (v) has been denoted as,

$$v = \sqrt{\frac{MG}{R}} \tag{2}$$

4. CALCULATION OF THE ELLIPTICAL ORBIT

In elliptical orbit center mass is located in one of focus. The main characteristics of the elliptical orbit have been its eccentricity, the semi major axis length and the velocity of object at perigee and apogee. The velocity of object at any point has been determined by two methods 1) Conservation of Mechanical Energy and 2) Conservation of Angular Momentum.

This article deals with the determination of the time dilation between the equatorial surface of the Earth and the elliptic orbit where the acceleration of both frames varies and so does the time. The accelerations are same with respect to each other because of frame of reference. So the time dilation could never occur. The position of satellite has to be defined initially, before we calculate the time dilated that occurs in elliptical orbit .For that the main characteristics that has to be defined are True Anomaly (v), Mean Anomaly (M) and Eccentric Anomaly (E). These values help to find the position of the satellite at a given time interval. The expression to predict these values are,:

$$M = \frac{t}{T} 2\pi \tag{3}$$

The mean anomaly which is the angle between the perigee and the satellite position on a circular orbit at a given time (t) whereas orbital time period (T) can be devised for a given interval of time. The Eccentric Anomaly is the angle between the perigee and the position of the satellite projected onto circle perpendicular to the semi major axis.

$$M = E - e\sin E \tag{4}$$

The above equation is used to relate the mean anomaly (M) and the eccentric anomaly (E).

$$\theta = 2\tan^{-1}\left(\left(\sqrt{\frac{1+e}{1-e}}\right)\tan\frac{E}{2}\right) \tag{5}$$

The above relation from E gives the direction of the satellite in polar coordinate. The True Anomaly is the angle between the semi major axis and the position of the satellite from its focal point.

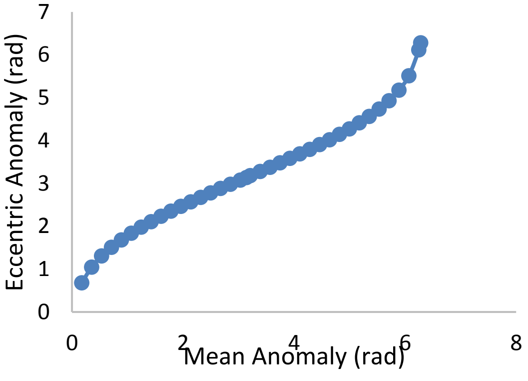

Fig. 3 - Mean and Eccentric anomaly relation for eccentricity of 0.8

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \tag{6}$$

It clearly results from the above trigonometric expression that the position of satellite can be traced for a given time interval in a polar coordinate system (r, θ). Fig. 3 shows the mean and eccentric anomaly relation for eccentricity of 0.8. The tracing position of the satellite for a given period is represented as graph in Fig. 4 at a time interval of 100 seconds.

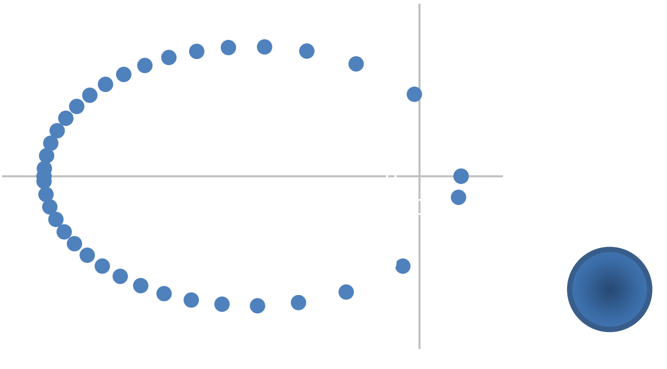

Fig. 4 - Orbit Position with time interval of 100 seconds

The relation of the conservation of mechanical energy as well as the conservation of the angular momentum gives the instantaneous velocity of an orbiting object as follows

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)} \tag{7}$$

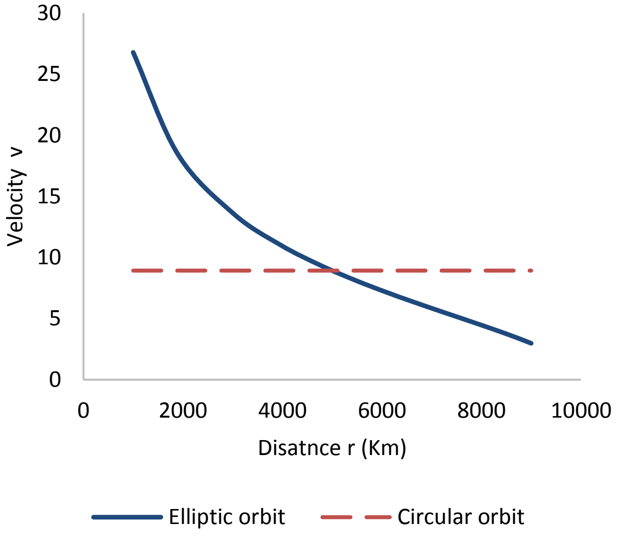

Fig. 5 - velocity with respect to distance

Fig. 5 shows that the variation of velocity in elliptical varies with respect to radial distance from the center of the planet whereas the circular orbit has constant velocity.

5. ESTIMATION OF TIME DILATION

Einstein's theory of relativity evolved from his special theory of relativity as general theory of relativity.

Time dilation can be possibly occurring in two ways, namely the Relativistic time dilation and the Gravitational time dilation.

5.1 Relativistic time dilation

Special theory of relativity has been used to predict the Relativistic time dilation and a function called Lorentz factor was determined.

This theory proves that the increase in velocity of one frame of reference could results in slowing of time as compared to an inertial frame of reference.

Lorentz factor indication shows that for object moving at 85% of speed of light the slowing of time has been doubled and as speed reaches 100% speed of light and time has been bound to be freeze.

The expression for relativistic time dilation has been determined by Lorentz factor

$$t' = t\sqrt{1 - \frac{v^2}{c^2}} \tag{8}$$

5.2 Gravitational time dilation

Gravitational effect has been predicted by the general theory of relativity; this theory proves that time tends to slow down at center of the mass such as planet or star and the flow of time increases gradually as it moves away from center.

$$t' = t\sqrt{1-\frac{2GM}{Rc^2}} \tag{9}$$

For example, if a clock has been set to be launch to outer space from the Earth surface the speed of the clock starts to be slower and slower linearly as the clock approaches the space, due to the linear variation in distance R from the center of the Earth.

6. PLOT FOR TIME DILATION IN ELLIPTICAL ORBIT

The Position of the satellite indicates the time dilated after each equal time interval. The following graph shows the variation of time dilated for a satellite orbiting in an elliptical orbit which has a semi major axis of 5000 km and an eccentricity of 0.8.

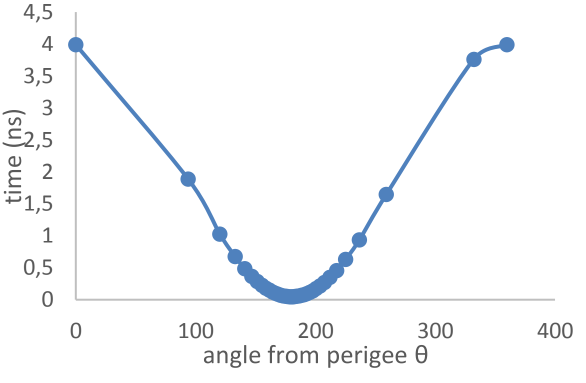

Fig. 6 - Relativistic time dilation with respect to θ

Fig. 6 represents the amount of time (one second) dilated at each position with respect of time on the Earth surface. This time dilation has been purely a relativistic time dilation i.e. on the basis of satellite's velocity.

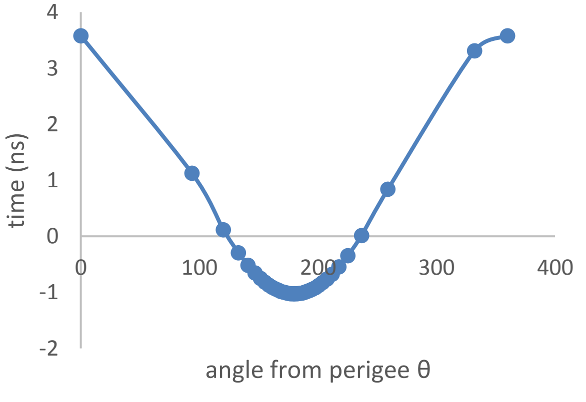

Fig. 7 - gravitational time dilation with respect to θ

Fig. 7 indicates the amount of time by which time dilated due to the satellite altitude from the center of earth for one second on earth surface.

The time flow gets affected by both gravitational and relativistic time dilation in any frame of reference in opposite manner, which means the relativistic time dilation slows down the time and the gravitational time dilation causes the time to speed up. Total time dilation has been the resultant of these time dilations.

6.1 Analysis of Time Delay

Consider that flight velocity of an aircraft reaches up to $\sqrt{3}c/2$,which is 260,000 km/s; to the people on the ground it seems that the time interval t' of the aircraft would be consequentially equivalent to 1/ 2 of the surface interval time T regardless of the direction in which the aircraft has been traveling.

This is an incontestable conclusion of special relativity. Hence if one of the lights outside the aircraft is on for 1 percent of a millisecond, then the people on the ground will consequentially observe that the light is on for 2 percent of a millisecond. Since the conclusion on time delay will not be changed as a result of the magnitude of the aircraft flight velocity [8].

Further this applications can be used to monitor the forest regions using Global Positioning Systems[9].

7. CONCLUSION

The main aspect of estimating the time dilation for a satellite in elliptical orbit for any futuristic mission. For effective data communication the time accuracy is necessary. Also for any future colonization mission on any other planet of the solar system the time dilation needs to be compensated as every planet is orbiting in an elliptical motion around the Sun. The time dilation for equally accelerated frames at low speed is absent while in a gravitational potential it has a specific value, not a null value[10].

Future work involves the implementation of the test results in a computer simulated software such as MATLAB and carrying out the experiment for various scenarios such as Lagrange orbit and also the determination of the time dilation of a planet with respect to other planet.

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