Velocity distribution as a result of the interaction between organizational processes in shear flows

Stefan N. SAVULESCU[1], Florin BALTARETU*,[2]

*Corresponding author

[1]INCAS – National Institute for Aerospace Research "Elie Carafoli"
B-dul Iuliu Maniu 220, Bucharest 061126, Romania

*,[2]Technical University of Civil Engineering Bucharest,
Department of Thermal Engineering,
Bd. Pache Protopopescu nr. 66, 021414, Bucharest, Romania
flbaltaretu@yahoo.com, florin.baltaretu@instal.utcb.ro

DOI: 10.13111/2066-8201.2014.6.4.8

Abstract: *This paper continues the recent research of the authors concerning the interaction between the micro and macro flow structure domains (further on called IPmMD). The authors propose a probabilistic type approach of the IPmMD, in plane boundary layer, channel and pipe flows. This process, purely theoretical expressed by using transformations associated to transversal coordinate, tends to ensure the equipartition of the kinetic energy and to lead to a minimum in information transmission in the space-time ensemble. Taking into account several velocity distributions as not mutually exclusive events, new distributions are obtained by a probabilistic type union of the events. These new distributions present important aspects related to the classical transition from the laminar-to-turbulent regime. In this way, as an example, the parabolic-laminar and the linear distributions lead to a velocity distribution very close to experimental data corresponding to the rough pipe flow.*

Key Words: *shear flow, probabilistic approach, velocity distribution.*

1. INTRODUCTORY REMARKS

In our previous paper [1] we introduced the existence of some physical interaction processes between micro and macro fluid structures domains (IPmMD), based on our research [2], [3]. In this paper we present a fundamental support to the IPmMD by the conditions to fulfill the entropy principle, and to minimize the energy involved.

The mathematical description is based on the transformation of the coordinate across the flow main direction.

For instance, the given distribution (as a velocity profile), is transformed in other distribution ranging from the very near to very far by comparison with the given initial distribution.

The number of such new distribution is very great at the micro structure scale and very small at the macro structure scale.

It results that the mathematical model for the IPmMD cannot be deterministic, belonging essentially to the probability field of knowledge.

The deterministic real fluid flow mathematical description (Navier-Stokes equations), in spite of its generality, stands for an approximation (very useful) of IPmMD. We have to mention, accordingly, the difficulty to specify the minimum space-time scales of the validity of this deterministic description.

For instance, the hot wire can measure the velocity at a scale much smaller that the pressure device measurement. Usually, we introduce the boundary layer approximation for the velocity profile and the constant pressure across the layer.

Concerning the critical Reynolds number in transition, the wall roughness, as well as the chemical fluid nature, offer very different values. Similarly, the great turbulent fluctuations near wall stand for some terrifying aviation accidents during the landing or takeoff of the very modern planes.

In biomechanics, there are many examples of the IPmMD (like the instinct notion or the bird ability to flow in a turbulent atmosphere).

The humanity presents also many examples of individuals' abilities which can be explained by particular aspects of the IPmMD.

We have to remark that, in order to minimize the bulk energy, the IPmMD acts on a finite space-time domain like the turbulent spot in real fluid flows.

It is also important to mention the fundamental difference between the IPmMD approach and the classical, or quantic-relativistic approach. Accordingly, we have to mention the studies of complexity [4], as well as other probabilistic approaches [5], [6].

2. THE PROBABILISTIC NATURE OF THE IPmMD

Due to the very great number of the micro elements (necessary to the fulfillment of the entropy principle), and of their organizations in a finite domain (necessary to minimize the energy involved), we explain the probabilistic nature of the IPmMD.

Furthermore, the expression of the probability combination stands for an adequate mathematical description.

By considering two organization events A and B, having their probabilities P(A) and P(B), we use the relation of the joint probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \tag{1}$$

in order to get a new organization probability, i.e. a new IPmMD.

If the events are considered as independent, we have:

$$P(A \cap B) = P(A) \cdot P(B) \tag{2}$$

It seems very interesting to remark that eq. (1) preserves its typical form with a maximum depending of the many known and unknown parameters.

A qualitative numerical study of the generalized form:

$$P_k(A \cup B) = k_A \cdot P(A) + k_B \cdot P(B) - k_{AB} \cdot P(A \cap B) \tag{3}$$

points out the existence of this maximum as shown in fig. 1a and fig. 1b, where the occurrences of the particular values of the integer number:

$$N = k_1 + k_2 - k_3 \tag{4}$$

were counted.

A possible corresponding probability of this simple example is:

$$P(N) = \frac{N}{N_{\max}} = \frac{k_1 + k_2 - k_3}{N_{\max}} \tag{5}$$

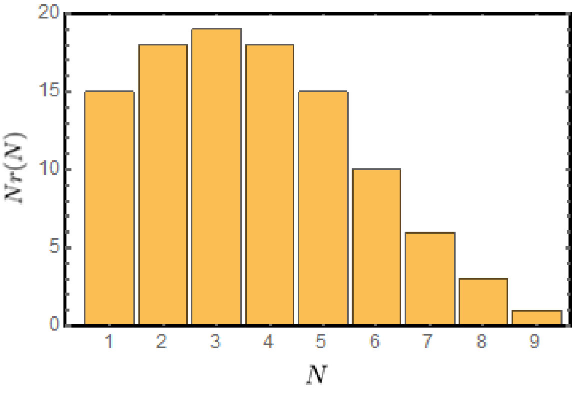

Fig. 1 a) $Nr(N)$, $k_1, k_2, k_3 \in \{1 \ldots 5\}$, $N_{\max} = 9$

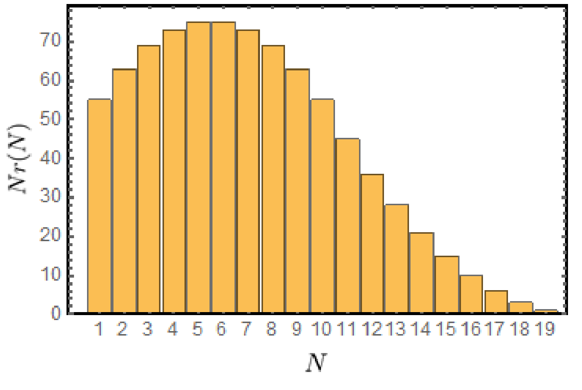

Fig. 1 b) $Nr(N)$, $k_1, k_2, k_3 \in \{1 \ldots 10\}$, $N_{\max} = 19$

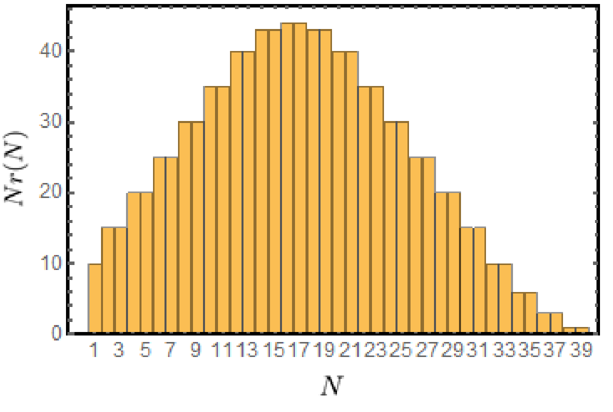

Fig. 1 c) $Nr(N)$, $k_1, k_2, k_3 \in \{1 \ldots 10\}$, $N_{\max} = 39$

Another interesting example is pointed out in figure 1 c), where, considering that $P(A) = P(B) = 1/2$, $P(A \cap B) = P(A) \cdot P(B)$ and using a corresponding multiplication factor to obtain integer numbers, the integer number N is expressed as:

$$N = 4 \cdot \left(k_1 \cdot \frac{1}{2} + k_2 \cdot \frac{1}{2} - k_3 \cdot \frac{1}{4} \right) \tag{6}$$

Now, let consider the A and B as velocity distributions events, or as particular IPmMD elements in a real fluid flow (boundary layer, channel, pipe flows), and the simple case:

$$k_A = k_B = k_{AB} = 1. \tag{7}$$

Starting now with a characteristic velocity distribution:

$$P(A) = 1 - (1-\eta)^{N-1} \tag{8}$$

and with the linear distribution (that corresponds to a minimum information transmission):

$$P(B) = \eta, \tag{9}$$

we get the “joint” result:

$$U_a = 1 - (1-\eta)^N, \tag{10}$$

as a typical velocity distribution, depending only on the parameter N.

The transformed velocity distribution [1] points out the features of the organizational processes in shear flows.

Another interesting characteristic is the influence of the solid wall roughness, as revealed in fig. 2.

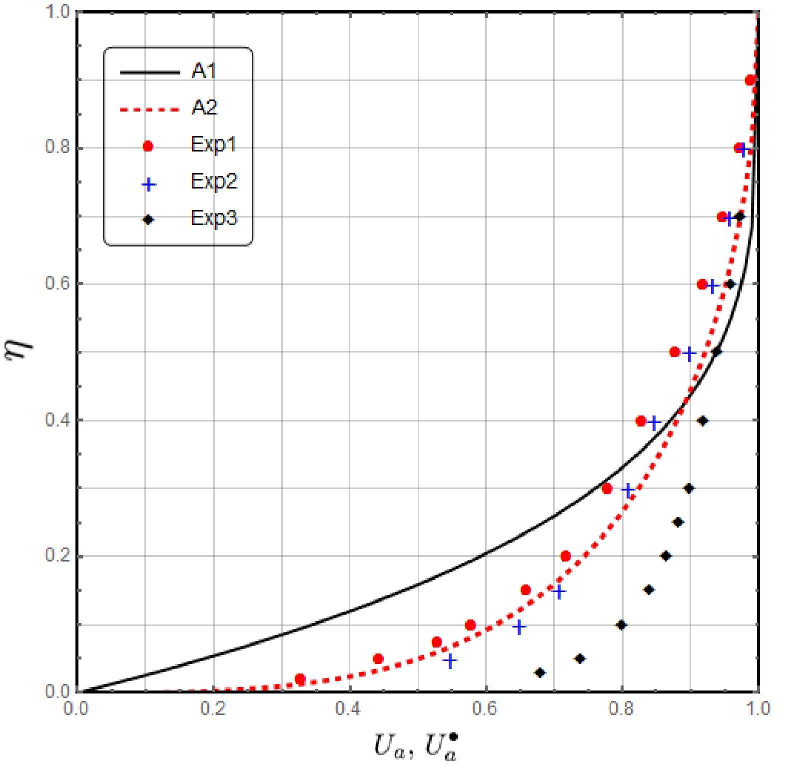

Fig. 2 Analytical and experimental velocity distributions for a rough pipe flow.
A1 – characteristic velocity distribution – eq. (11),
A2 – transformed velocity distribution – eq. (12),
Exp1 – water, rough pipe flow,
Exp2 – air, rough pipe flow,
Exp3 – air, *smooth* pipe flow.

The starting velocity distribution used in the example shown in figure 2 is:

$$U_a = 1 - (1-\eta)^4 \tag{11}$$

(corresponding to $N = 4$), that leads to the transformed velocity distribution (curve A2):

$$\eta(U_a^{\bullet}) = 1 - (1 - U_a^{\bullet})^{1/2}\left[1 + \frac{1}{2}U_a^{\bullet} + \frac{3}{8}(U_a^{\bullet})^2\right] \tag{12}$$

which is a good approximation for the experimental data [7], where the roughness parameter $k/R = 1.6\ldots4.5\times10^{-2}$.

However, where the pipe wall has no roughness ($k/R < 10^{-2}$), the velocity distribution stands for the well-known η^n description, with $n \cong 1/9$ (we mention that $U_a^{\bullet} = \eta^{\frac{n}{1+2n}}$ - transformation corresponding to $n \cong 1/7$). The results can be visualized in figure 3.

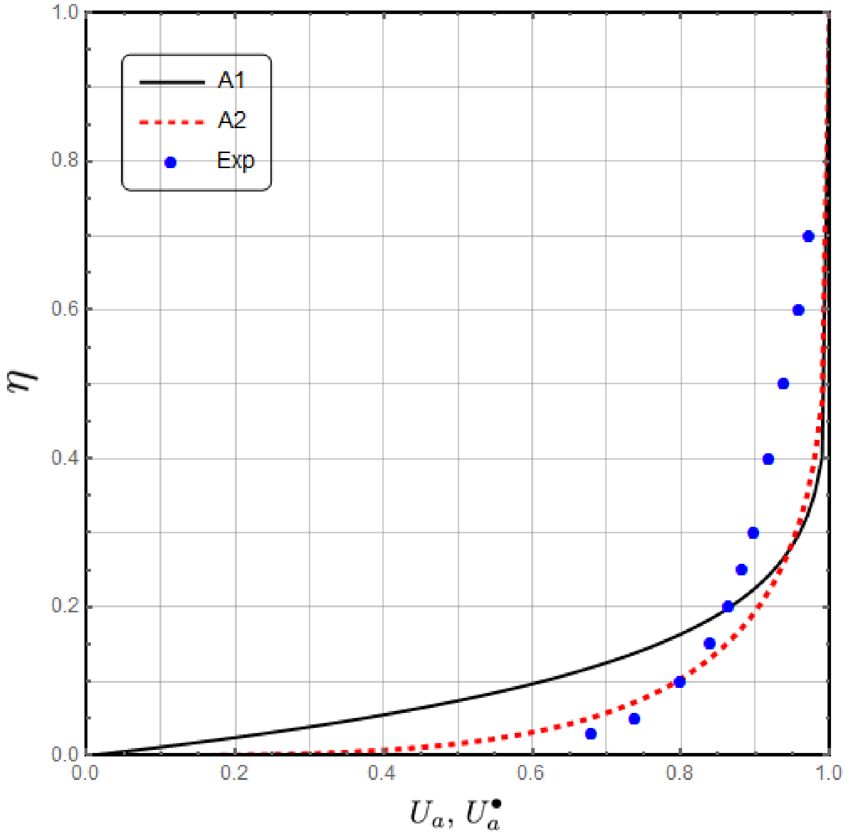

Fig. 3 Analytical and experimental velocity distributions for a smooth pipe flow.
A1/A2 – characteristic/transformed analytical velocity distribution, Exp – experimental data.

It results that IPmMD exists, and the wall roughness level is a parameter influencing the organization IPmMD and consequently the fluctuation level ($\varphi_U \cong 5\cdot10^{-3}$ with roughness and $\varphi_U \cong 3\cdot10^{-3}$ without roughness). The theoretical determination of these limits is a matter of future research.

Another very interesting feature of the IPmMD lies on its interpretation as a classical expression of some deterministic fluid flow phenomena. As our studies reveal, the IPmMD has its specific nature.

It acts as a particular competition between the micro structure domain and the corresponding macro structure domain. Due to this particular connection, it acts on finite domains (boundary layer thickness, spot development) and as unusual time variation like the turbulent fluctuation.

Naturally, our observations depend on the physical instrumentation the technology can offer. As an example, our deterministic classical description reveals the velocity profiles by hot wire measurements and a constant pressure across the boundary layer due to the corresponding pressure measurement device.

Contrary to this deterministic - semi-empirical approximation, the IPmMd acts always in its specific nature, as a probabilistic possibility which has a maximum for some given macroscopic parameters.

3. CONCLUSIONS

The present paper is devoted to IPmMD, by proving its probabilistic existence as a general relation (interaction processes between micro and macro domains).

In order to explain this concept, we notice that, for the usual knowledge, the microstructure domain belongs to the quantic description and the macrostructure domain belongs to the continuum description. Between these two knowledge descriptions, there are a lot of semi-empirical suppositions.

Our IPmMD introduces the ensemble of space-time coordinate transformation as an adequate probability connection between these structure domains. It is based on the entropy principle, the minimum of the energy in the micro-macro structure domain, probabilistic existence and, eventually, on the energy equipartition in the probabilistic space-time structure domains at some adequate scale.

The IPmMD needs further adequate studies in order to quantify and to have an accurately description of a lot of phenomena, actually unknown.

REFERENCES

[1] Șt. N. Săvulescu, Fl. Băltărețu, Correlations between the theoretical Fluons model and the physical experimental results, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, Vol. **6**, Special Issue, DOI: 10.13111/2066-8201.2014.6.S1.16, pp. 143-150, 2014.

[2] Șt. N. Săvulescu, Fl. Băltărețu, Analysis of the microscopic-macroscopic structure in real fluid flow, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol. **5**, no. 1, DOI: 10.13111/2066-8201.2013.5.1.11, pp. 95-102, 2013.

[3] Șt. N. Săvulescu, Fl. Băltărețu, H. Dumitrescu, The analysis of the process tending towards equilibrium in transitional and turbulent wall flows, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol. **2**, no. 2, DOI: 10.13111/2066-8201.2010.2.2.14, pp. 100-113, 2010.

[4] Al. Boutot, *L'invention des formes - Chaos, catastrophes, fractales, structures dissipatives, attracteurs étranges*, Odile Jacob, Paris, 1993.

[5] C.-L. Chiu, S.-M. Hsu, Probabilistic approach to modeling of velocity distributions in fluid flows, *Journal of Hydrology*, DOI: 10.1016/j.jhydrol.2005.04.011, vol. **316**, Issues 1–4, pp 28–42, 2006.

[6] G. Marini, G. De Martino, N. Fontana, M. Fiorentino, V. Singh, Entropy approach for 2D velocity distribution in open-channel flow, *Journal of Hydraulic Research*, DOI: 10.1080/00221686.2011.635889, vol. **49**, no. 6, pp. 784-790, 2011.

[7] F. Raemy, *Mesure de la turbulence dans un ecoulement d'eau en conduit circulaire a paroi rugueuse*, These EPFZ, Zurich, Diss ETH 6076, 1978.