Absolute stability for the lateral-directional BWB model with rate limited actuator

Claudia STATE*,[1], Ionel IORGA[1]

*Corresponding author
*,[1]University of Craiova
Alexandru Ioan Cuza Str. 13, 200585 Craiova, Romania
cldstate@yahoo.com

DOI: 10.13111/2066-8201.2012.4.2.12

Abstract: *In this paper the authors present a study regarding the interaction between the human pilot and the aircraft which may result in a dangerous phenomenon called Pilot Induced Oscillations (PIO), in the context of the lateral directional motion. The theoretical model of the airplane used is a Blended Wing Body (BWB) configuration and the human operator is expressed by the Synchronous Pilot Model (represented by a simple gain). The Popov criterion, in the case of the infinite parameter, is applied in order to investigate the absolute stability of the pilot-airplane linearized system in the presence of the rate saturation of the actuator.*

Keywords: *oscillations; Lurie Control Problem; Popov Criterion; Blended Wing Body configuration*

1. INTRODUCTION

The pilot induced oscillations (shortly PIO) are a potentially dangerous phenomenon which is described in the scientific literature as "sustained or uncontrollable oscillations resulting from the efforts of the pilot to control the airplane" (***, 1980)[1], or, for example, as a "misadaptation between the pilot and the aircraft during some task in which tight closed loop control of the aircraft is required from the pilot" (Amato, 1999)[2].

1.1 TYPES OF PIO

The following types of PIO are known (the reader can consult [6]):

- **Category I**: Linear oscillations of the pilot-vehicle system, resulted from excessive lags introduced by filters, actuators, feel system and digital system time delays.
- **Category II**: Quasi-linear oscillations which are mainly due to actuator rate limiting.
- **Category III**: Severe life-threatening PIO which are caused by nonlinearities and transitions in pilot or effective airplane dynamics.
- **Category IV**: Highly non-linear oscillations which are theoretically considered and less studied.

1.2 AN EXAMPLE OF PIO: JAS-39

In February 1989 the first JAS-39 crash due to PIO was happened during landing approach. The event was initiated by an inadvertent response of the pilot-aircraft system to a lateral

turbulence: the lateral PIO coupled in the pitch axis which resulted in a crash. After the impact the test pilot suffered a broken arm and the aircraft was destroyed.

Figure 1 – JAS-39 Gripen in flight

The second JAS-39 accident took place in August 1993, in the mid flight, and the same test pilot escaped by ejecting himself from the airplane.

The actuator rate limiting played an important role in both accidents of the JAS-39 but, from parametric variations of the stick inputs in both pitch and roll, later, a criterion was developed in order to allow the pilot - aircraft system to work near the margins of the rate limit (for this type of aircraft a "fix" subsystem has been developed in order to prevent the rate limiting – the reader can consult [2], pg. 13).

Both accidents showed the PIO II phenomenon.

2. THE BWB CONCEPT

Blended Wing Body (BWB) aircraft, which may also simply be called Blended Wing aircraft model, has an airframe design which incorporates design features of futuristic fuselage with a flying wing.

An intuitive presentation of the Blended Wing aircraft concept can be found, in [7] pp. 71-92. Also, several researches regarding the BWB are, for example, [8] and [4].

Among the advantages of the BWB configuration there are: less fuel consumption and reduced radar signature (in the absence of the conventional tail).

The original BWB configuration conceived by McDonnel Douglas team can be seen in the next figure.

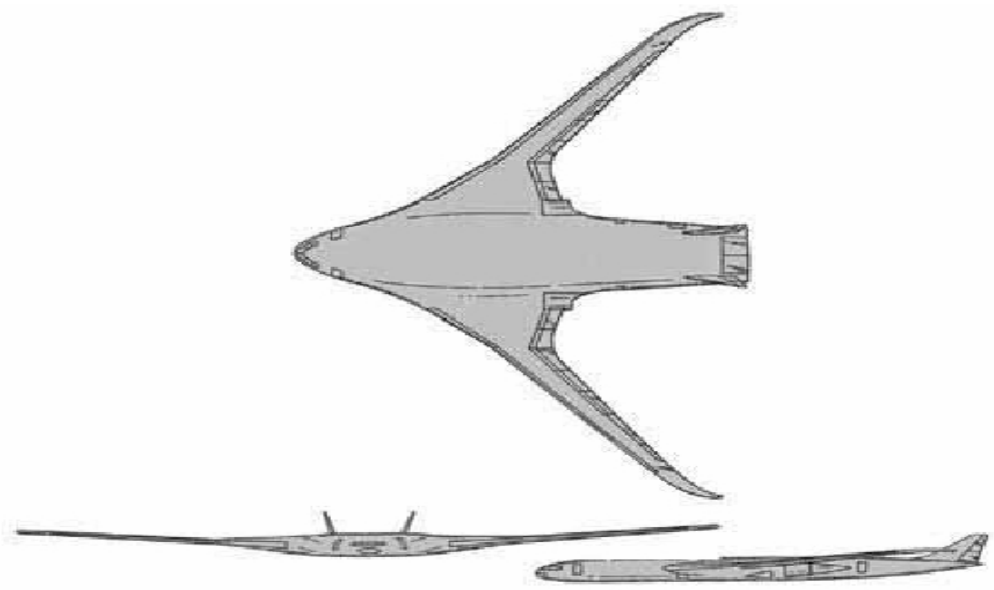

Figure -2 – First BWB configuration (pg. 78 from [7])

2.1 THE LOW-ORDER LATERAL-DIRECTIONAL BWB SYSTEM

From [9], pg. 121, the following lateral-directional BWB system is considered:

$$\begin{cases} \dot{v} = Y_v v - rU_1 + pW_1 + Y_p p + Y_r r + g\cos\theta_1 \sin\varphi + Y_{\delta_r}\delta_r \\ \dot{p} = L_v v + L_p p + L_r r + L_{\delta_a}\delta_a + L_{\delta_r}\delta_r \\ \dot{r} = N_v v + N_p p + N_r r + N_{\delta_a}\delta_a + N_{\delta_r}\delta_r \\ \dot{\varphi} = p + r\cos\varphi\tan\theta_1 \end{cases} \tag{1}$$

where:

- U_1 - axial velocity;
- v - lateral velocity components of the cg (center of gravity);
- W_1 - normal velocity;
- φ - roll angle of the aircraft (rad);
- p - roll rate (rad/s);
- r - yaw rate (rad/s).

The numeric values for the side force (Y), rolling moment (L) and yawing moment (N) are given bellow:

- $Y_v = -0.0247$, $Y_p = -2.862676$, $Y_r = -1.5129453$
- $L_v = -0.00751354$, $L_p = -1.980448$, $L_r = 0.791021$
- $N_v = -0.00138587$, $N_p = -0.406887$, $N_r = -0.04860798$

- $W_1 = 25.2547\dfrac{m}{s}$, $U_1 = 101.1471\dfrac{m}{s}$, $\theta_1 = 0.1907$
- $Y_{\delta_r} = 0.5510$, $L_{\delta_r} = 0.1162$, $N_{\delta_r} = -0.0404$

$$Y_v \approx 0,\ L_v \approx 0,\ N_v \approx 0$$
$$\dot{\delta}_r = \omega_0 \psi\left(\bar{\delta}_r - \delta_r + k_r\left(\bar{r} - r\right)\right) \tag{2}$$

The system analysed is rewritten as the system below - with one input (δ_r). This is partly motivated by the fact that the BWB configuration doesn't have a conventional tail, but lateral fins tails are used instead.

$$\begin{cases} \dot{v} = (W_1 + Y_p)p + (Y_r - U_1)r + g\cos\theta_1 \sin\varphi + Y_{\delta_r}\delta_r \\ \dot{p} = L_p p + L_r r + L_{\delta_r}\delta_r \\ \dot{r} = N_p p + N_r r + N_{\delta_r}\delta_r \\ \dot{\varphi} = p + r\cos\varphi\tan\theta_1 \end{cases} \tag{3}$$

The above system is linearized around the trim point $(\bar{v}, \bar{p}, \bar{r}, \bar{\varphi})$ and, for the system obtained only the second and third equations are kept

$$\begin{cases} \dot{\pi} = L_p\pi + L_r\rho + L_{\delta_r}\Delta\delta_r \\ \dot{\rho} = N_p\pi + N_r\rho + N_{\delta_r}\Delta\delta_r \end{cases} \tag{4}$$

where: $\pi = \Delta p = p - \bar{p}$; $\rho = \Delta r = r - \bar{r}$ and $\Delta\delta_r = \delta_r - \bar{\delta}_r$.

For the above system the entities from the state - space representation are: $\boldsymbol{x}_\beta = \begin{pmatrix} \pi \\ \rho \end{pmatrix}$, $\boldsymbol{A}_\beta = \begin{pmatrix} L_p & L_r \\ N_p & N_r \end{pmatrix}$, $\mathbf{b}_\beta = \begin{pmatrix} L_{\delta_r} & N_{\delta_r} \end{pmatrix}^{\mathrm{T}}$ and $\boldsymbol{c}_\beta^{\mathrm{T}} = \begin{pmatrix} 0 & 1 \end{pmatrix}$, hence the transfer function is

$$\mathrm{G}(s) = \boldsymbol{c}_\beta^{\mathrm{T}}\left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)^{-1}\boldsymbol{b}_\beta = \boldsymbol{c}_\beta^{\mathrm{T}}\frac{1}{\det(s\boldsymbol{I} - \boldsymbol{A}_\beta)}\left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)^{*}\boldsymbol{b}_\beta \tag{5}$$

where

$$\det(s\boldsymbol{I} - \boldsymbol{A}_\beta) = (s - L_p)(s - N_r) - N_p L_r \tag{6}$$

resulting

$$\tilde{\rho}(s) = \frac{sN_{\delta_r} + N_p L_{\delta_r} - L_p N_{\delta_r}}{\det\left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)}\tilde{\Delta}\delta_r(s) \tag{7}$$

For the system (4) the input $\Delta\delta_r$ is introduced as a state, resulting

$$\begin{cases} \dot{\pi} = L_p \pi + L_r \rho + L_{\delta_r} \Delta\delta_r \\ \dot{\rho} = N_p \pi + N_r \rho + N_{\delta_r} \Delta\delta_r \\ \dot{\Delta\delta}_r = -\omega_0 \psi\left(k_r \rho + \Delta\delta_r\right) \end{cases} \tag{8}$$

with output

$$y(t) = k_r \rho(t) + \Delta\delta_r(t) \tag{9}$$

For the above system the elements from the state - space representation are:

$$\boldsymbol{x}_\alpha = \begin{pmatrix} \pi \\ \rho \\ \Delta\delta_r \end{pmatrix}, \ \mathbf{A}_\alpha = \begin{pmatrix} L_p & L_r & L_{\delta_r} \\ N_p & N_r & N_{\delta_r} \\ 0 & 0 & 0 \end{pmatrix}, \ \boldsymbol{b}_\alpha = \begin{pmatrix} 0 & 0 & \omega_0 \end{pmatrix}^{\mathrm{T}} \text{ and } \mathbf{c}_\alpha{}^{\mathrm{T}} = \begin{pmatrix} 0 & k_r & 1 \end{pmatrix}$$

Remark 2.1.1 The nonlinear function ψ denotes a saturation function**,** ω_0 has the value of *20 rad/sec*, $k_r = 1.673$ and the following notation is made

$$\psi\left(y(t)\right) = -u(t) \tag{10}$$

The transfer function is computed from

$$\mathrm{T}(s) = \boldsymbol{c}_\alpha{}^{\mathrm{T}} \left(s\boldsymbol{I} - \boldsymbol{A}_\alpha\right)^{-1} \boldsymbol{b}_\alpha \tag{11}$$

$$\tilde{y}(s) = \boldsymbol{c}_\alpha^{\mathrm{T}} \frac{1}{\det\left(s\boldsymbol{I} - \boldsymbol{A}_\alpha\right)} \left(s\boldsymbol{I} - \boldsymbol{A}_\alpha\right)^* \begin{pmatrix} 0 \\ 0 \\ \omega_0 \end{pmatrix} \tilde{u}(s) \tag{12}$$

Remark 2.1.2 $\tilde{y}(s)$ is computed in a general form in the last relation of the subsection 3.2 and in detail in the first relation of the subsection 4.1.

2.2 THE DEFLECTION OF THE FLIGHT CONTROL SURFACE

In Figure 3 below, the first order transfer function of an actuator is shown

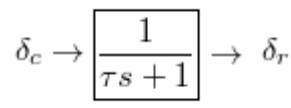

Figure 3 – First order transfer function of an actuator

where:

- τ is a time constant ;
- δ_c represents the input due to the stick;
- δ_r is the deflection of the flight control surface.

The following is used

$$\frac{\delta_r}{\delta_c} = \frac{1}{\tau s + 1} = \frac{\frac{1}{\tau}}{s + \frac{1}{\tau}} = \frac{\omega_0}{s + \omega_0} \tag{13}$$

resulting

$$\dot{\delta}_r + \omega_0 \delta_r = \omega_0 \delta_c \tag{14}$$

which is equivalent to:

$$\dot{\delta}_r = \omega_0 (\delta_c - \delta_r) \tag{15}$$

The rate saturation of the deflection of the flight control surface δ_r is defined in the following:

$$\dot{\delta}_r = \begin{cases} \left|\dot{\delta}_r\right|, & if \quad \left|\dot{\delta}_r\right| < V_L \\ V_L, & if \quad \dot{\delta}_r \geq V_L \\ -V_L, & if \quad \dot{\delta}_r \leq -V_L \end{cases} \tag{16}$$

where V_L is the rate limit value (in the test case we assume it equal to 1.57 $\frac{rad}{s}$).

3. ABSOLUTE STABILITY PROBLEM

Absolute stability refers to the global asymptotic stability of the zero equilibrium point of the general nonlinear system

$$\dot{\boldsymbol{x}}_\alpha(t) = \boldsymbol{A}_\alpha \boldsymbol{x}_\alpha(t) - \boldsymbol{b}\psi(\boldsymbol{c}_\alpha{}^{\mathrm{T}} \boldsymbol{x}_\alpha(t)) \tag{17}$$

having sector restricted nonlinearities of the form (see Figure 6):

$$0 \leq \underline{\psi} \leq \frac{\psi(y)}{y} \leq \overline{\psi} \leq \infty, \ \psi(0) = 0 \tag{18}$$

and the property of the equilibrium being valid for all the linear and nonlinear functions verifying (17).

For further reading one can consult, for example, the paper [14].

Without entering in further details, the definition of the global asimptotic stability property of dynamical system can be taken from the reference [13], pg. 66.

The Popov Criterion is used in order to answer the question if the linearized pilot-airplane system associated to the nonlinear one – in the case under discussion, the system (4) which is linearised from (3) – is absolutely stable in the presence of saturations (imposed by the dynamics of the actuator – specifically the rate limiter), for the mentioned conditions.

3.1 GENERIC BLOCKS OF THE PILOT-AIRCRAFT SYSTEM

General stabilization of the pilot-aircraft system, using the rate limiter is shown in Figure 4, in which the limiter is of "AIAA type" (as in [5], p. 166) and χ represents the output of the system.

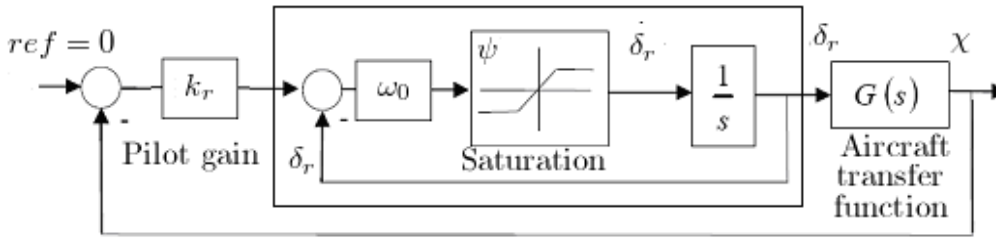

Figure 4 – Generic pilot-aircraft coupled block system with rate limiter

One should be aware that in this paper δ_r represents the rudder deflection (the flight control surface is the rudder).

The plane transfer function $G(s)$ is expressed by:

$$G(s) = \boldsymbol{c}_\beta^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A}_\beta)^{-1}\boldsymbol{b}_\beta \tag{19}$$

and, throughout this paper, by ψ a nonlinear function is denoted (a saturation, like in Figure 6 below), which fulfills the following sector condition:

$$\underline{\psi} \le \frac{\psi(y)}{y} \le \overline{\psi} \tag{20}$$

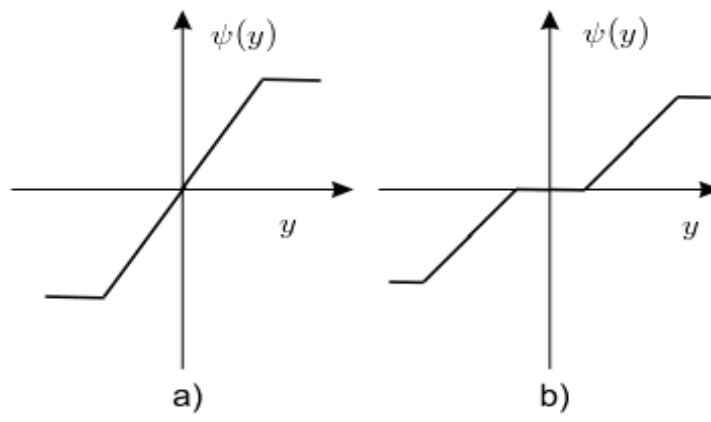

Figure 5 – Saturation nonlinearities: a) standard; b) with deadzone

An examination of the graphics of the above figure shows that these functions are *sector restricted*, i.e., belong to the class described by the next figure.

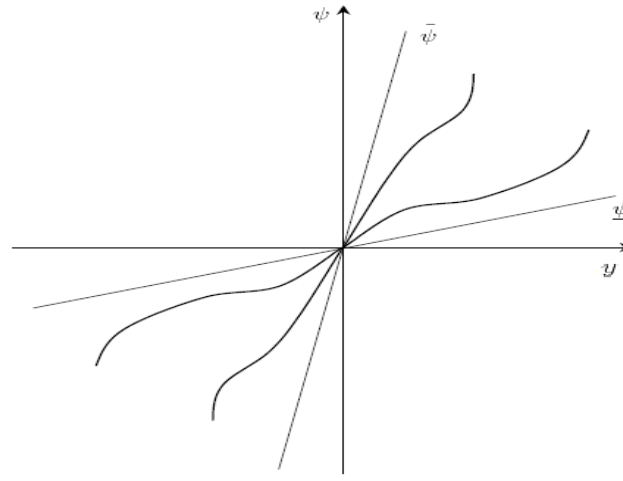

Figure 6 – Sector restricted nonlinearity

3.2 LURIE CONTROL PROBLEM

The following transformation (adapted from [12], pp. 57-58) of the system who has rate saturation can be written as in Figure 7 below. In the example considered in this paper, the system (4)+(2) is rewritten using the equivalence (as a form) between (21) and (22), resulting the system (8)

$$\dot{\boldsymbol{x}}_\alpha(t) = \boldsymbol{A}_\alpha \boldsymbol{x}_\alpha(t) - \boldsymbol{b}_\alpha \psi(y(t)) \tag{21}$$

where

$$y(t) = \boldsymbol{c}_\alpha{}^{\mathrm{T}} \boldsymbol{x}_\alpha(t)$$

and

$$\begin{cases} \dot{\boldsymbol{x}}_\beta(t) = \boldsymbol{A}_\beta \boldsymbol{x}_\beta(t) + \boldsymbol{b}_\beta \delta_e(t) \\ \dot{\delta}_e(t) = -\omega_0 \psi(\boldsymbol{c}_\beta{}^{\mathrm{T}} \boldsymbol{x}_\beta(t) + \delta_e(t)) \end{cases} \tag{22}$$

where

- $\boldsymbol{A}_\beta$ is a Hurwitz matrix;
- $\boldsymbol{c}_\beta$, $\boldsymbol{A}_\beta$, $\boldsymbol{b}_\beta$ are smaller in dimension than in (21).

Remark 3.2 In the example of this subsection $\boldsymbol{c}_\beta{}^{\mathrm{T}}$ has the following form

$$\boldsymbol{c}_\beta{}^{\mathrm{T}} = \begin{pmatrix} 0 & k_r \end{pmatrix}$$

For the system

$$\dot{\boldsymbol{x}}_\beta(t) = \boldsymbol{A}_\beta \boldsymbol{x}_\beta(t) + \boldsymbol{b}_\beta \delta_e(t) \tag{23}$$

δ_e is introduced as a state:

$$\begin{cases} \dot{\boldsymbol{x}}_\beta(t) = \boldsymbol{A}_\beta \boldsymbol{x}_\beta(t) + \boldsymbol{b}_\beta \delta_e(t) \\ \dot{\delta}_e(t) = -\omega_0 \psi(y(t)) \end{cases} \tag{24}$$

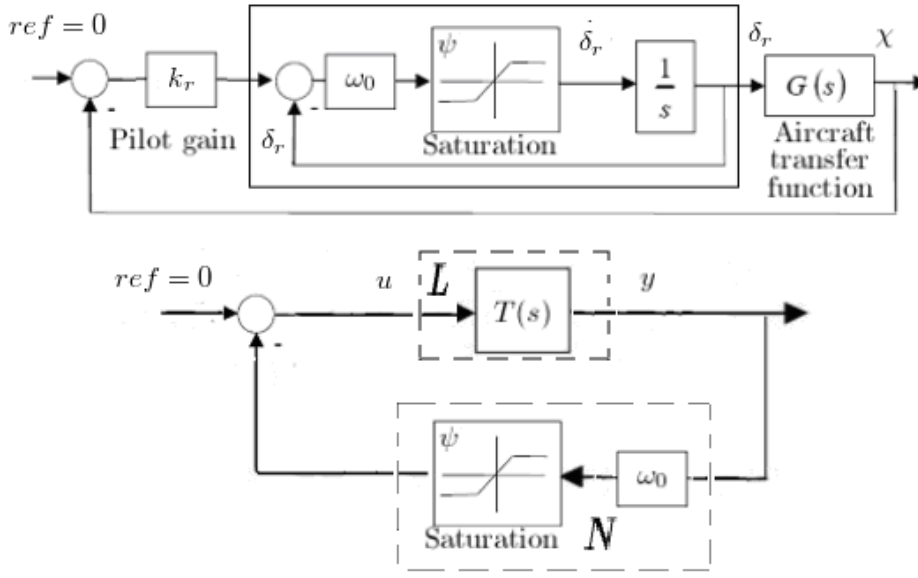

Figure 7 – Equivalence between systems who have rate saturation

where the following notations were used:

$$\begin{cases} y(t) = \boldsymbol{c}_\beta^{\mathrm{T}} \boldsymbol{x}_\beta(t) + \delta_e(t) \\ \psi(y(t)) = -u(t) \end{cases} \tag{25}$$

Applying the Laplace transform

$$\begin{cases} s\tilde{\boldsymbol{x}}_\beta(s) = \boldsymbol{A}_\beta \tilde{\boldsymbol{x}}_\beta(s) + \boldsymbol{b}_\beta \tilde{\delta}_e(s) \\ s\tilde{\delta}_e(s) = \omega_0 \tilde{u}(s) \end{cases} \tag{26}$$

$$\tilde{y}(s) = \boldsymbol{c}_\beta^{\mathrm{T}} \tilde{\boldsymbol{x}}_\beta(s) + \tilde{\delta}_e(s) \tag{27}$$

From (26) results:

$$\begin{cases} \tilde{\boldsymbol{x}}_\beta(s) = (s\boldsymbol{I} - \boldsymbol{A}_\beta)^{-1} \boldsymbol{b}_\beta \tilde{\delta}_e(s) \\ \tilde{\delta}_e(s) = \dfrac{\omega_0}{s} \tilde{u}(s) \end{cases} \tag{28}$$

From the above system is obtained

$$\tilde{\boldsymbol{x}}_\beta(s) = \frac{\omega_0}{s} (s\boldsymbol{I} - \boldsymbol{A}_\beta)^{-1} \boldsymbol{b}_\beta \tilde{u}(s) \tag{29}$$

From (27), (28) and (29) results

$$\tilde{y}(s) = \boldsymbol{c}_\beta^{\mathrm{T}} (s\boldsymbol{I} - \boldsymbol{A}_\beta)^{-1} \boldsymbol{b}_\beta \frac{\omega_0}{s} \tilde{u}(s) + \frac{\omega_0}{s} \tilde{u}(s) \tag{30}$$

Using the notation

$$\mathrm{G}(s) = \boldsymbol{c}_\beta^{\mathrm{T}} \left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)^{-1} \boldsymbol{b}_\beta = \begin{pmatrix} 0 & k_r \end{pmatrix} \left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)^{-1} \boldsymbol{b}_\beta \tag{31}$$

from (30) and (31) the following relation is determined

$$\tilde{y}(s) = \mathrm{G}(s)\frac{\omega_0}{s}\tilde{u}(s) + \frac{\omega_0}{s}\tilde{u}(s) \tag{32}$$

which is equivalent to

$$\begin{aligned} \tilde{y}(s) &= \mathrm{T}(s)\tilde{u}(s) = \omega_0\left(\mathrm{G}(s)\frac{1}{s} + \frac{1}{s}\right)\tilde{u}(s) \\ &= \omega_0\left(\begin{pmatrix} 0 & k_r \end{pmatrix}\left(s\boldsymbol{I} - \boldsymbol{A}_\beta\right)^{-1}\boldsymbol{b}_\beta\frac{1}{s} + \frac{1}{s}\right)\tilde{u}(s) \end{aligned} \tag{33}$$

From (33) we can see that the transfer function is in the critical case of one simple pole in origin (the transfer function denominator – the characteristic polynomial - has all the roots in C^- with the exception of one which is zero).

4. THE POPOV CRITERION, APPLICATION AND CONCLUSIONS

For practical considerations the following expression of the Popov criterion is used (from [10], pg. 246):

Theorem 4.1. *Consider a Lurie system with a nonlinearity* ψ *in the sector* $[0,\ k]$. *The equilibrium in the origin is globaly asymptotically (exponentially) stable, provided that there exists* $\xi > 0$ *such that the following is true:*

$$\frac{1}{k} + \mathrm{Re}\left[\left(1 + j\omega\xi\right)\mathrm{T}\left(j\omega\right)\right] > 0, \forall\omega \in \mathbb{R} \tag{34}$$

The Popov condition express a frequency condition for the global asymptotically stability property of a dynamically system in the condition of the Lurie problem (a good reference for the Lurie problem is [11], pp. 263-264).

Remark 4.1 From the general theory of functions of a complex variable with real coefficients (for example rational meromorphic functions) it is known that the real part of a transfer function is even (in ω) and the imaginary part is odd (but when multiplied with ω it is also even), so results that the above relation is even and then the condition $\omega \geq 0$ is not restrictive:

$$\frac{1}{k} + \mathrm{Re}\left[\left(1 + j\omega\xi\right)\mathrm{T}\left(j\omega\right)\right] > 0, \forall\omega \geq 0 \tag{35}$$

One should note that in relation (35) by multiplying with $\frac{1}{\xi}$, which is a positive quantity, the following inequality is equivalent.

$$\frac{1}{k\xi}+\frac{1}{\xi}\mathrm{Re}\left[\left(1+j\omega\xi\right)\mathrm{T}\left(j\omega\right)\right]=$$
$$\frac{1}{k\xi}+\frac{1}{\xi}\mathrm{Re}\left(\mathrm{T}\left(j\omega\right)\right)-\omega\,\mathrm{Im}\left(\mathrm{T}\left(j\omega\right)\right)>0 \tag{36}$$

For the above formally obtained relation, applying the limit $\xi\to\infty$ yields to:

$$-\omega\,\mathrm{Im}\left[\mathrm{T}\left(j\omega\right)\right]>0,\forall\omega\geq 0 \tag{37}$$

Remark 4.2 One should note that even when the limit is applied, the condition $\xi>0$ is valid, so the obtained frequency condition is legitimate (for use examples of the 'infinite Popov parameter' the reader can consult the paper [15]).

Remark 4.3 k is the length of the sector defined by

$$k=\overline{\psi}-\underline{\psi}=V_L-0=V_L \tag{38}$$

4.1 THE POPOV CRITERION APPLIED TO BWB MODEL

From the last relations of subsections 3.2 and 2.1 the BWB transfer function associated to the Lurie problem in the case of rate limiter is:

$$\mathrm{T}\left(s\right)=\frac{\omega_0}{s}+k_r\omega_0\frac{\left(s-L_p\right)N_{\delta_r}+N_pL_{\delta_r}}{s\left[\left(s-L_p\right)\left(s-N_r\right)-N_pL_r\right]} \tag{39}$$

The notations below are made

$$\begin{cases}k_1=N_pL_{\delta_r}-L_pN_{\delta_r}\\ k_2=-\left(N_r+L_p\right)\\ k_3=L_pN_r-N_pL_r\end{cases} \tag{40}$$

and the BWB transfer function is rewritten as:

$$\mathrm{T}(s)=\frac{\omega_0}{s}+k_r\omega_0\frac{sN_{\delta_r}+k_1}{s\left(s^2+k_2s+k_3\right)} \tag{41}$$

$$\mathrm{T}(j\omega)=-\frac{j\omega_0}{\omega}+k_r\omega_0\frac{\left(j\omega N_{\delta_r}+k_1\right)\left(-j\omega\right)\left[\left(k_3-\omega^2\right)-j\omega k_2\right]}{\omega^2\left[\left(k_3-\omega^2\right)^2-\omega^2k_2^{\,2}\right]} \tag{42}$$

The above relation is equivalent to:

$$\mathrm{T}(j\omega)=k_r\omega_0\frac{\omega^2\left(N_{\delta_r}k_3-k_2k_1\right)-\omega^4N_{\delta_r}}{\omega^2\left[\left(k_3-\omega^2\right)^2+\omega^2k_2^2\right]}-j\left[k_r\omega_0\frac{\omega^3\left(N_{\delta_r}k_2-k_1\right)+\omega k_1k_3}{\omega^2\left[\left(k_3-\omega^2\right)^2+\omega^2k_2^2\right]}+\frac{\omega_0}{\omega}\right] \tag{43}$$

The frequency domain condition (37) in the presented case can be written as:

$$-\omega Im[\mathrm{T}(j\omega)] = k_r\omega_0 \frac{\omega^2\left(N_{\delta_r}k_2 - k_1\right) + k_1k_3}{\left(k_3 - \omega^2\right)^2 + \omega^2 k_2^2} + \omega_0 > 0, \forall\omega \geq 0 \tag{44}$$

Numerically,

$$k_2 N_{\delta_r} - k_1 \approx 0.049 \tag{45}$$

$$k_1k_3 \approx -0.052 \tag{46}$$

The limit below is computed

$$\lim_{\omega\to\infty} -\omega Im[\mathrm{T}(j\omega)] = \omega_0 > 0 \tag{47}$$

If the following notation is made

$$g(\omega) = \omega_0(k_r \frac{\omega^2(N_{\delta_r}k_2 - k_1) + k_1k_3}{\mathrm{P}(\omega)} + 1) \tag{48}$$

where

$$\mathrm{P}(\omega) = (k_3 - \omega^2)^2 + \omega^2 k_2^2 \tag{49}$$

the first derivative of $g(\omega)$ is

$$g^{'}(\omega) = \omega_0 \frac{2\omega\overline{\kappa}_1\mathrm{P}(\omega) - \left(\omega^2\overline{\kappa}_1 + \overline{\kappa}_2\right)\mathrm{P}'(\omega)}{\mathrm{P}(\omega)^2} \tag{50}$$

where

$$\overline{\kappa}_1 = k_2 N_{\delta_r} - k_1$$

$$\overline{\kappa}_2 = k_1k_3$$

and $\mathrm{P}'(\omega)$ is obtained by derivation from (49)

$$\mathrm{P}'(\omega) = 4\omega^3 + 2\omega\left(k_2^{\ 2} - k_3\right) \tag{51}$$

From (50) and (51) it results that, for $\hat{\omega} = 0$, $g^{'}(\hat{\omega}) = 0$ and, also $\hat{\omega} = 0$ which is a minimum of $g(\omega)$ (moreover, this is the only minimum because all the others zeros of $g'(\omega)$ are complex – by dividing the numerator of relation (50) by ω and by solving the associated quadratic equation results that her roots are complex) and if $g(\hat{\omega}) = \omega_0(k_r \frac{\hat{\omega}^2\left(N_{\delta_r}k_2 - k_1\right) + k_1k_3}{\left(k_3 - \hat{\omega}^2\right)^2 + \hat{\omega}^2 k_2^2} + 1) > 0$ then the relation (44) is true.

In order to verify the above assertion the following limit is computed

$$\lim_{\omega->0} -\omega \operatorname{Im}\left[\mathrm{T}\left(j\omega\right)\right] = \omega_0 (k_r \frac{k_1}{k_3} + 1) = 9.78 > 0 \tag{52}$$

From (44), (47) and (52) it results that (37) is true, so the BWB configuration with rate limiter actuator is absolutely stably using the Popov Criterion for the parameter $\xi = \infty$, in the specified conditions.

5. CONCLUSIONS

The absolute stability of the pilot-aircraft system, in the case of systems with rate limiter, for the BWB configuration presented, was proved in the specified conditions, using the Popov criterion - for $\xi = \infty$ parameter – in the context of the associated Lurie problem.

As a future work a time delay associated to the human pilot and/or to the digital processing units of the aircraft might be considered.

REFERENCES

[1] ****Flying Qualities of Piloted Airplanes*, (U.S. Military Specification) MIL-F-8785C, Nov. 1980.

[2] F. Amato, R. Iervolino, S. Scala and L. Verde, *Actuator design for aircraft robustness versus category PIO*, Proc. of the 7th Mediterranean Conference on Control and Automation, Haifa, Israel, June 28-30, 1999.

[3] *** *Flight Control Design - Best practices*, NATO Technical Report 29, 2000.

[4] H. Smith and Y. Abbasi, *The MOB Blended Wing Body Reference Aircraft*, Technical report, Cranfield University, UK, 2004.

[5] V. Răsvan, *Systems with monotone and slope restricted nonlinearities*, Tatra Mountains Mathematical Publications, Volume **48**, 2011, No.1: 165-187, Aivalable online at http://www.sav.sk/journals/uploads/0614105916rasvan.pdf

[6] D. T. McRuer, D. H. Klyde and T. T. Myers, *Development of a Comprehensive PIO Theory*, AIAA paper 96-3433, (1996), pp. 581-597.

[7] J. R. Chambers, *Innovation in flight*, NASA SP-2005-4539, 2005, Available on-line at http://history.nasa.gov/monograph39/mon39_a.pdf

[8] N. U. Rahman, J. F. Whidborne, *A Lateral Directional Flight Control System for the MOB Blended Wing Body Platform*. UKACC Int. Conf. CONTROL 2008, CDROM paper 67, Manchester, U.K., September 2008.

[9] C. A. State, A Linear Analysis of a Blended Body (BWB) Aircraft Model, *INCAS BULLETIN*, Vol. **3**, Issue 3, 2011, ISSN 2066-8201.

[10] S. Sastry, *Nonlinear Systems; Analysis, Stability and Control*, ISBN 0-387-98513-1, Spinger Verlag, 1999.

[11] H. K. Khalil, *Nonlinear Systems*, 3rd edition, ISBN 0-13-067389-7, Prentice Hall, 2002.

[12] V. Răsvan, *Absolute Stability of Control Systems with Delay (in Romanian)*, Editura Academiei, Bucureşti, 1975.

[13] M. Voicu, *Stability Analysis Techniques of the Automatic Control Szstems (in Romanian)*, Editura Tehnică, Bucureşti, 1986.

[14] D. Danciu, *Absolute stability conditions for some scalar nonlinear time-delay systems with monotone increasing nonlinearity*, Annals of the University of Craiova, Series Automation, Computers, Electronics and Mechatronics, ISSN 1841-06267, (34), 2, 2010, pp. 7-12.

[15] V. Răsvan, D. Danciu and D. Popescu, *On absolute (robust) stability: slope restrictions and stability multipliers*, Int. J. Robust. Nonlinear Control (2011), Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/rnc.1821.