Correlations between the theoretical Fluons model and the physical experimental results

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Abstract: *This paper continues the recent research of the authors concerning the Fluons mathematical model, as a theoretical basis for the interaction between the micro and macro flow structure domains (further on called IPmMD). Correlations with physical experimental results are pointed out, by revealing some traces of the IPmMD and its influence to the field of fluctuations.*

Key Words: *integro-differential formulation, Fluons model, turbulence.*

1. INTRODUCTION

In our previous paper [1] we presented the mathematical aspects concerning the Fluons concept. The present paper analyzes the physical experimental support of this mathematical model. Essentially we discuss the connection between the Fluons model and the field of fluctuations associated to the mean velocity distributions in boundary-layer, channel and pipe flows.

This analysis stands for another point of view, which does not need to start with the empirical decomposition:

$$\vec{V} = \underbrace{\left(\overline{\vec{V}}\right)}_{mean} + \underbrace{\left(\vec{V}'\right)}_{fluctuation} \tag{1}$$

in order to appreciate the flow as laminar, transitional or turbulent.

We try to point out the existence of a physical process concerning the interaction between the micro and macro flow structure domains (further on called IPmMD), revealed experimentally as a fluctuation field. As a theoretical basis for the IPmMD we propose the Fluons model [2]. Some qualitative results concerning the global intensity and the distributions form of the fluctuations associated to a given velocity distribution raises the necessity to investigate qualitatively and quantitatively the field of wall layers flow fluctuations revealed by experiments. Accordingly, we will take into consideration the physical principles of mass conservation, increasing entropy generation and equipartition of the kinetic energy at the various time-space scales, required by IPmMD. The Fluons model allows explaining the physical existence of some finite domain of fluctuation (like the turbulent spot), the tendency towards the minimum of the global energy in some macroscopic domains, as well as the connection between micro and macro scales by successive transformations.

The successive transformations attain the scale of the molecular chaos, i.e. the discrete structure of the fluids, by using physically the same integral algorithm. Let's consider a given distribution $f = f(\eta)$, with $0 \le f(\eta) \le f_e = f(1)$, $0 \le \eta \le 1$ which provides the inversion $\eta = \eta(f)$ (analytically or numerically). By using the transformation:

$$\eta_2(\eta) = \frac{\int_0^\eta f^2(\eta) d\eta}{\int_0^1 f^2(\eta) d\eta} \tag{2}$$

and by writing $\eta_2 \to \eta$ and $f(\eta) \to f^*(\eta)$, we get the distribution $f^*(\eta)$, with $0 \le f^*(\eta) \le 1$, for $0 \le \eta \le 1$. Also, by using the transformation:

$$f^{\oplus}(\eta) = \frac{\int_0^\eta \left[\int_1^\eta f(\eta) d\eta \right] d\eta}{\int_0^1 \left[\int_1^\eta f(\eta) d\eta \right] d\eta} \tag{3}$$

we get the distribution $f^{\oplus}(\eta)$, with $0 \le f^{\oplus}(\eta) \le 1$, for $0 \le \eta \le 1$. Using these integral algorithms we obtain, therefore, the distributions $f^*(\eta)$ and $f^{\oplus}(\eta)$, as associate distributions of $f(\eta)$.

By successive use of the same algorithms, this time for $f^*(\eta)$, and correspondingly for $f^{\oplus}(\eta)$, we get a set of distributions which attain some level of invariance for each function $F_k^*(\eta)$ and $F_k^{\oplus}(\eta)$:

$$F_k^*(\eta) = \left(F_{k-1}^*(\eta)\right)^*, \; F_1^*(\eta) = f^*(\eta) \tag{4}$$

$$F_k^{\oplus}(\eta) = \left(F_{k-1}^{\oplus}(\eta)\right)^{\oplus}, \; F_1^{\oplus}(\eta) = f^{\oplus}(\eta) \tag{5}$$

but which show also an invariant difference between these distributions.

The Fluons, expressed as differences, are qualitatively and quantitatively related to the field of experimental fluctuations.

The essential difficulty to prove that this Fluons theoretical model is completely typical for IPmMD lies in our physical impossibility to cover by observation the range from molecular chaos (space-time scales Å and μs) up to the usual fluid flows experimental devices (space-time scales μm and ms).

As an example, for the velocity field, the hot-wire measurements are limited to a space scale of $\mu m = 10^{-6}\,m$; for the pressure field, the corresponding transducers need a $10^{-3}\,m$ scale in order to give reliable results.

At the molecular scale, both these measuring devices are inappropriate to provide direct information about the existence of some particular process connecting the micro and macro structure domains. However, the analysis of space-time variations of fluctuations reveals some traces of the IPmMD and its relation with the Fluons model and therefore stands for a physical interpretation of the measurements.

2. THE EXPERIMENTAL FIELD OF FLUCTUATIONS AND THE FLUONS MODEL

Let's consider a given mean velocity distribution $U(y)$ across a flowing layer, with $0 < U < U_e$, $0 \le y \le y_e$ and the normalized distribution $U_a(\eta)$, where $\eta = y/y_e$, $U_a = U/U_e$. Physically, in a space-time domain between the microscopic and the macroscopic scales, the distributions $U(y,t)$ can be very complicated and practically impossible to be determined. However, our IPmMD leads to some deterministic organization into the ensemble of instantaneous distributions. The integral algorithm used in the Fluons mathematical expressions seems to be appropriate to provide a deterministic description of the IPmMD. For the normalized velocity distribution $U_a(\eta)$ we consider the following Fluons expressions:

$$\Delta_h(\eta) = \frac{\int_0^\eta U_a^2 \,\mathrm{d}\eta}{\int_0^1 U_a^2 \,\mathrm{d}\eta} - \eta(U_a) \tag{6}$$

$$\Delta_e(\eta) = \left[U_a^*(\eta)\right]^2 - U_a^2(\eta) \tag{7}$$

$$\Delta_0^*(\eta) = U_a(\eta) - U_a^*(\eta) \tag{8}$$

$$\Delta_0^\oplus(\eta) = U_a(\eta) - U_a^\oplus(\eta) \tag{9}$$

$$\Delta_*^\oplus(\eta) = U_a^*(\eta) - U_a^\oplus(\eta) \tag{10}$$

where the distributions $U_a^*(\eta)$ and $U_a^\oplus(\eta)$ have been previously presented and can be analytically and numerically obtained by using the integral algorithm from the IDF (integro-differential formulation) of the Navier-Stokes equations [1] [2].

Consequently, we present the qualitative and the quantitative results of the comparison between the Fluons mathematical model and the various experimental data concerning the field of fluctuations associated to mean wall layer velocity distribution across the layer. These fields of fluctuations are experimentally provided as mean values or space-time variations in the classical laminar, transitional and turbulent shear flows.

Qualitatively, we can write:

$$\frac{\overline{v'v'}}{\left(\overline{v'v'}\right)_{\max}}(\eta) \approx \frac{\Delta h}{(\Delta h)_{\max}}(\eta) \tag{11}$$

$$\frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)_{\max}}(\eta) \approx \frac{\Delta e}{(\Delta e)_{\max}}(\eta) \approx \frac{\overline{u'u'}}{\left(\overline{u'u'}\right)_{\max}}(\eta) \tag{12}$$

where u', v', w' are the corresponding fluctuations of the turbulent $U_T(\eta)$ distribution; these relations were confirmed by the experimental fluctuations measurements [4] [5] [6]. It is necessary to make the following remark: when we choose the normalized coordinate η, we get a single $e(\eta)$ distribution, but for $h(\eta)$ we get also a second distribution $h^{-}(\eta)$, with T, as shown in figure 1. The distributions $h(\eta)$ and $h^{-}(\eta)$ are similar, but the location of their maximum is different. It is very interesting to notice the existence of two maxima for some $\overline{v'v'}(\eta)$ measurements [4]. However, one needs a lot of careful measurements in order to confirm the generality of this observation.

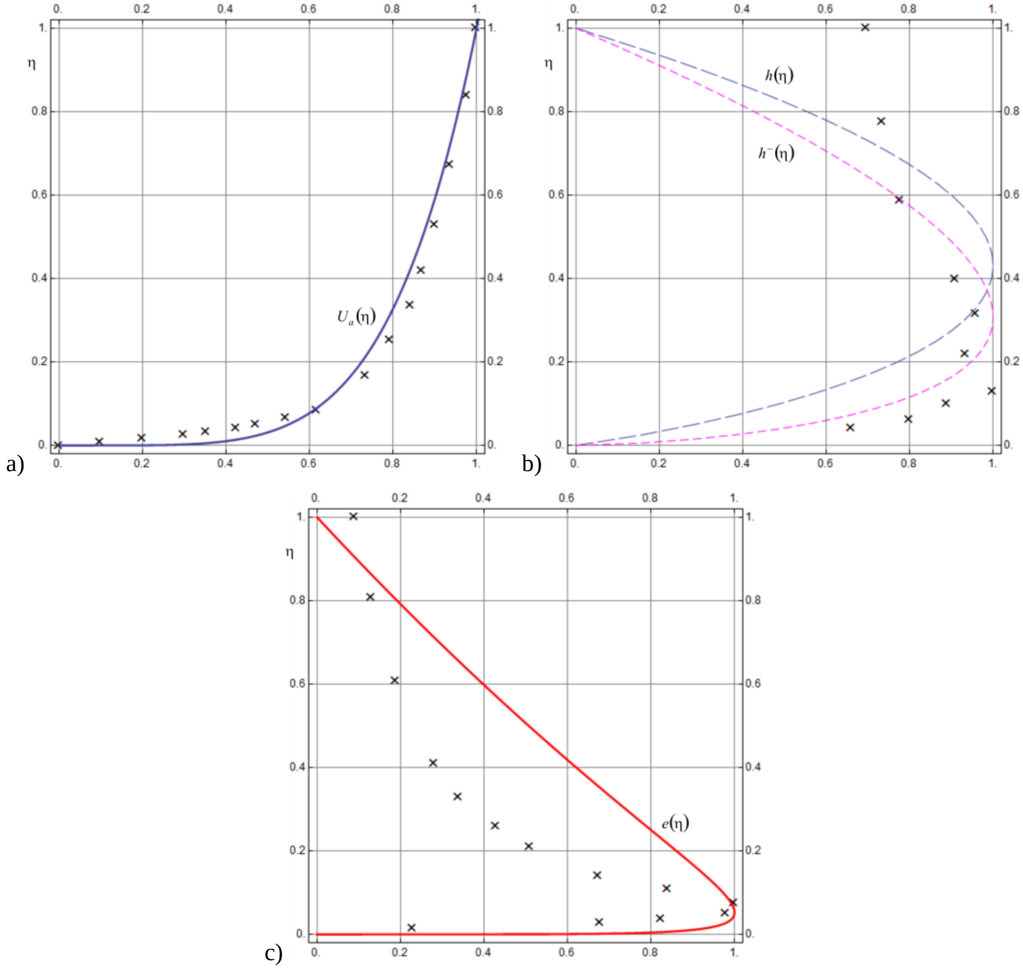


Fig. 1 – Correlations between the Fluons model and the experimental results for a turbulent pipe flow [4]
a) experimental mean velocity profile and approximated $U_a(\eta) = \eta^n$
b) experimental $\overline{v'v'}(\eta)$ and corresponding theoretical $h(\eta)$ and $h^{-}(\eta)$ distributions
c) experimental $\overline{u'u'} + \overline{v'v'} + \overline{w'w'}$ and corresponding theoretical $e(\eta)$ distribution

Also, we analyze qualitatively the hot-wire oscillograms for various wall layer flows by using the mass conservation principle, valid at any scale. We can identify some traces of the IPmMD, as it follows:

a) The general rule of intersection of the distributions $U_a(\eta)$, $U_a^{*}(\eta)$ and $U_a^{\oplus}(\eta)$ in a η_i point, which physically shows the change of the $\Delta(\eta)$ sign and is quoted as *intermittency* change;

b) The general tendency of fluctuations to behave as Δ, i.e. $U_e^{/} < \overline{U_e}$ at $\eta = 1$ (center of the channel, pipe and δ_L in boundary-layer) and $U^{/} > \overline{U}$ near walls [5];

c) The general tendency of time-variation, which has a bursting type for the $\mathrm{d}\Delta_0^*/\mathrm{d}t$, especially near walls where $|\Delta_0^*|$ ($\eta < \eta_i$) attains its maximum and needs a short time in order to belong to the associate distribution $U_a^*(\eta)$ [5]. To the contrary, the return from $U_a^*(\eta)$ towards $U_a(\eta)$ is slower, because $U_a(\eta)$ is a mean from many instantaneous distributions, each of which needed some time in order to "belong" to it. The oscillograms show this type of "return" as step-by-step.

d) Finally, we have to mention the symmetry of $\Delta(\eta)$ around $\Delta = 0$ for some $U_a(\eta)$ distributions, like $U_a(\eta) = \eta(2-\eta)$ for the wall flows (boundary-layer, 2D channel, pipe flows), which means a minimum of the fluctuations associated to these $\Delta(\eta)$, and quoted in the usual flow physics as laminar.

This symmetry of the $\Delta(\eta)$ distributions is approximately preserved for the corresponding $U_a^*(\eta)$, given by the expression:

$$\eta = \frac{1}{8} X^*\left[20 - 15X^* + 3(X^*)^2\right],\ X^* = 1 - \sqrt{1 - U_a^*} \tag{13}$$

and which is very near of a $U_{a,T}(\eta)$ distribution, quoted as turbulent (figure 2).

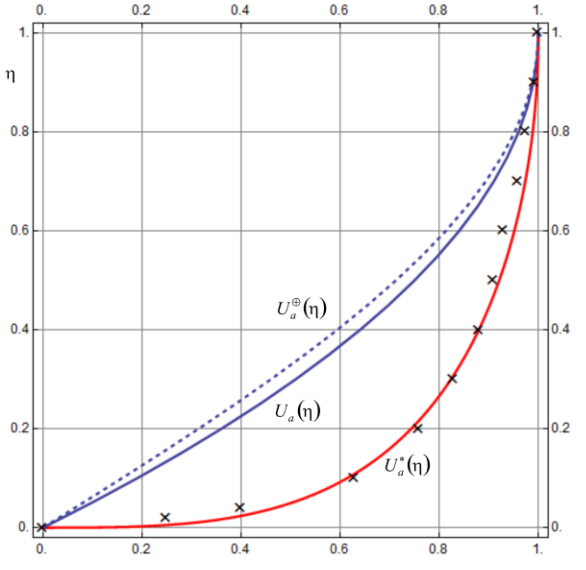


Fig. 2 – Correlation between the $U_a^*(\eta)$ and *turbulent* experimental data [5]

Below we present some qualitative results concerning the field of fluctuations [4] [5] and the Fluons model as a mathematical tool for IPmMD.

First of all, the existence of a finite domain for the physical field of turbulent fluctuations (like the observed turbulent spot) can be explained by the limited space/volume necessary to transform the laminar mean velocity into fluctuations. Therefore we propose the relation:

$$\frac{\text{Spot volume}}{\delta_L^3} \approx \frac{\int_0^\delta U_L^2 \mathrm{d}y}{\int_0^\delta \left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right) \mathrm{d}y} \tag{14}$$

for a boundary-layer flow. The experimental results [5] confirm this relation.

In order to have a quantitative value of the integral:

$$k_I = \frac{1}{2}\int_0^\delta \left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right) \mathrm{d}y, \tag{15}$$

we refer to our previous works [1] [2]. Essentially, by using the mass flow conservation at the respective domain ($0 < y < \delta$ for a boundary layer flow, $0 < y < b$ for a 2D channel flow, $0 < y = R - r < R$ for a pipe flow), we can calculate the variation of the kinetic energy $E = \rho U^2/2$, quoted as a measure of fluctuations. We get the relations:

a) for a boundary layer flow ($U_e^* = U_e$, $\delta^* \neq \delta$):

$$\frac{\delta^*}{\delta} = \frac{\int_0^1 (1 - U_a)\mathrm{d}\eta_0}{\int_0^1 (1 - U_a^*)\mathrm{d}\eta_0^*} \tag{16}$$

$$\frac{\Delta E}{\delta U_e^2} = \frac{\delta^*}{\delta}\int_0^1 \left[1 - (U_a^*)^2\right]\mathrm{d}\eta_0^* - \int_0^1 (1 - U_a^2)\mathrm{d}\eta_0 = \varphi_{U,BL} \tag{17}$$

b) for a 2D channel flow ($U_e^* \neq U_e$, $\delta^* = \delta = b$):

$$\frac{U_e^*}{U_e} = \frac{\int_0^1 U_a \,\mathrm{d}\eta_0}{\int_0^1 U_a^* \,\mathrm{d}\eta_0} \tag{18}$$

$$\frac{\Delta E}{b U_e^2} = \int_0^1 U_a^2 \,\mathrm{d}\eta_0 - \frac{U_e^*}{U_e}\int_0^1 (U_a^*)^2 \,\mathrm{d}\eta_0 = \varphi_{U,CH} \tag{19}$$

c) for a pipe flow ($U_e^* \neq U_e, \delta^* = \delta = R$):

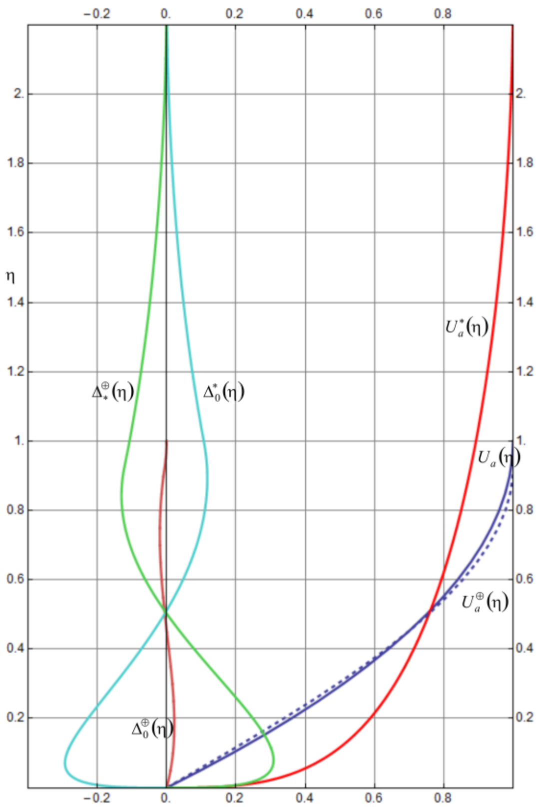
$$\frac{U_e^*}{U_e} = \frac{\int_0^1 (1 - \eta_0) U_a \,\mathrm{d}\eta_0}{\int_0^1 (1 - \eta_0) U_a^* \,\mathrm{d}\eta_0} \tag{20}$$

$$\frac{\Delta E}{\pi R^2 U_e^2} = 2\int_0^1 (1 - \eta_0) U_a^2 \,\mathrm{d}\eta_0 - 2\int_0^1 (1 - \eta_0)(U_a^*)^2 \,\mathrm{d}\eta_0 = \varphi_{U,PIPE} \tag{21}$$

The experimental results generally confirm this method to evaluate k_I, given by equation (15), as well as δ^*/δ, given by equation (16).

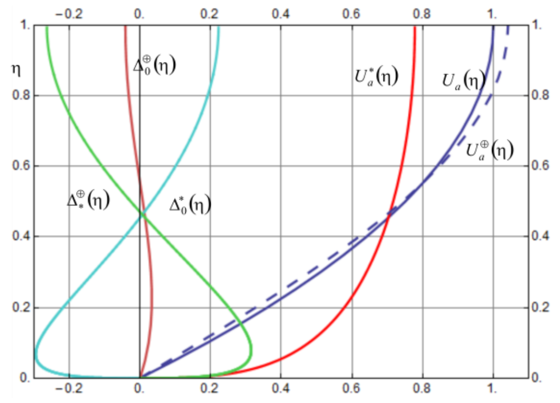
We have to mention that we can quantitatively obtain results confirmed by the experimental data concerning the fluctuations field only by analytical calculations. Accordingly, we start with the laminar velocity distribution, for example $U_a(\eta) = \eta(2-\eta)$, and get $U_{a,T}(\eta)$, which can be used in the above formulae. This is a theoretical method, which doesn't need semi-empirical expressions for $\overline{u'v'}$ and the appropriate constants.

The results for the flat plate boundary layer flow, the 2D channel flow, and the pipe flow are presented in figure 3.

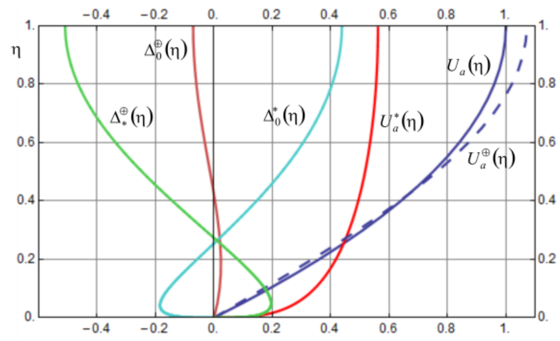


a) Flat plate boundary layer flow

$$\frac{\delta^*}{\delta} = \frac{7}{3}, \frac{\delta^{\oplus}}{\delta} = \frac{25}{27}$$



b) 2D Channel flow



c) Pipe flow

Fig. 3 – Theoretical results using the initial distribution $U_a(\eta) = \eta(2-\eta)$

a) Flat plate boundary layer flow ($U_e^* = U_e$, $\delta^* \neq \delta$)

b) 2D Channel flow ($U_e^* \neq U_e$, $\delta^* = \delta = b$)

c) Pipe flow ($U_e^* \neq U_e$, $\delta^* = \delta = R$)

3. CONCLUSIVE REMARKS

This paper presents the analysis of the field of turbulent wall flows fluctuations provided by the usual experimental devices and the mathematical Fluons model, which stands for special properties of our IDF of Navier-Stokes equations [1] [2]. The main conclusion points out the existence of a physical interaction between the micro and the macro flow domain structures (IPmMD), in spite of the difficulties to identify it by the actual experimental technologies.

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