Solid Topology Optimization Using Truss Lattices

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Abstract: *This paper presents the development, results and conclusions obtained with a solid topology optimization code, using truss lattices as an alternative to continuum elements. Truss elements are used for the simplicity of the implementation in a new code, in all aspects: boundary conditions, loadings, elemental stiffness matrix. Blocked structures, well posed mechanical equilibrium problems are assured by a proper truss arrangement, lattice driven, similar to structured or unstructured meshing.*

Key Words: *topology optimization, SIMP, truss lattice, FEM, optimality criteria*

1. SOLID ISOTROPIC MATERIAL WITH PENALIZATION

SIMP is the method adopted in this paper. The design space consists in the material properties in the mesh. Each element has a set of material properties, Poisson ratio, Young modulus as in linear elasticity.

The method considers a fictive material density, constant over each element, which is used to properly adapt the relevant stiffness variable, which is Young modulus. A power law is adopted, $E = x_e^p E_0$, where E_0 is the nominal value, E the effective value, x_e is the design variable (fictive density), p is the power, chosen constant, and physical density is $\rho = x_e \rho_0$.

Power law for continuum has to be smaller than 3, for a Poisson ratio smaller than 1/3, as in [1]. Solution uniqueness has to be accompanied by a numerical filtering technique, as presented in [2].

The design problem is formulated as an optimization problem, where structure's compliance is to be minimized.

A volume constrained is imposed, followed by lower and upper bounds to ensure physical values for density.

$$\begin{cases} \min : c(\rho) = U^T K U = \sum_{e=1}^{N} x_e^p u_e^T k_0 u_e \\ \quad a.i. : \dfrac{V(x)}{V_0} = f \\ \quad KU = F \\ \quad 0 < x_{\min} \le x \le 1 \end{cases} \tag{1}$$

U and F are the global displacement and force vectors, K is the global stiffness matrix, u_e and k_e are elemental displacement and stiffness matrix.

2. OPTIMALITY CRITERIA

Optimality criteria is the adopted method to solve the problem, as in [1] and [3]. The Lagrange function is built, where λ_0 and λ_1 are scalar and respectively vector multipliers, λ_{2e} and λ_{3e} are lower and upper bound multipliers, removed from the formulation (set to zero) and replaced by an ad-hoc fix in the code.

$$L(x) = c(x) + \lambda_0\left(V(x) - fV_0\right) + \lambda_1^T\left(KU - F\right) + \sum_{e=1}^{N}\lambda_{2e}(x_{\min} - x_e) + \sum_{e=1}^{N}\lambda_{3e}(x_e - x_{\max}) \quad (2)$$

Minimization condition

$$\frac{\partial L}{\partial x_e} = \frac{\partial c}{\partial x_e} + \lambda_0\frac{\partial V}{\partial x_e} + \lambda_1^T\frac{\partial KU}{\partial x_e} = \frac{\partial U^T}{\partial x_e}KU + U^T\frac{\partial K}{\partial x_e}U + U^TK\frac{\partial U}{\partial x_e} \quad (3)$$

Knowing that $\frac{\partial KU}{\partial x_e} = \frac{\partial K}{\partial x_e}U + K\frac{\partial U}{\partial x_e}$, we have:

$$\begin{aligned}\frac{\partial L}{\partial x_e} &= \frac{\partial U^T}{\partial x_e}KU + U^T\frac{\partial K}{\partial x_e}U + U^TK\frac{\partial U}{\partial x_e} + \lambda_0\frac{\partial V}{\partial x_e} + \lambda_1^T\left(\frac{\partial K}{\partial x_e}U + K\frac{\partial U}{\partial x_e}\right)\\ &= U^T\frac{\partial K}{\partial x_e}U + \lambda_1^T\frac{\partial K}{\partial x_e}U + \frac{\partial U}{\partial x_e}\left(2U^TK + \lambda_1^TK\right) + \lambda_0\frac{\partial V}{\partial x_e}\end{aligned} \quad (4)$$

An arbitrary value could be set to λ_1^T, so choosing $-2U^T$ the derivative $\frac{\partial U}{\partial x_e}$ is eliminated and a simple optimality condition is obtained:

$$\frac{\partial L}{\partial x_e} = -p(x_e)^{p-1}u_e^Tk_0u_e + \lambda_0V_e = -p(x_e)^{p-1}q_c + \lambda_0V_e = 0 \quad (5)$$

Noting $q_c = u_e^Tk_0u_e$, basically the deformation potential energy of an element with $x_e = 1$, a new formulation appears: $\frac{px_e^{p-1}q_c}{\lambda_0V_e} = 1$. The right hand side of this equation can be seen as the limit of an iterative process, like $\frac{x_e^{(n+1)}}{x_e^{(n)}}$, so that the iteration takes the form:

$$\frac{p(x_e^{(n)})^{p-1}q_c}{\lambda_0V_e} = \left(\frac{x_e^{(n+1)}}{x_e^{(n)}}\right)^{1/\varsigma} \quad (6)$$

The power $1/\varsigma$ in equation (6) has the property to accelerate the convergence of this fixed point like iteration. The volume constraint multiplier λ_0 must satisfy equation (7), which in our code is solved with the secant method, where *f* is the volume fraction.

$$g(\lambda_0) = V\left(x(\lambda_0)\right) - fV_0 = 0 \quad (7)$$

3. IMPLEMENTATION

Truss lattice modeling of a solid media has been inspired by [4] and [5]. The mechanical energy equivalence is used in some civil engineering design codes, in order to provide quick results, with basic input. What appeared to be promising is the flexibility expected in optimal shaping of a structural member, using a simple, fast finite element modeling. Smaller details were supposed to result from the optimization, in comparison with a hexahedral mesh. Shell/plate elements may be later identified from the optimized lattices, via elemental energy equivalence.

The code is written in FORTRAN 95, and is structured as a typical FEM code. The well known truss stiffness matrix, as in [10] is adopted. A graph oriented data structure – Compressed Row Storage is introduced for all the data: topology graph, stiffness matrix. The first developed linear solver is SOR. It always converges, with a very poor rate. Its replacement with the conjugated gradient method as in [11], dramatically improved convergence and running time.

Secant method is used to compute the volume constraint multiplier. For all considered cases, the effort to solve this algebraic equation is higher than the linear solver itself, due to computation of the density and effective volume.

OpenMP parallelization is implemented for the conjugate gradient method and other routines, but its efficiency has not been proved. Output in vtk format, readable by Paraview is the proven method to represent truss/beam elements with assigned field values.

Validation of the FEM solver has been done against a commercial code, by direct generation of the input code in 2 and 3D cases, with very good results.

Altough **SIMP/OC** is the driver in our code, the process could be seen as **size optimization**, specific to truss/beam problems.

A fully unstructured, tetrahedral/triangular approach is fully supported by the code. In this respect, a tetrahedral/triangle is just another blocked, valid lattice.

4. RESULTS AND CONCLUSIONS

Bi-dimensional cantilever beams have been considered. The 2D functionality is provided, by modifying the stiffness matrix and right hand side, to enable a zero value for the out of plane DOF. Filtering has been applied with various radii.

A number of key issues have been identified:

- power law has to be 0.5, which is radically different than in SIMP, where is around 3; however, solution depends on the value;
- ζ from eq. (6) could be 1, instead of 0.5 as in SIMP;
- without filtration (radius smaller then minimum truss element), results are clearly mesh dependent, as in Fig. 1 and **Error! Reference source not found.**, where the same problem is solved; lattice shape/diagonal orientations provides global topology as by the load transmitting mechanism;
- filtration eliminates small structures;
- anisotropic unstructured mesh offers the greatest mesh insensitivity, but also has an inherent filtration capability Fig. 4;

Further work will be dedicated to investigation of a proper filtration and to replace truss with beam elements (with or without twisting), which may have a significant effect in the diffusivity.

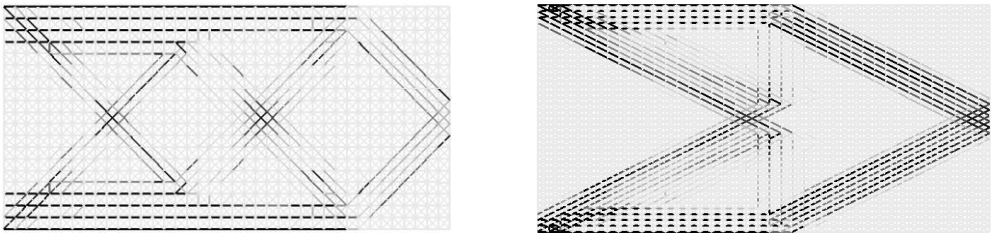

Fig. 1 Solutions for 40 x 20 and 40 x 40 lattices

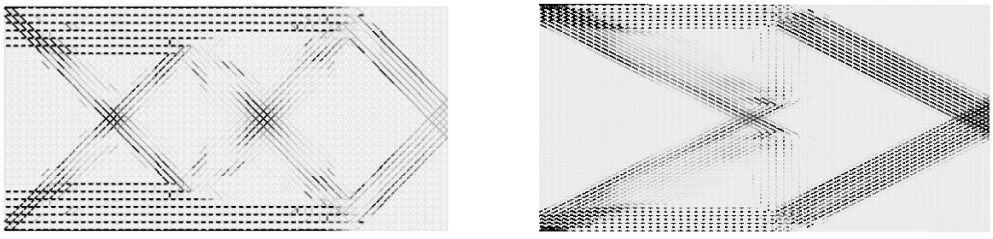

Fig. 2 Solutions for 60 x 30 and 60 x 60, square lattice with diagonals

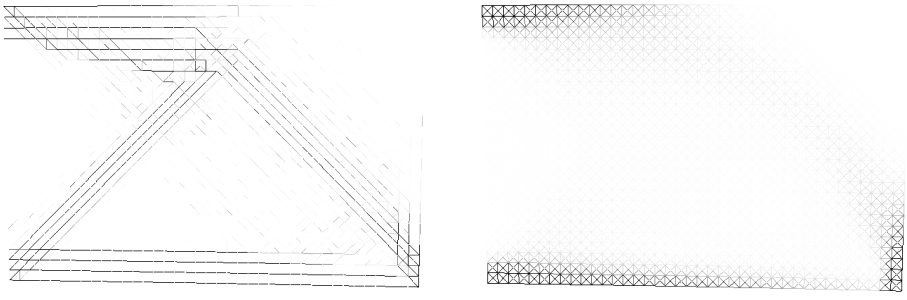

Fig. 3 Left: no filtration (null radius), right: filtration (radius = 2 x truss edge)

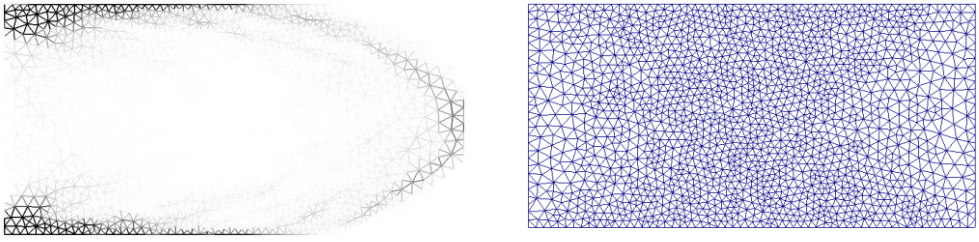

Fig. 4 unstructured mesh lattices, 60 x 30 nodes on the boundary

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