Comparative analysis of kill probability one of the main features of Air Defense Integrated Systems

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Abstract: *The combat features of the Ground Based Air Defence Systems represent the potential of search, discovery, indicate, combat and destruction of the enemy's air assets and the ability to manoeuvre of forces and combat means, for the purpose of capturing the enemy's airspace and avoid actions and attack to defend objectives (of troops) assigned in the area of responsibility tacking into account the conditions established by the mission.*

The paper is focused on a comparative study on the possibilities of target destruction of the Air Air Defence Systems (antiaircraft artillery and Surface-To-Air Missiles).

Two situations were chosen: for the first case, related to S_1, S_2 and S_3, we've assumed the presence of a flying target describing a uniform rectilinear trajectory both in the presence and in the absence of the enemy's electronic jamming. For the second case concerning S_4 we've assumed that the target changed its angle of flight.

Key Words: *Ground Based Air Defense, Integrated Systems, Single shoot hit probability.*

1. INTRODUCTION

As a result of the increasing share of aviation decisive action, studies are conducted in most armies and upgraded and new types of weapons and military equipment are utilized along with improved combat tactical methods and concepts in order to permanently maintain a high performance combat artillery and Surface-To-Air Missiles subunits.

According to military experts improving the quality of Air Defense relies on increasing the probability of hitting aerial targets up to one hundred percent.

The developed Western countries allocate huge funds for research and production of the air defense military equipment, and maintaining in this purpose the high technological capabilities.

To ensure a possible effective antiaircraft riposte to all heights, the future trend will consist in integrating anti-aircraft guns and rockets in the next generation defense systems.

The system of aerial targets destruction performs, in fact, the fundamental mission of retaliatory anti-aircraft system, finishing the task of fighting and destruction of the enemy during the fight exercise.

For fighting the air targets there are used both artillery systems and anti-aircraft missile systems.

2. GROUND BASED AIR DEFENSE SYSTEM - S_1

2.1 Without using the radioelectronic jamming by the enemy

Considering the characteristics of the target, on the one hand, and those of the system S_1 [13], [14], on the other hand, according to table 1, it results the possbilities of destruction without using the radioelectronic jamming.

For all three systems (antiaircraft artillery) we considered the fire rate to be a half of theoretical fire rate in order to achieve results as close as possible to the actual situation.

Table 1 – Initial elements

Characteristics	System of units	**S_1**	**S_2**	**S_3**
Rate of fire	[hits/min]	500	550	60
Shooting range on target	[m]	3500	3500	3500
Target speed	[m/s]	300	300	300
Subsystems	Number of pieces	6	4	6
Single shoot hit probability	%	0,0021	0,0022	0,0032

According to relation [5]:

$$t = \frac{D_{ZT}}{V_T} \tag{1}$$

where:

- D_{ZT} = the flight distance of the target in the fire area of the system, [m];
- V_T = the flight speed of the target, [m/s].

The flight duration of the target in the action area (t) is 11,667 [sec].

$$t = \frac{3500}{300} = 11{,}667\, s$$

The rate of fire, [hits/sec] 8.

$$\frac{500}{60} = 8{,}333 \cong 8$$

The number of projectiles fired by S_1 is obtained by multiplying the duration of the flight target in the action area (11,677 s) by the rate of fire (8 hits/s) and the number of guns (6 guns), and it is equal to 93.

$$N = t \; \zeta \; n_t \tag{2}$$

when applied relation (2),

$$11{,}677 \cdot 8 \cdot 6 = 560{,}496 \cong 560 \text{ fired projectiles}$$

To calculate the hit probability of aerial target with „n” projectile we *applied* relation (3)

$$P_{N(D)/A} = 1 - e^{-N \cdot P_1} \tag{3}$$

where:

- $P_{N(D)/A}$ = probability of destruction of aerial target with „n" projectiles;
- P_1 = single shot hit probability;
- N = the number of the executed shots;
- e = 2,71828.

Hence we get:

$$P_{560} = 1 - e^{-560 \cdot 0,0021}, \quad n \cdot P_1 = 560 \cdot 0,0021 = 1,176, \quad P_{560} = 1 - e^{-1,176} \quad but...e^{-n} = \frac{1}{e^n}$$

$$e^{-1,176} = \frac{1}{e^{1,176}}, \quad e^{1,176} = 3,241, \quad \frac{1}{3,241} = 0,309, \quad P_{560} = 1 - 0,309 = 0,691$$

The following result shows that the aerial target is not destroyed because the probability of destruction is smaller than 0,8.

2.2 Using of the radioelectronic jamming by the enemy

Comparing the characteristics of the target, and those of the system S_1 and the coefficients involved in the use of jamming by the enemy, we obtain the possibilities of destroying with S_1, in jamming conditions, as presented in table 2.

The probability of indicating, and preparing for the fight situation, and the coefficient of stability at passive jamming are also presented in table 2.

Table 2 – Initial elements for jamming conditions [1], [2], [5]

Characteristics		S_1	S_2	S_3
Rate of fire	[hits/min]	500	550	60
Distance of shooting on target	[m]	3500	3500	3500
Target speed	[m/s]	300	300	300
Subsystems	Number of pieces	6	6	6
Single shoot hit probability	%	0,0021	0,0021	0,0032
Probability of indicating		1	0,6	0,6
Preparing for fight situation	%	0,8	0,8	0,8
The coefficient of jamming stability	%	0,75	0,75	0,75

Regarding the previous data, note that:

- the flight time of the target in the action system area (t) is 11,667 s.
- the rate of fire, in hits/s is 8.
- the number of projectiles fired by the system is 560.
- probability of destroying for 560 projectile, without using the radioelectronic jamming is 0,691.

Replacing in relation:

$$P_{N(D)_{CB/A}} = P_{N(D)/A} \cdot P_I \cdot \varphi_{SPL} \cdot \varphi_{CRT} \tag{4}$$

where:

- $P_{N(D)_{CB/A}}$ = probability of destruction of aerial target with „n” projectiles, in radioelectronic jamming conditions;
- P_I = probability of indicating, with values:
  - 0,6 ÷ 0,8 under non-automated arrangements;
  - 0,8 ÷ 0,9 under automated ones;
- φ_{SPL} = preparing for fight coefficient of the fire control system;
- φ_{CRT} = the coefficient of stability at jamming.

The possibilities of destroying an aerial target – number of destroyed aerial target – result from relation (5):

$$T_{N(D)/A} = N_{T/A} \cdot P_{N(D)/A\,\mathrm{sau}\,N(D)_{CB/A}} \tag{5}$$

where:

- $T_{N(D)/A}$ = possibility of destroying an aerial target;
- $P_{N(D)/A\,\mathrm{sau}\,N(D)_{CB/A}}$ = probability of destroying an aerial target that does not use a radioelectronic jamming and probability of destroying an aerial target that uses a radioelectronic jamming, respectively.

Hence we obtain:

$$0{,}691 \cdot 1 \cdot 0{,}8 \cdot 0{,}75 = 0{,}415$$

The aerial target is not destroyed because the probability of destruction is smaller than 0,8.

3. GROUND BASED AIR DEFENSE SYSTEM – S_2

3.1 Without using the radioelectronic jamming by the enemy

Based on data related to S_2 centralized in table 1, the possibilities of annihilation are obtained without the use of radioelectronic jamming by the enemy, as presented in table 3.

According to relation 3, the flight duration of the target in the action area (t) is 11,667 s.

$$t = \frac{3500}{300} = 11{,}667\,s$$

The rate of fire, in hits/s is 9.

$$\frac{550}{60} = 9{,}167 \cong 9$$

The number of projectiles fired by system S_2 is 420.

$$11{,}677 \cdot 9 \cdot 4 = 420{,}372 \cong 420 \text{ fired projectiles}$$

According to relation 3 the probability of destroying with 420 projectiles is 0,603.

$$P_{420} = 1 - e^{-420 \cdot 0{,}0022} \qquad P_{420} = 1 - 0{,}397 = 0{,}603$$

The aerial target is not destroyed because the probability of destruction is smaller than 0,8.

3.2 Using the radioelectronic jamming by the enemy

Following the same algorithm based on data centralized in table 2, the possibilities of annihilation obtained with the use of radioelectronic jamming by the enemy, are presented in table 3.

The probability of indicating,and preparing for fight situation, and the coefficient of stability at passive jamming are presented in table 2,

Regarding the previous data, note that:

- The flight duration of the target in the action area (t) is 11,667 s.
- The rate of fire, in hits/s is 9.
- The number of projectiles fired by the subunit is 420.
- Probability of destroying for 420 projectile, without using the radioelectronic jamming is 0,603.

The probability of destroying with 420 projectiles is 0.217.

$$0{,}603 \cdot 0{,}6 \cdot 0{,}8 \cdot 0{,}75 = 0{,}217$$

The aerial target is not destroyed because the probability of destruction is smaller than 0,8.

4. THE CALCULATION OF POSSIBILITIES OF DESTRUCTION FOR SYSTEM S_3

4.1 Without using the radioelectronic jamming by the enemy

Based on data related in table 1,with S_3 the possibilities of annihilation are obtained without the use of radioelectronic jamming by the enemy, presented in table 3.

The rate of fire is 1.

$$\frac{60}{60} = 1$$

The number of projectiles fired by system S_3 is 70.

$$11{,}677 \cdot 1 \cdot 6 = 70{,}062 \cong 70 \text{ fired projectiles}$$

The probability of destroying with 70 projectiles is 0,201

$$P_{70} = 1 - e^{-70 \cdot 0{,}0032} \qquad P_{70} = 1 - 0{,}799 = 0{,}201$$

The aerial target is not destroyed because the probability of destruction is smaller than 0,8.

4.2 Using the radioelectronic jamming by the enemy

Data showed in table 1, are the source for calculating the possibilities of annihilation with the use of radioelectronic jamming, presented in table 3.

Regarding the previous data, note that:

- The flight duration of the target in the action area (t) is 11,667 s.
- The rate of fire, in hits/s is 1.
- The number of projectiles fired by the subunit is 70.
- The probability of destroying for 70 projectile, without using the radioelectronic jamming is 0,201.

The probability of destroying the target with 70 projectiles is 0.072.

$$0{,}201 \cdot 0{,}6 \cdot 0{,}8 \cdot 0{,}75 = 0{,}072$$

The aerial target is not destroyed because the probability of destruction is smaller than 0,8. According to the above data and calculations for each systems we made a compilation of them so the comparative analysis is based on the data in table 3.

Tabel 3. Possibilities of destroying

	without jamming			with jamming		
System	S_1	S_2	S_3	S_1	S_2	S_3
Number of projectils necessary for the destruction of the target	837	735	504	1395	2205	1380
Number of fired projectiles	560	420	70	560	420	70
Probability of destruction	**0,691**	**0,603**	**0,201**	**0,415**	**0,217**	**0,072**
Is the target destroyed?	**No**	**No**	**No**	**No**	**No**	**No**
Number of guns necessary for the destruction of the target	9	7	42	15	21	115
Number of additional guns for the destruction of the target	3	3	36	9	7	109

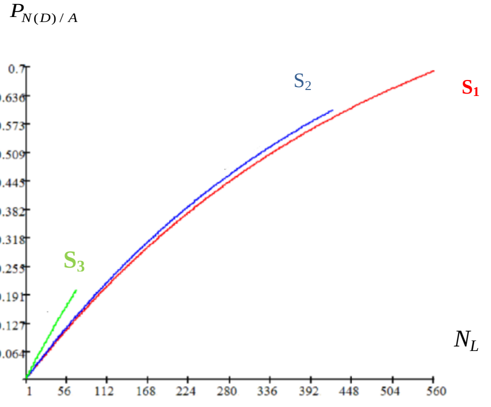


Fig. 1 Probability of destruction by the number of projectiles fired, without jamming

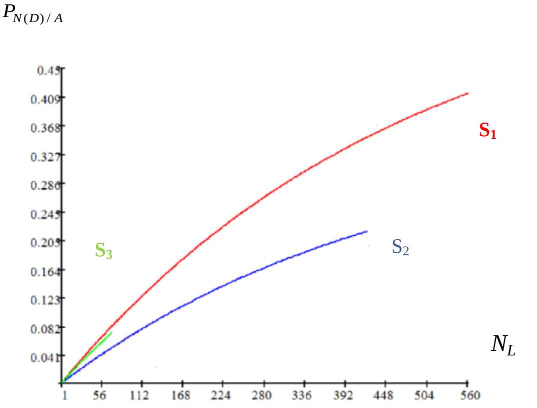


Fig. 2 Probability of destruction for each system depending on the number of projectiles fired using the radioelectronic jamming by the enemy

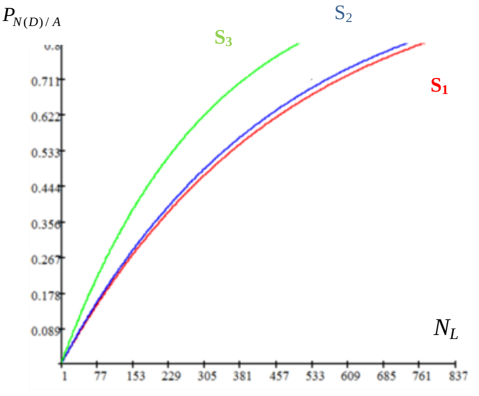


Fig. 3 Probability of destruction for each systems depending on the number of projectiles fired without the radioelectronic jamming

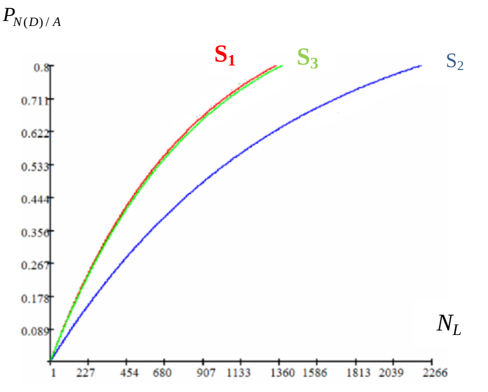
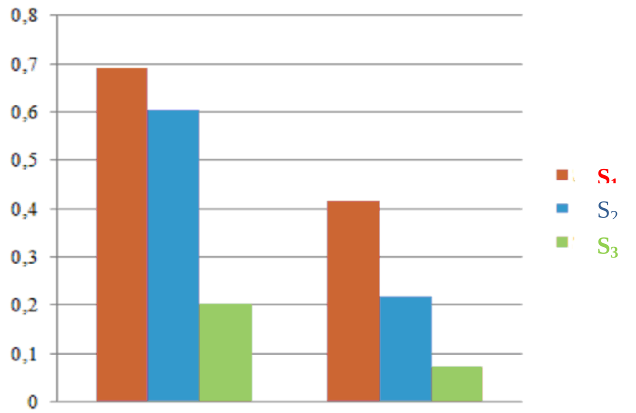


Fig. 4 Probability of destruction for each systems depending on the number of projectiles fired using the radioelectronic jamming



a) without jamming b) with jamming

Fig. 5 a, b Probability of destruction for each system

5. THE TRAJECTORY SIMULATION USING THE CATCH UP CURVE GUIDANCE METHOD IMPLEMENTED PROGRAM FOR THE SYSTEM S_4 (SURFACE - TO - AIR MISSILE)

Unlike others three systems, S_4 is one of the Surface - to - Air Missile Systems which can be easily integrated with the other three ones.

The catch up curve guidance method means the law of approaching missile to the target, that at any time the missile speed vector is oriented in the direction of the target. In this method the command signal is proportional to the size of the advance angle (between the missile speed vector and missile-target line) that constitutes the variance parameter.

The target trajectory and missile speed as functions of time for flight conditions should be known for drawing the graphic.

The Missile kinematic trajectory is drawn only in the vertical guidance plane for the linear and uniformly target flight situation, and evenly to a report\ V_R / $V_T = 2$.

It is found that in the final portion of the trajectory the rocket turns tightly approaching to the target from the back of it. The trajectories are determined by their overload ratio η_d / η_n.

Assesment of the missile's trajectory characteristics

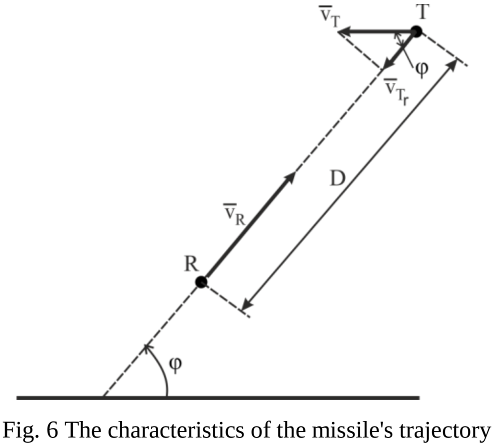


Fig. 6 The characteristics of the missile's trajectory

Using the notations in figure 1 we can write the kinematic equations of the catch up curve method for one of the guidance plans:

$$\dot{D} = -V_{T_R} - V_R = -V_T \cdot \cos\varphi \tag{6}$$

$$D\dot{\varphi} = V_T \cdot \sin\varphi \tag{7}$$

where:

$\dot{D}$ – the distance variation between the missile and the target;

V_{T_R} – the radial speed of the target (target speed component oriented in the direction of the missile);

$D\dot{\varphi}$ – linear speed of the distance line rotation.

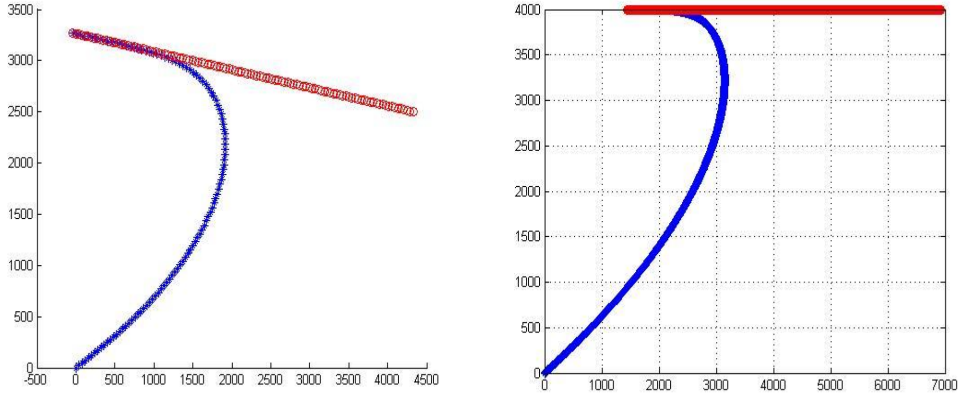


Fig. 7 a, b The case of the impact between the two aerodynamics vectors

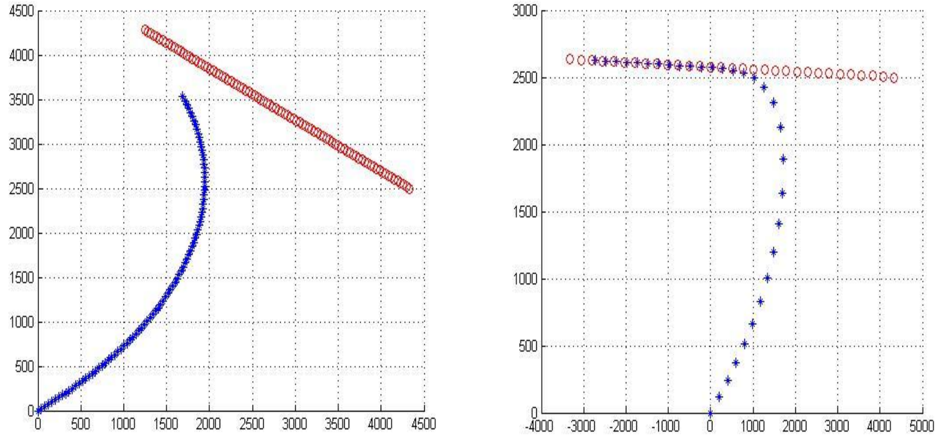


Fig. 7 c, d The case of mising the target

In order to search an optimization for the missile interception path, a program which allows to study different approaching trajectories when some parameters are modified proves to be very useful. This has implemented the catch up with curve guidance method [4] with general characteristics as follows:

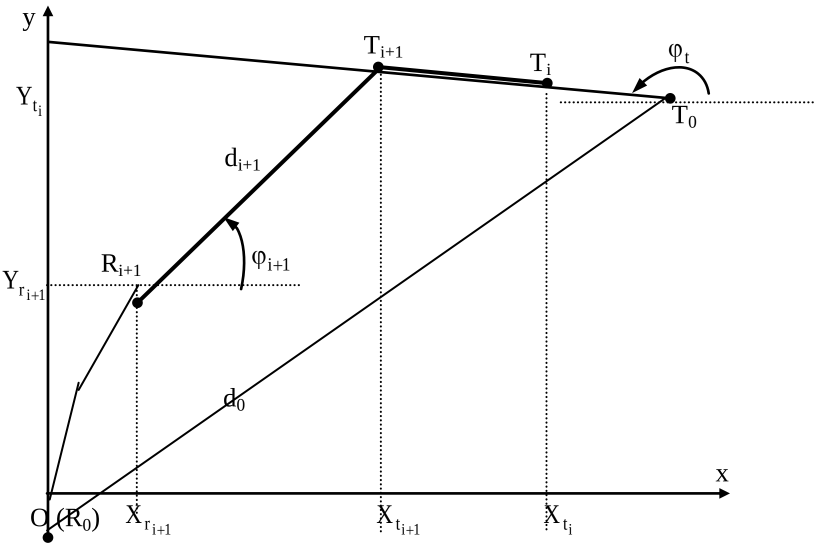


Fig. 8 The method cinematic schedule

After performing of the accounting from the *i* step, all the characteristics with the *i* index are determined (R_i and T_i are the annotations for the missile and the target position at the moment, representing the points on these trajectories, respectively). The succession of the calculus at the *i+1* step is the following:

At the *i* step there are known the missile and target positions R_i and T_i and we have to determine the following:

- The slant range d_{i+1} is determined

$$\Delta t: \ d_1 = v_r \cdot \Delta t$$

- The target coordinates T_{i+1} are determined, using the relations:

$$X_{t_{i+1}} = X_{t_i} - v_t \cdot \Delta t \tag{8}$$

$$Y_{t_{i+1}} = Y_{t_i} \tag{9}$$

- The R_{i+1}, $X_{r_{i+1}}$ and $Y_{r_{i+1}}$ coordinates of the position of the missile are calculated:

$$X_{r_{i+1}} = X_{r_i} + d_1 \cdot \cos\varphi_i \tag{10}$$

$$Y_{r_{i+1}} = Y_{r_i} + d_i \cdot \sin\varphi_i \tag{11}$$

- The pitch angle is determined:

$$\varphi_{i+1} = \begin{cases} arctg\left(\dfrac{Y_{t_i} - Y_{r_i}}{X_{t_i} - X_{r_i}}\right) & if \quad X_{t_i} - X_{r_i} \geq 0 \\ \pi + arctg\left(\dfrac{Y_{t_i} - Y_{r_i}}{X_{t_i} - X_{r_i}}\right) & if \quad X_{t_i} - X_{r_i} < 0 \end{cases} \tag{12}$$

- The slant range between new positions R_{i+1} and T_{i+1}:

$$d = \sqrt{\left(X_{t_{i+1}} - X_{r_{i+1}}\right)^2 + \left(Y_{t_{i+1}} - Y_{r_{i+1}}\right)^2} \tag{13}$$

6. CONCLUSIONS

If the enemy does not use jamming, according to figure 1 – (that we performed using the program Mathcad 15, by generating the schedule based on the number of projectiles fired by each system and the probability of their destruction - and figure 5 - that I made in Microsoft Excel 2010 based on probabilities of annihilation (destruction) of each subunit) - we can see that S_1 and S_2 systems, have close values of the probability of destruction (without jamming 0.691 and 0,602).

As for S_3, its destruction probability is very small, only 0,201 with a total of 70 projectiles fired.

The great disadvantage of this system is the low rate of fire per gun, which determines the probability of destruction to decrease a lot from the other systems.

According to table 3, no system was unable to destroy the target because the probability of destroying is smaller than 0,8.

The specified system in the firing session is S_1 because it has the highest probability of destruction.

For an increased probability of destruction we can focus the fire to achieve integrated defense.

To focus fire all systems are indicated because the probability of destruction is very high, of about 0.902.

Regarding to the missile trajectory simulation, by modifying the φt angle corresponding to a target maneuver in altitude (H), different situations of interception can be relatively easy studied.

Figures 7a and 7b present the case of impact for the basic hypothesis or when the target has/executes a maneuver in altitude, respectively.

In figures 7c and 7d are presented the case of missing the target for the hypothesis when the target maneuvers in H or when the target maneuvers in H and increases the velocity from 300 m/s to 600 m/s respectively, so exceeding the Air Defense system effective range.

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