Common aspects and differences in the behaviour of classical configuration versus canard configuration aircraft in the presence of vertical gusts, assuming the hypothesis of an elastic fuselage

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Abstract: *The paper analyzes, in parallel, common aspects and differences in the behavior of classical configuration versus canard configuration aircraft in the presence of vertical gusts, assuming the hypothesis of an elastic fuselage. The effects of the main constructional dimensions of the horizontal empennage on lift cancelling and horizontal empennage control are being analyzed.*

Key Words: *vertical gust, canceling, lift, empennage.*

1. THE CANCELLING OF HORIZONTAL EMPENNAGE LIFT IN THE CLASSICAL CONFIGURATION [6]

1.1 Classical configuration aircraft. Hypothesis, notations.

Let us consider a classical configuration aircraft, with an elastic fuselage and an arrow angle χ at the line of the focuses, entering an atmosphere with vertical ascending/ descending gusts with the intensity w, as in Fig. 1 the influence of the wing upon the horizontal empennage being null.

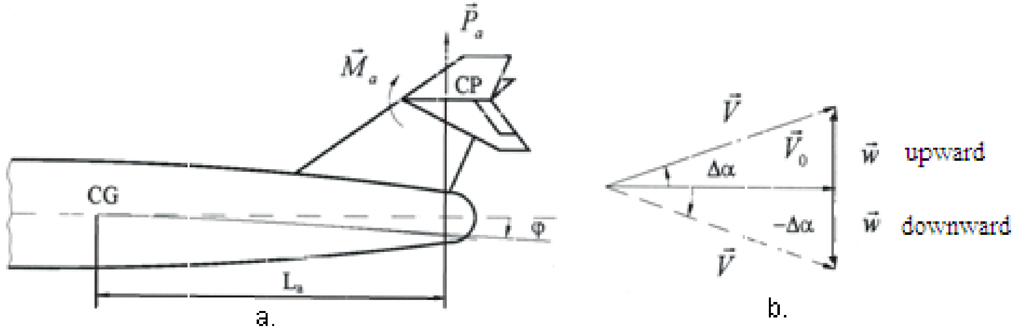

Fig. 1. Aircraft in the disturbed atmosphere:
a - loads on the deformable elastic fuselage; b - velocity triangle for upward and downward gusts

Using the established notations [1, 2, 3], K_1 and K_2 being the elastic constants for the load P_a and aerodynamic moment M_a of the horizontal empennage, the rotation angle of the fuselage along with the horizontal empennage is:

$$\varphi = -\frac{P_a}{K_1} + \frac{M_a}{K_2}. \tag{1}$$

The aerodynamic moment obtained by deflecting the elevator at an angle β is

$$M_a = qS_a c_a \frac{dC_m}{d\beta}\beta \tag{2}$$

With the established notations: $q = \frac{\rho}{2}V^2$ - dynamic pressure;

S_a -surface of the horizontal empennage;

$c_a = \frac{2}{S_a}\int_0^b c^2(y)dy$ - average aerodynamic string of the horizontal empennage;

$\frac{dC_m}{d\beta}$ - angle of the lift coefficient of the horizontal empennage, which is, as a rule, negative.

1.2 Lift of the horizontal empennage in the presence of vertical gusts

Considering the effect of a vertical gust which manifests itself by modifying angularity by the angle $\Delta\alpha = \pm w / V_0$ (where the upper sign is for the ascending gusts and the lower one for the descending gusts) the formula for the lift of the empennage is

$$P_a = qS_a\left[\frac{\boldsymbol{d}C_z}{\boldsymbol{d}\alpha}(\varphi \pm \Delta\alpha) + \frac{\boldsymbol{d}C_z}{\boldsymbol{d}\beta}\beta\right] \tag{3}$$

Lift is determined according to the angularity change due to the gust and the elevator deflection:

$$\begin{aligned} P_a = {} & qS_a\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0 \cos\chi \left[1 \mp \frac{\frac{qS_a}{K_1}\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0 \cos\chi}{1 + \frac{qS_a}{K_1}\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0 \cos\chi}\right]\Delta\alpha + \\ & + qS_a\left[\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0 \cos\chi \frac{\frac{qS_a}{K_2}c_a\frac{\boldsymbol{d}C_m}{\boldsymbol{d}\beta} - \frac{qS_a}{K_1}\frac{\boldsymbol{d}C_z}{\boldsymbol{d}\beta}}{1 + \frac{qS_a}{K_1}\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0 \cos\chi} + \frac{\boldsymbol{d}C_z}{\boldsymbol{d}\beta}\right]\beta. \end{aligned} \tag{4}$$

In (4) we considered that from [5] we already have known that:

$$\left(\frac{dC_z}{d\alpha}\right)_\chi = \left(\frac{dC_{zn}}{d\alpha_n}\right)_0 \cos\chi \tag{5}$$

where $\left(\frac{\boldsymbol{d}C_z}{\boldsymbol{d}\alpha}\right)_\chi$ is the angle of the lift coefficient, $\left(\frac{\boldsymbol{d}C_{zn}}{\boldsymbol{d}\alpha_n}\right)_0$ is the angle of the lift coefficient along the normal to the foci line, and $\alpha_n = \frac{\alpha}{\cos\chi}$, the rest of the values are known.

1.3 Critical lift cancelling speed in the case of ascending vertical gusts

Obviously, what interests us is the value of the lift cancelling speed on the horizontal empennage, which is called critical speed from this point, under the stress of an ascending vertical gust. Also, it's important for this to be as high as possible, so that the aircraft never reaches it. After cancelling lift in (2), the value of the dynamic pressure under which this phenomenon is realized is determined as follows:

$$q_a = -\frac{K_2\left(\left(\frac{dC_{zn}}{d\alpha_n}\right)_0 \Delta\alpha\cos\chi + \frac{dC_m}{d\beta}\beta\right)}{cS_a\beta\frac{dC_m}{d\beta}\cdot\left(\frac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi} \tag{6}$$

Critical cancelling speed for the horizontal empennage lift is obtained by intersecting the graphs of the following functions

$$g_1(V_0) = V_0^2 \tag{7}$$

$$h_1(V_0) = -\frac{K_2}{\frac{1}{2}\rho S_a c_a\frac{dC_m}{d\beta}\beta}\cdot\frac{w}{V_0} - \frac{2K_2\frac{dC_z}{d\beta}}{\frac{1}{2}\rho S_a c_a\frac{dC_m}{d\beta}\left(\frac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi} - w^2 \tag{8}$$

As in fig.2

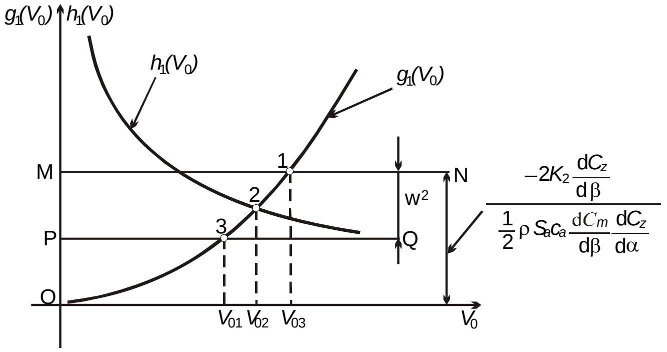

Fig. 2. Determining critical lift cancelling speed in the case of ascending gusts

Point 1 corresponds to the situation where an aircraft flies in a calm atmosphere, without vertical gusts $\vec{w} = \vec{0}$, the fuselage being elastic.

In this situation the critical cancelling speed is identical to that in [1,3]

$$V_{0\,crt}=\sqrt{-\frac{K_2\dfrac{dC_z}{d\beta}}{\dfrac{1}{2}\rho S_a c_a\dfrac{dC_m}{d\beta}\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi}}\tag{9}$$

Fig. 2 şi (9) suggest measures concerning both horizontal empennage construction and aircraft piloting so that critical cancelling speed is as high as possible:

a. Empennage and fuselage construction parameters are chosen as follows:

- K_2 as high as possible;
- angle of the aerodynamic moment coefficient $\boldsymbol{dC_m/d\beta}$ as small as possible;
- angle of the lift coefficient $\boldsymbol{dC_z/d\alpha}$ as small as possible;
- average aerodynamic string c_a and horizontal empennage surface S_a as small as possible;
- arrow angle of the horizontal empennage as high as possible.

b. the aircraft must be flown at the highest possible altitudes because of lower air density ρ.

1.4 Critical horizontal empennage lift cancelling speed in the event of descending gusts

In this case, the dynamic lift cancelling speed as obtained from (4) is

$$q_d=-\frac{\Delta\alpha\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi+\dfrac{dC_z}{d\beta}\beta}{S_a\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi\left[\dfrac{c_a}{K_2}\cdot\dfrac{dC_m}{d\beta}\beta+\dfrac{2}{K_1}\Delta\alpha\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi\right]}\tag{10}$$

Critical lift cancelling speed is obtained by intersecting the graphs of the functions

$$g_2(V_0)=V_0^2+\frac{K_2\dfrac{dC_z}{d\beta}}{\dfrac{1}{2}\rho S_a\dfrac{c_a}{K_2}\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi\dfrac{dC_m}{d\beta}}\tag{11}$$

$$h_2(V_0)=-\frac{\rho\dfrac{S_a}{K_1}\left(\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi\right)w^2+1}{\dfrac{1}{2}\rho S_a\dfrac{c_a}{K_2}\dfrac{dC_m}{d\beta}\beta}\frac{w}{V_0}-\frac{2\dfrac{K_2}{K_1}\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0\cos\chi}{c_a\dfrac{dC_m}{d\beta}\beta}wV_0-w^2\tag{12}$$

The graphical representation of the two functions is given in Fig.3, for a certain gust speed $w=$ **ct.** and not null. The function $h_2(V_0)$ admits as an asymptote the straight-line with the equation $h_2(V_0)=mV_0$ where

$$m = h_2(V_0) = -\frac{2\dfrac{K_2}{K_1}\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0 \cos\chi}{c_a\dfrac{dC_m}{d\beta}\beta}w \tag{13}$$

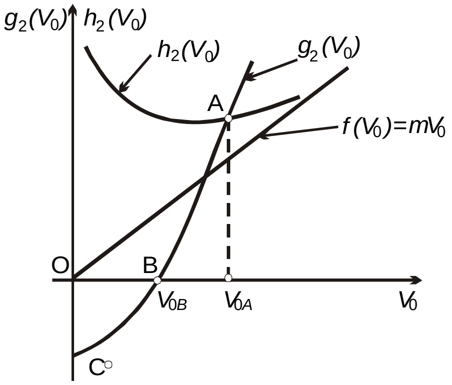

Fig. 3. Determining critical lift cancelling speed in the case of descending gusts

In Fig. 3. the following points are shown:

- A, corresponding to the case where $h_2(V_0) = g_2(V_0)$. The speed V_{0A} corresponds to the present situation;
- B, corresponding to the case where the descending gust speed is null $(w = 0)$.

Here, $h_2(V_0) = g_2(V_0) = 0$ and

$$V_{0B} = \sqrt{\frac{K_2\dfrac{dC_z}{d\beta}}{\dfrac{1}{2}\rho S_a c_a\left(\dfrac{dC_{zn}}{d\alpha_n}\right)_0 \cos\chi\dfrac{dC_m}{d\beta}}} \tag{14}$$

The above is the formula for the horizontal empennage lift cancelling speed where the fuselage is elastic [1, 3] and flight is achieved in a calm atmosphere.

- C, corresponding to the case where the aircraft is immobile and the issue is moot.

From Fig. 3. we deduce that in order to increase the critical horizontal empennage lift cancelling speed in the hypothesis of an elastic fuselage and a descending gust, the following measures must be taken:

a. The build parameters of the horizontal empennage are chosen just like in the previous case;

b. The aircraft is to be flown at the highest possible altitudes in order to benefit from the low air density. The deflection angle β of the elevator is chosen so that the angle of the asymptote of the curve $h_2(V_0)$ is as high as possible (from (13)).

Concerning the horizontal empennage lift cancelling in the presence of an elastic fuselage and vertical gusts, a few conclusions may be drawn:

1) Ascending gusts are more dangerous than descending gusts for a classical configuration aircraft at the same gust speed, because critical lift cancelling on the horizontal empennage is lower in the case of an ascending gust than in that of a descending gust.
2) Critical horizontal empennage lift cancelling speed in the presence of an ascending gust is lower than in case of gusts absence. The presence of a descending gust has a positive effect because the critical horizontal empennage lift cancelling speed is higher than the same speed without the gust.
3) The aircraft which are most at risk of horizontal empennage lift cancelling are the large aircraft (Jumbo class) when flying at high speeds at low altitudes because their c_a and S_a are high.

2. HORIZONTAL EMPENNAGE LIFT CANCELLING IN CANARD CONFIGURATION AIRCRAFT [7]

2.1 Canard configuration aircraft. Hypothesis, notations.

Let us consider a canard configuration aircraft with an elastic fuselage and arrow angle χ at the line of the foci, entering an atmosphere perturbed by vertical ascending/descending gusts of the intensity w, as in fig. 4.

The notations are the same as in paragraph 1.1.

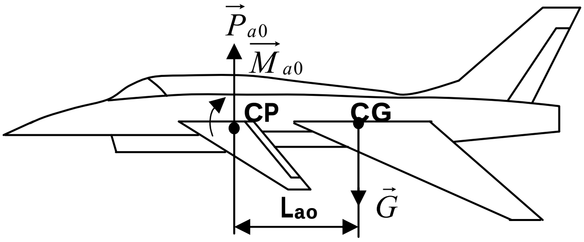

Fig. 4. Canard configuration aircraft

In this case, the fuselage and horizontal empennage rotation angle is $\varphi = \dfrac{P_{ao}}{K_1} + \dfrac{M_{ao}}{K_2}$.

2.2 Horizontal empennage lift in the presence of vertical gusts

Modifying the incidence because of vertical gusts with the angle $\Delta\alpha = \pm w / V_0$ (with + for ascending and – for descending vertical gusts), leads to the following formula for horizontal empennage lift

$$P_{ao} = qS_{ao}\left[\frac{dC_{zao}}{d\alpha}(\varphi \pm \Delta\alpha) + \frac{dC_{zao}}{d\beta}\beta\right] \tag{15}$$

After calculations, through replacing the value of the aerodynamic moment, the formula for lift is obtained as follows

$$P_{ao} = qS_{ao}\left\{\frac{dC_{zao}}{d\alpha}\left[qS_{ao}(A + B) \pm \Delta\alpha\right] + C\right\} \tag{16}$$

where

$$A=\frac{\frac{1}{K_1}\frac{dC_{zao}}{d\alpha}\Delta\alpha}{1-\frac{qS_{ao}}{K_1}\frac{dC_{zao}}{d\alpha}} \qquad B=\frac{\frac{1}{K_1}\frac{dC_{zao}}{d\beta}+\frac{c_{ao}}{K_2}\frac{dC_{mao}}{d\beta}}{1-\frac{qS_{ao}}{K_1}\frac{dC_{zao}}{d\alpha}}\beta \qquad C=\frac{dC_{zao}}{d\alpha}\beta \tag{17}$$

The critical horizontal empennage lift cancelling speed is obtained by intersecting the graphs of the functions

$$g_1(V_0)=-\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}\beta w^2\mp\frac{1}{V_0}\frac{dC_{zao}}{d\alpha}w$$

$$g_2(V_0)=\left(\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}V_0^2+\frac{dC_{zao}}{d\beta}\right)\beta \tag{18}$$

2.3 The critical lift cancelling speed for ascending vertical gusts

By taking the “+” sign corresponding to the asending gust, the critical horizontal empennage lift cancelling speed is obtained by intersecting the graphs of the functions.

$$g_1(V_0)=-\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}\beta w^2-\frac{1}{V_0}\frac{dC_{zao}}{d\alpha}w \tag{19}$$

$$g_2(V_0)=\left(\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}V_0^2+\frac{dC_{zao}}{d\beta}\right)\beta \tag{20}$$

Two cases are possible:

a) The graphs intersect under the abscissa as in Fig. 5.

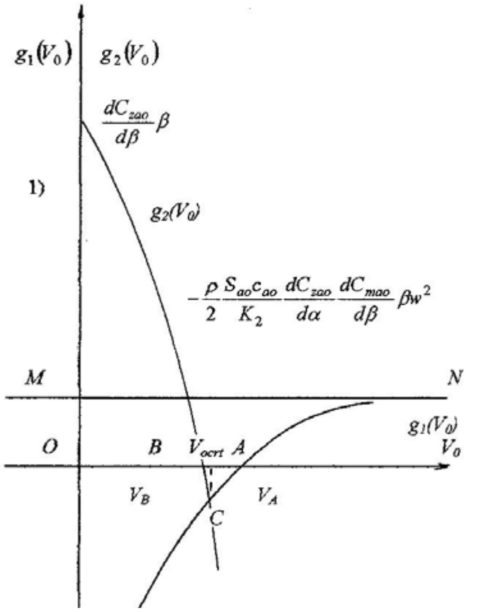

Fig. 5. Determining critical speed in case a)

Flight speed on the *Ox* axis is:

$$V_A = \frac{K_2}{-\frac{\rho}{2} S_{ao} c_{ao} \frac{dC_{mao}}{d\beta} \beta w} \tag{21}$$

$$V_B = \sqrt{\frac{K_2}{-\frac{\rho}{2} S_{ao} c_{ao} \frac{dC_{zao}}{d\alpha} \frac{dC_{mao}}{d\beta}}} \tag{22}$$

From fig. 5 şi and formulas (21) and (22) a few conclusions concerning critical horizontal empennage lift cancelling speed are drawn:

- Critical lift cancelling speed drops with the increase in the turning angle β of the elevator;

- V_A contains constructive and flight paratemeters as do β, w, but V_b contains only constructive paratemeters

- Critical horizontal empennage lift cancelling speed is has a lower limit at V_B, that is the aircraft's constructive parameters.

The second case of graph intersection of $g_1(V_0)$and $g_2(V_0)$ is shown in fig. 6.

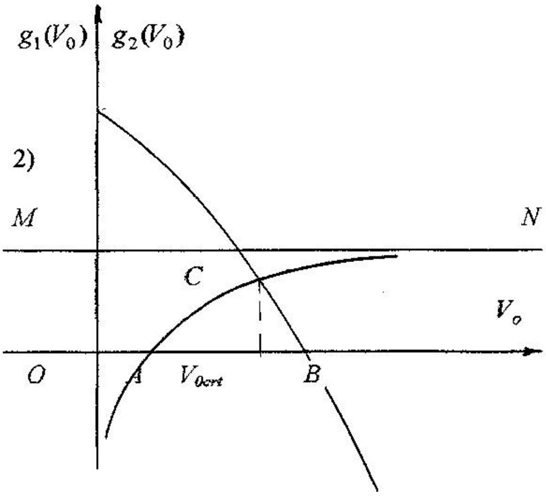

Fig. 6. Determining critical speed in case b)

From Fig. 6, V_B, which is expressed the same as in (22), places an upper limit on flight speed. V_B contains constructive parameters of the horizontal empennage.

The effects and interpretations of the values are the same as in the previous case.

In the limit-case of $V_A = V_B$, then $V_A = V_B = V_C$, which leads to limiting elevator turning angle possibilities for the horizontal empennage depending on gust speed

$$\beta w = \sqrt{\frac{K_2 \frac{dC_{zao}}{d\beta}}{-\frac{\rho}{2} S_{ao} c_{ao} \frac{dC_{zao}}{d\alpha} \frac{dC_{mao}}{d\beta}}} \tag{23}$$

2.4 Critical lift cancelling speed in the case of a descending gust.

With (18) in the sign corresponding to the descending gust, the critical horizontal empennage lift cancelling speed is obtained by intersecting the graphs of the following functions

$$g_1(V_0) = -\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}\beta w^2 + \frac{1}{V_0}\frac{dC_{zao}}{d\alpha}w \tag{24}$$

$$g_2(V_0) = \left(\frac{\rho}{2}\frac{S_{ao}c_{ao}}{K_2}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}V_0^2 + \frac{dC_{zao}}{d\beta}\right)\beta \tag{25}$$

Depending on the aircraft constructive parameters, there are three distinct cases.

a) The graphs of $g_1(V_0)$ and $g_2(V_0)$ intersect as in fig. 7.

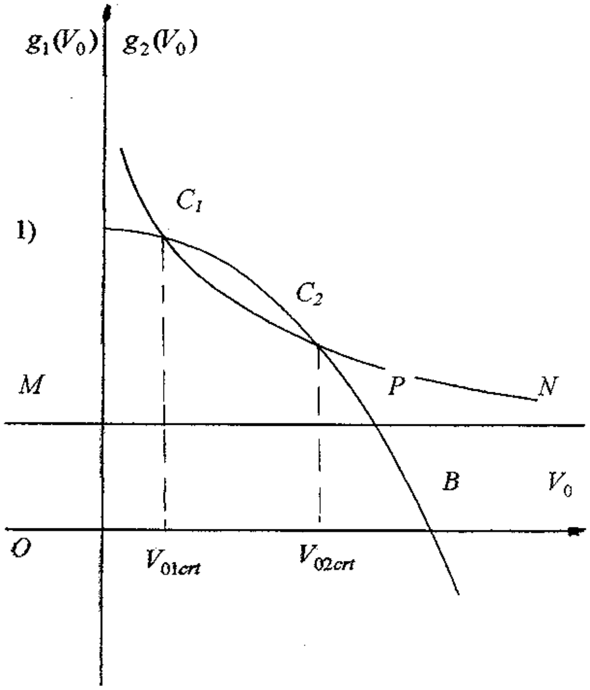

Fig. 7. The existence of two critical speeds in case a)

In this case there are two critical horizontal empennage lift cancelling speeds, V_{01} and V_{02}. These values are inferior to the speed corresponding to the intersection point of the parable $g_2(V_0)$ with the abscissa.

$$V_B = \sqrt{\frac{K_2\frac{dC_{zao}}{d\beta}}{-\frac{\rho}{2}S_{ao}c_{ao}\frac{dC_{zao}}{d\alpha}\frac{dC_{mao}}{d\beta}}} \tag{26}$$

The V_{02} upper limit may be further limited by determining the speed corresponding to the intersection point of the parable $g_2(V_0)$ with the asymptote M-N. Thus,

$$V_{oP} = \sqrt{-\left(w^2 + \frac{K_2 \frac{dC_{zao}}{d\beta}}{\frac{\rho}{2} S_{ao} c_{ao} \frac{dC_{zao}}{d\alpha} \frac{dC_{mao}}{d\beta}} \right)} \tag{27}$$

b) The curves $g_1 = g_1(V_0)$ and $g_2 = g_2(V_0)$ are tangent in a point C, as in Fig. 8.

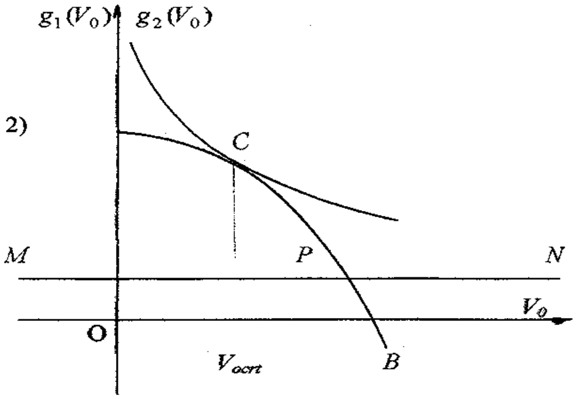

Fig. 8. The existence of a critical speed in case b)

In this case the critical horizontal empennage lift cancelling speed of a canard configuration aircraft is

$$V_{0crt} = \sqrt[3]{-\frac{K_2 w}{\rho S_{ao} c_{ao} \frac{dC_{mao}}{d\beta} \beta}} \tag{28}$$

So the gust speed has a positive effect, raising the critical horizontal empennage lift cancelling speed.

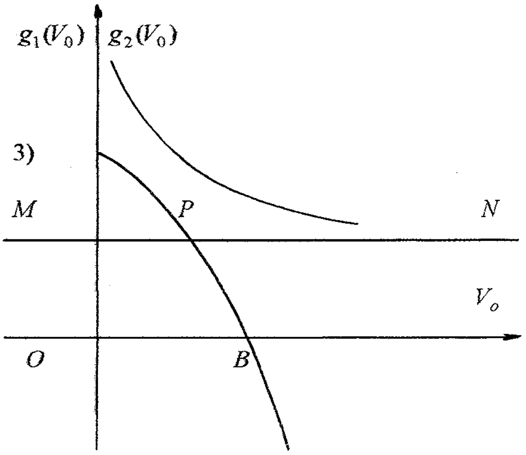

Fig. 9. Absence of any critical speed

The curves $g_1 = g_1(V_0)$ and $g_2 = g_2(V_0)$ don’t intersect, as shown in Fig. 9.

This is an ideal situation, when there are no critical horizontal empennage lift cancelling speeds.

Observations:

- The critical horizontal empennage lift cancelling speed of a canard configuration aircraft drops with the increase in elevator turning angle β ;
- The critical horizontal empennage lift cancelling speed of a canard configuration aircraft drops with the increase in gust speed w;
- The critical horizontal empennage lift cancelling speed of a canard configuration aircraft has an upper limit set by the build parameters and flight altitude of the aircraft, except for the case of an ascending gust (a);
- The critical horizontal empennage lift cancelling speed of a canard configuration aircraft rises with the increase in arrow angle χ .

3. CONCLUSIONS

1. Constructive parameters of the horizontal empennage place an upper limit on the critical horizontal empennage lift cancelling speed; shown previously the effect of each of those.
2. No matter the aircraft configuration, when traversing areas with vertical gusts, flight speed must be decreased in order to travel below critical speed.
3. No matter the aircraft configuration, when traversing areas with vertical gusts, the elevator turning angle β must be decreased.

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REFERENCES

[1] A. Petre, *Teoria aeroelasticităţii. Statica*. Editura Academiei, Bucureşti, 1966.

[2] A. Petre, *Calculul structurilor de aviaţie.* Editura tehnică, Bucureşti, 1984.

[3] V. Avadani, Calculul de rezistenţă al avionului, Vol **II**, Editura Academiei Tehnice Militare, Bucureşti, 1980.

[4] R. L. Bisplinghoff, H. Ashley, Principles of aeroelasticity, John Wiley and sons, Inc., 1956.

[5] V. N. Constantinescu, St. Găletuşe, *Mecanica fluidelor şi elemente de aerodinamică*, Editura Didactică şi Pedagogică, Bucureşti, 1983.

[6] O. Preotu, *Studiu asupra vitezei critice de anulare a portanţei ampenajului orizontal în prezenţa deformaţiilor fuzelajului şi a rafalelor verticale.* A XXVIII-a Sesiune de comunicări ştiinţifice cu participare internaţională, Bucureşti, 21-22.10.1999.

[7] O. Preotu, *Étude sur la vitesse critique de l’annulation de la portance de ’empennage horizontal des aéronefs en configuration canard* –8th International Conference on Applied and Theoretical Electricity, ICATE2006, Pages 362-366, october 26-28, Baile Herculane, 2006.