The analysis of the process tending towards equilibrium in transitional and turbulent wall flows

Ştefan N. SĂVULESCU, Florin BĂLTĂREȚU*, Horia DUMITRESCU

*Corresponding author
Technical University of Civil Engineering
Bd. Pache Protopopescu 66, Bucharest 021414, Romania
flbaltaretu@yahoo.com
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Abstract: *Starting from the integro-differential formulation (IDF) [1], [2] of the basic conservation equations in wall flows, the authors perform a qualitative analysis of the microscopic-macroscopic domain. According to IDF, the kinetical energy presents a mean distribution and a lot of similar terms called FLUONS. Their various combinations lead to physical fluctuations and their dynamical equilibrium concerns the existence of an associated distribution of the kinetical energy, which is a solution of IDF. The global contributions of the fluctuations are, therefore, estimated as analytical formulae for the wall flows. The experiments confirm these results.*

Key Words: integro-differential formulation (IDF), transitional and turbulent wall flows, fluons.

1. INTRODUCTORY REMARKS

The quantity $k(\eta_0)=\left(\overline{u'^2}+\overline{v'^2}+\overline{w'^2}\right)/U_e^2$, associated to the mean distribution $U_a(\eta_0)$ in wall flows is determined numerically [3] and experimentally [4], [5], [6]. Theoretically, the numerical integration of the Navier-Stokes equations, essentially semi-empirical, is very laborious and dependent from ad-hoc models and constants. Also, the experiments involve a lot of work and high-tech instrumentation, able to give good results in various flows conditions (wall roughness, pollution of the fluid, the turbulence of the incident flow etc.).

Therefore, it seems interesting to obtain some analytical formulae for $k(\eta_0)$, when $U_a(\eta_0)$ is provided. It is also interesting to analyse the physical aspects related to these formulae.

According to figure 1, a quantity of fluid from a boundary layer involves N molecules with their chaotic motion [7], [8], forced to fulfill some macroscopic requirements. In this respect the basic idea refers to the continuum hypothesis, which allows to introduce arbitrarily selected particles (composed by many molecules) and to assign them quantities like velocity, pressure, temperature etc., able to ensure the mathematical conditions for the conservation laws. This situation is presented in the figure 2, for the quantity $K(n)$.

Our knowledge becomes very obscure in the domain between $K_{microscopic}$ and $K_{macroscopic}\equiv\overline{K}$, indicated by dots in figure 2. The classical statistical physics [7], [8] solves this difficult problem by using the probability theory and the concept of molecular equilibrium, leading to $\overline{K}$ and to the justification of the continuum hypothesis. However, by observing the anisotropy of velocity fluctuations in wall flows it seems that the process $K_{microscopic}\to\overline{K}$ is much more complicated.

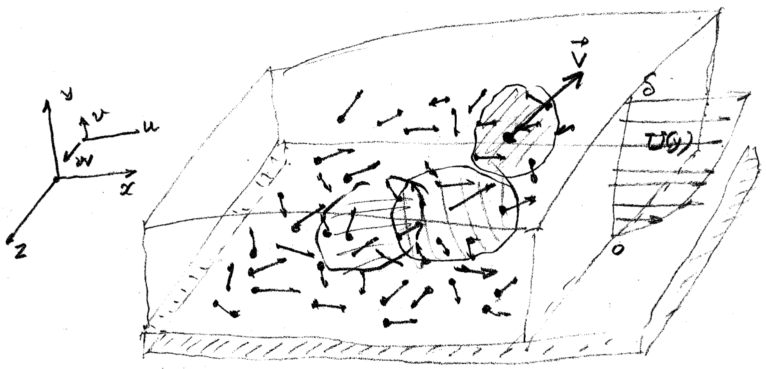

Fig. 1 – A volume of fluid in the boundary layer, characterized by the thickness $\delta(x,z,t)$ and the global velocity profile $U(y)$, with microscopic (molecular) structure and macroscopic particles (in the continuum hypothesis)

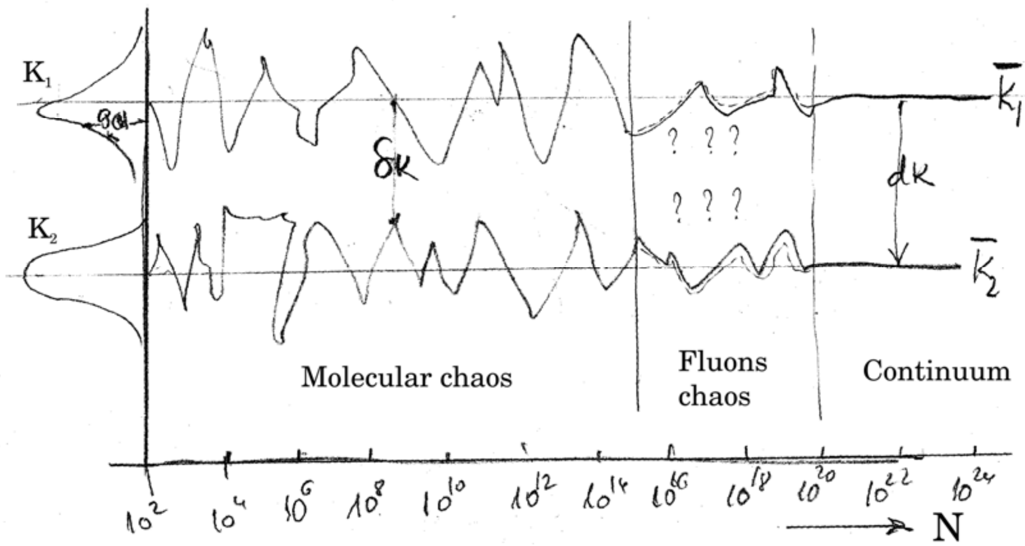

Fig. 2 – The K_1 and K_2 quantities expressed in function of the number of molecules (particles), the probability P_K, the probabilistic mean $\langle K^r \rangle = \int_{-\infty}^{+\infty} P_K K^r dK$ and the fluctuation $\langle K^r \rangle - \langle K \rangle^r$

Essentially these processes stand for the tendency to some local molecular continuum equilibrium, because in the wall flows the molecules belonging to particles having different velocities, pressure, temperature etc. interact in a much more complicated way than the usual gradient-type transfer. Can we have a look in this complex transition type equilibrium in wall flows? Are there some traces of these processes in the conservation laws we are using for wall flows?

The aim of this paper is to present a contribution to this difficult problem by using IDF.

2. QUALITATIVE ANALYSIS OF THE INTERACTION BETWEEN THE MICROSCOPIC AND THE MACROSCOPIC DOMAIN IN THE WALL FLOWS

For a wall flow, let's $K(x,y,z,t)$ be a scalar quantity which has to satisfy the general conservation equations:

$$\frac{\partial\rho}{\partial t}+\nabla(\rho\vec{\boldsymbol{v}})=0 \quad \text{or} \quad \nabla\vec{\boldsymbol{v}}=-\frac{1}{\rho}\frac{D\rho}{Dt} \tag{1}$$

$$\frac{DK}{Dt}=\alpha_K\nabla^2 K+\Sigma \tag{2}$$

where ρ represents the fluid density, and $\vec{\boldsymbol{v}}=u\vec{\boldsymbol{i}}+v\vec{\boldsymbol{j}}+w\vec{\boldsymbol{k}}$ is the velocity vector.

We used the mathematical operators:

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+(\vec{\boldsymbol{v}}\,\nabla),\quad \nabla=\frac{\partial}{\partial x}\vec{\boldsymbol{i}}+\frac{\partial}{\partial y}\vec{\boldsymbol{j}}+\frac{\partial}{\partial z}\vec{\boldsymbol{k}}\,,\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \tag{3}$$

The initial condition and the boundary conditions are:

$$\begin{cases} t=0 & K=K_{t=0}(x,y,z) \\ y=0 & K=K_0(x,z,t),\ \vec{\boldsymbol{v}}=\vec{\boldsymbol{v}}_0(x,z,t) \\ y=\delta(x,z,t) & K=K_e(x,z,t),\ \vec{\boldsymbol{v}}=\vec{\boldsymbol{v}}_e(x,z,t) \end{cases} \tag{4}$$

The term $\Sigma(x,y,z,t)$ stands for the supplementary contributions due to external fields or other effects.

For instance, by taking $K=\vec{\boldsymbol{v}}$, $\alpha_K=\nu$, $\Sigma=-\frac{1}{\rho}\nabla p$ we get the usual Navier-Stokes equations for the incompressible and isothermal wall flows (for $K=U$, $\alpha_K=\nu$, $\Sigma=-\frac{1}{\rho}\frac{\partial p}{\partial x}$ we have the x-direction Navier-Stokes equation).

Similarly, for $\alpha_K=\Delta x^2/\Delta t$, $\vec{\boldsymbol{v}}=0$ we get the one-dimensional (x) Markov process, where Σ can involve additional terms by series development of the quantities $K_{j,j}^{(n+1)}-K_{j,j}^{(n)}$ and $K_{j+1,j}^{(n)}-K_{j,j}^{(n)}$, where $j-1$, j, $j+1$ are the usual discrete steps and $(n+1)-(n)$ discrete time intervals. By writing $\frac{\Delta x^2}{\Delta t}=\Delta x\cdot\left(\frac{\Delta x}{\Delta t}\right)=\lambda_{mol}\cdot v_{mol}$ we get an evaluation of the fluid viscosity ν as the product of the molecular distance λ_{mol} and the molecular velocity v_{mol}. This is considered as a physical property and introduced in the Navier-Stokes equations.

To solve analytically these equations in continuum, one has to drastically neglect many terms (like $curl\,\vec{\boldsymbol{v}}=0$ for potential flows and $\frac{\partial^2}{\partial x^2}<<\frac{\partial^2}{\partial y^2}$ for boundary layers) or to add arbitrarily some unknown quantities (like $u=\overline{u}+u^{/}$) in the perturbative methods.

Is it possible to get some idea about the tendency towards equilibrium in the microscopic-macroscopic domain by stating from the macroscopic conservation laws?

We try to answer to this question by means of the IDF. For the equation (4) with the boundary conditions (4), using the IDF transformation of coordinates - equation (A1) in Appendix -, we get:

$$\frac{\partial}{\partial\eta}\left(\nu_a^*\frac{\partial I_K}{\partial\eta}\right)=\sum_{r=1}^{6} f_r(\psi_\delta)\sigma_r \tag{5}$$

with the corresponding boundary conditions:

$$\begin{cases}\eta=0 & I_K=I_{K,0}\\ \eta=1 & I_K=I_{K,1}\end{cases} \tag{6}$$

where:

$$\psi_\delta=\int_0^\delta \varphi\, d\eta\,,\ \frac{\partial I_K}{\partial\eta}=\varphi\frac{\partial K}{\partial\eta}\,,\ \nu_a^*=1+y_\xi^2+y_\zeta^2 \tag{7}$$

and the expressions for f_r and σ_r are given in the Appendix.

The two-fold integration $\int_0^\eta\int_1^\eta[(\ldots)d\eta]d\eta$ and the boundary conditions fulfillment lead to the relationships:

$$\nu_a^* I_K=\left(\nu_a^* I_K\right)_0+\left|\nu_a^*\frac{\partial I_K}{\partial\eta}\right|_1\eta+\int_0^\eta I_K\frac{\partial\nu_a^*}{\partial\eta}d\eta+\sum_{r=1}^{6} f_r(\psi_\delta)(\sigma_r)_1^{\oplus}(\sigma_r)^{\oplus} \tag{8}$$

$$\left(\nu_a^* I_K\right)_1=\left(\nu_a^* I_K\right)_0+\left|\nu_a^*\frac{\partial I_K}{\partial\eta}\right|_1+\int_0^\eta I_K\frac{\partial\nu_a^*}{\partial\eta}d\eta+\sum_{r=1}^{6} f_r(\psi_\delta)(\sigma_r)_1^{\oplus} \tag{9}$$

where:

$$f^{\oplus}=\int_0^\eta\left(\int_1^\eta f\,d\eta\right)d\eta\Bigg/\int_0^1\left(\int_1^\eta f\,d\eta\right)d\eta \tag{10}$$

The relation (8) and (9) stands for the IDF for the quantities I_K and ψ_δ.

At a first look, one has to point out their complexity by comparison to the relative simplicity of the original equations. However, the idea of combining both (8) and (9) leads to a qualitatively fruitful result. In order to obtain the equation corresponding to this combination, we eliminate a term, say f_r, $r\in\{1,2,\ldots,6\}$. We get:

$$I_K = \frac{1}{\nu_a^*}\left(\nu_a^* I_K\right)_1 \left(\sigma_{r,i}\right)^{\oplus} + \frac{1}{\nu_a^*}\left(\nu_a^* I_K\right)_0 \left[1-\left(\sigma_{r,i}\right)^{\oplus}\right] + \frac{1}{\nu_a^*}\left(\nu_a^* \frac{\partial I_K}{\partial \eta}\right)_1 \left[\eta-\left(\sigma_{r,i}\right)^{\oplus}\right] +$$
$$+\frac{1}{\nu_a^*}\int_0^1 I_K \frac{\partial \nu_a^*}{\partial \eta} d\eta \left[\int_0^{\eta} I_K \frac{\partial \nu_a^*}{\partial \eta} d\eta \Big/ \int_0^1 I_K \frac{\partial \nu_a^*}{\partial \eta} d\eta - \left(\sigma_{r,i}\right)^{\oplus}\right] + \tag{11}$$
$$+\frac{1}{\nu_a^*}\sum_{r=1}^{6} f_r\left(\psi_\delta\right)\left[\left(\sigma_r\right)^{\oplus} - \left(\sigma_{r,i}\right)^{\oplus}\right]\left(\sigma_r\right)_1^{\oplus}$$

that is an expression with very interesting mathematical features.

Firstly, we notice the decomposition of the quantity I_K into a “mean” and a lot of similar “differences” $f^{\oplus} - g^{\oplus} \sim \eta(1-\eta)$, which we called FLUONS [2]. Because f_r can be arbitrarily chosen, it results that there are 2^r fluons. If $r = 10^2$ we get $2^{100} \sim 10^{30}$ fluons, a figure which is similar to the number of molecules per 1 mole of gas. By realizing some equilibrium, the physics of microparticles needs a great number of similar elements and the interaction between them, like “collision”. The fluons fulfill both these requirements. The macroscopic conditions (Reynolds number, viscosity values, wall roughness etc.) lead to the appearance or disappearance of some fluons and affect their interaction as microparticles. The IDF presents a lot of similar entities (fluons) and their arbitrary combinations in a finite space-time domain. Therefore, the IDF presents a fluons chaos, similar to the molecular chaos. Both processes are intended to minimize and to uniformly distribute the energy.

Secondly, we have to mention the differences between the probability aspects in the fluons chaos and in the molecular chaos. The last one, under normal temperature and pressure conditions, is symmetrical in the Maxwell-Boltzmann probability distribution; however, the probability distribution of the fluons chaos in wall flows is non-symmetrical. Moreover, the IDF points out the quantity $\nu\left(1 + y_\xi^2 + y_\zeta^2\right)$, which plays the role of “fluctuations viscosity”, essentially non-negative.

Finally, in a finite space-time domain (finite energy) the IDF for wall layers (boundary, channel, pipe flows) points out the existence of two energy distributions, U^2 and $\left(U^*\right)^2$, related by the transformation

$$\eta = \int_0^y U^2 dy \Big/ \int_0^\delta U^2 dy \tag{12}$$

These distributions are present alternatively in this finite space-time domain and influenced by the imposed conditions to the continuous fluid. Both distributions correspond to the particle activityunder the continuum hypothesis and, respectively, to the fluons activity in the IDF. By neglecting, ab initio, the fluons activity leads to incomplete and semi-empirical methods in order to describe the dynamics of wall layers. For instance, the IDF points out the importance of the boundary conditions as contribution to the number of fluons and, consequently, to their activity.

3. THE EVALUATION OF THE GLOBAL INTENSITY OF FLUCTUATIONS IN WALL LAYERS FLOWS

In spite of the mathematical difficulties related to the IDF, we present now some examples showing its capacity to provide quantitative results. The physical difficulties are related to the meaning of fluctuations, because the splitting $U=\overline{U}+u^{/}$ is arbitrarily made and the quantity $\overline{U^m}=\overline{U}^m+f\left(\overline{U},u^{/}\right)$ involves a lot of combinations $\overline{U}^r\left(u^{/}\right)^{m-r}$. However, the IDF of the 2D incompressible, unsteady wall layer (detailed calculation in Addenda) points out two expressions for U^{1+m}, obtained by elimination of $\frac{\partial}{\partial\xi}\left(\psi^2_{m,\delta}\right)$ and, respectively, $\frac{\partial}{\partial\tau}\left(\psi^2_{m,\delta}\right)$, where $\psi_{m,\delta}=\int_0^\delta U^m dy$. The main terms are $A_\xi=\frac{\partial U_a}{\partial\eta_m}\int_0^\delta U_a^{1-m}d\eta_m$ and, respectively $A_\tau=\frac{\partial U_a}{\partial\eta_m}\int_0^\delta U_a^{-m}d\eta_m$, where $\eta_m=\frac{1}{\psi_{m,\delta}}\int_0^y U^m dy$. The differences $\Delta_\xi=U_a^{1+m}-A_\xi^{\oplus}$ and $\Delta_\tau=U_a^{1+m}-A_\tau^{\oplus}$ stands for the fluons activity. The equilibrium of the fluons' chaos can be evaluated by taking the global quantity $\int_0^1\left(A_\xi^{\oplus}-A_\tau^{\oplus}\right)d\eta_m$ as a measure of their activity. This quantity must be zero for the deterministic uniform velocity distribution. However, in real wall bounded flows, the measured fluctuations can be related to the fluons activity as a sudden change of the kinetical energy around some minimum.

In order to have a quantitative estimation of the kinetical energy involved in the fluons activity, we propose the following method:

1. By knowing a velocity distribution $U(y)$, where $0<U<U_e$, $0<y<\delta$, we use the transformation of the transversal y-coordinate into η-coordinate, given by (12), in order to get the associated distribution $U^*(y)$, where $0<U^*<U_e^*$, $0<y<\delta^*$. For instance, for $U_a=\eta_0^n$, where $U_a=U/U_e$, $\eta_0=y/\delta$, we obtain $U_a^*=\left(\eta_0^*\right)^{\frac{n}{1+2n}}$, with $\eta_0^*=y/\delta^*$ for boundary layers and $\eta_0^*=\eta_0$ for channel and for axi-symmetrical pipe flows.

2. We suppose that fluons activity doesn't influence the mean convection in a section of a wall bounded flows, by writing:

$$\int_0^\delta \rho U\,dy+\rho\left(\delta^*-\delta\right)=\int_0^{\delta^*}\rho U^*\,dy^*\text{, for boundary layers} \tag{13}$$

$$\int_0^b \rho U\,dy=\int_0^b \rho U^*\,dy\text{, for channel flows} \tag{14}$$

$$\int_0^R \rho U\,2\pi r\,dr=\int_0^R \rho U^*\,2\pi r\,dr\text{, for pipe flows, with } r=R-y \tag{15}$$

3. We evaluate the variation of the kinetical energy, by writing:

$$\frac{2\Delta E_C}{\rho}=\int_0^{\delta}U^2\,dy+\left(\delta^*-\delta\right)U_e^2-\int_0^{\delta^*}\left(U^*\right)^2 dy \text{, for boundary layers} \quad (16)$$

$$\frac{2\Delta E_C}{\rho}=\int_0^{b}U^2\,dy-\int_0^{b}\left(U^*\right)^2 dy \text{, for channel flows} \quad (17)$$

$$\frac{2\Delta E_C}{\rho}=\int_0^{R}U^2\,2\pi r\,dr-\int_0^{R}\left(U^*\right)^2 2\pi r\,dr \text{, for pipe flows} \quad (18)$$

By using the notation $Q_{a,r}=\int_0^1 U_a^r\,d\eta_0$ and $Q_{a,r}^*=\int_0^1\left(U_a^*\right)^r d\eta_0$, we obtain the relationships:

$$\frac{2\Delta E_C}{\delta\rho U_e^2}=\frac{\left(1-Q_{1a}\right)\left(Q_{2a}-Q_{1a}\right)}{Q_{2a}-Q_{3a}}-\left(1-Q_{2a}\right)\text{, for boundary layers} \quad (19)$$

$$\frac{2\Delta E_C}{b\rho U_e^2}=Q_{2a}\left(1-\frac{Q_{1a}^2}{Q_{3a}^2}Q_{4a}\right)\text{, for channel flows} \quad (20)$$

For the pipe flow, the integrals $\int_0^1\eta_0 U_a^* d\eta_0$ and $\int_0^1\eta_0\left(U_a^*\right)^2 d\eta_0$ are much more complicated in a general $U_a\left(\eta_0\right)$ case.

Table 1. The analytical formulae for $U_a=\eta_0^n$.

	Boundary Layer	Channel Flow	Pipe Flow
$\frac{\Delta E_C}{\delta U_e^2}$	$\frac{4n^3}{(1+n)(1+2n)(1+4n)}$	$\frac{4n^3}{(1+n)^2(1+2n)(1+4n)}$	$\frac{2n^3\left(10+23n+4n^2\right)}{(1+n)^2(2+n)^2(1+2n)^2(1+4n)}$
$1-\frac{U^*}{U_e}$	$1-\left(\frac{1+n}{1+3n}\right)^{\frac{n}{1+2n}}$	$\frac{2n^2}{(1+n)(1+2n)}$	$\frac{2n^2\left(3+8n+2n^2\right)}{(1+n)(2+n)(1+2n)^2}$
$\frac{\delta^*}{\delta}$	$\frac{1+3n}{1+n}$	1	1
$\eta_{0,\text{int}}$	$\left(\frac{1+n}{1+3n}\right)^{\frac{1}{2n}}$	$\left[\frac{1+3n}{(1+n)(1+2n)}\right]^{\frac{1+2n}{2n^2}}$	$\left[\frac{(1+3n)(2+5n)}{(1+n)(2+n)(1+2n)^2}\right]^{\frac{1+2n}{2n^2}}$
k_a	$\frac{n}{1+n}\left(\eta_0^{\frac{2n}{1+2n}}-\eta_0^{2n}\right)$	$\frac{n}{(1+n)^2}\left(\eta_0^{\frac{2n}{1+2n}}-\eta_0^{2n}\right)$	$k_{a,PIPE}$ - equation (18)

For axial-symmetric pipe flow ($U_e^*\neq U_e$, $\delta^*=\delta=R$) we obtain:

$$k_{a,PIPE} = \frac{2n(1+3n)(10+23n+4n^2)}{(1+n)(1+2n)(2+n)^2(3+10n+4n^2)}\left(\eta_0^{\frac{2n}{1+2n}} - \eta_0^{2n}\right) \quad (21)$$

In the paper [1], the first application of the IDF leads to the normalized distribution $\frac{k}{k_{MAX}} \cong \frac{(U_a^2)^* - U_a^2}{\left[(U_a^2)^* - U_a^2\right]_{MAX}}$. Now we can evaluate k_{MAX} by the $\frac{\Delta E_C}{\delta U_e^2}$. Accordingly, $k_a(\eta_0)$ can be provided as a distribution of the physical measured quantity $\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)/U_e^2$. The last line in the above table indicate the corresponding formula for $U_a = \eta_0^n$. We have to remark the great sensitivity of the above formulae against the $U_a(\eta_0)$ distribution.

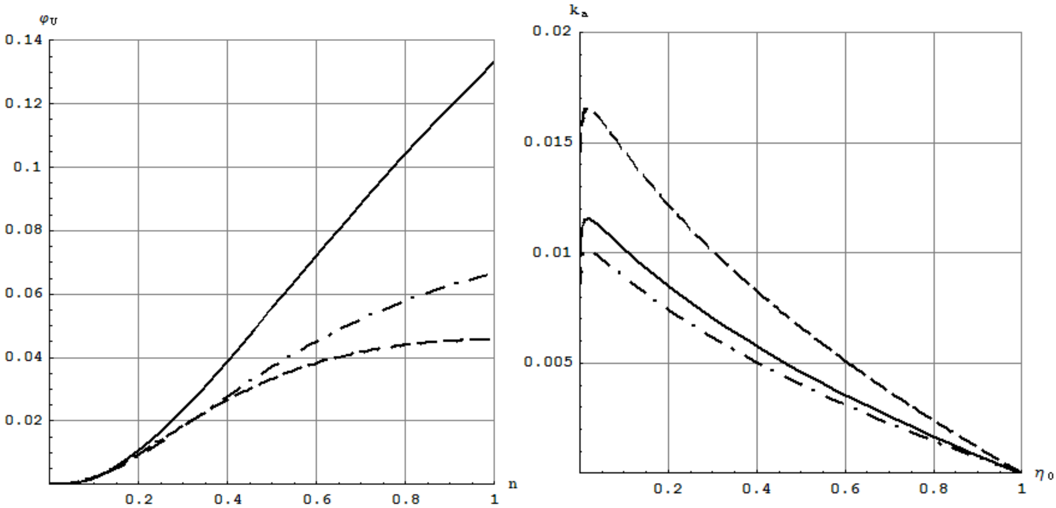

Fig. 3 – The $\varphi_U = \frac{\Delta E_C}{\delta U_e^2} = f(n)$ and the $k_a(\eta_0, n = 1/7)$ for $U_a = \eta_0^n$

——— Boundary layer
- — - — - — Channel Flow
— — — — — Pipe flow

4. CONCLUDING REMARKS

Qualitatively, the IDF of the conservation laws points out some features of the microscopic-macroscopic domain. The great number of the similar entities called fluons mimics the molecular dynamics. The physical equilibrium means a uniform distribution of a minimum energy as global quantity. The fluons activity explains the tendency toward chaotic structural dynamics (quoted as fluctuations in the continuum hypothesis) for every shear flow. The IDF shows, for instance, the importance of the macroscopic boundary conditions concerning the fluons activity ($2^{10} \sim 10^3, 2^{100} \sim 10^{30}$). In this respect, the perturbative methods ($f = \overline{f} + f'$) remain deterministic, and need arbitrary models and constants in order to evaluate the fluctuations. As an example, IDF explains the non-negativity of the correlation $-\overline{u'v'}$.

Quantitatively, the fluons activity in wall layers, with the velocity distribution $U(y)$, points out the existence of an associated distribution $U^*(y)$, obtained by the coordinate transformation $\eta = \int_0^y U^2 dy \Big/ \int_0^\delta U^2 dy$, where $\delta(t,x,z)$ stands for the finite thickness of the layer. The fluctuations are related to the chaotic change of the instantaneous velocity distribution between $U(y)$ and $U^*(y)$. This observation leads to the evaluation of the global intensity of fluctuations as well as their distribution. For the incompressible and isothermal wall layers which have the $U_a(\eta_0)$ distribution, we can provide analytical formulae for the quantity $k(\eta_0) = \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) / U_e^2$. In the particular case $U_a(\eta_0) = \eta_0^n$, the table 1 presents the corresponding analytical formulae, strongly depending on the positive n values. It seems that the turbulent boundary layers with low intensity of the external flow turbulence and low wall roughness can be well represented by the above formulae. As an illustrative example, we present in the figure 4 the $k_a(\eta)$ given by the analytical formula and the corresponding experimental data for the fully developed turbulent boundary layer [4]. In the case of a relaxing perturbed boundary layer [5], when $U_a(\eta_0)$ is given by experimental data, the integrals involved in the analytical formulae are numerically calculated. Both these cases present a satisfactory confirmation of the theoretical model. For the case of a slightly divergent channel flow [10], the increase of the quantity $K(x) = \int_0^1 k(\eta_0) d\eta_0$ is also satisfactorily confirmed by the experimental data.

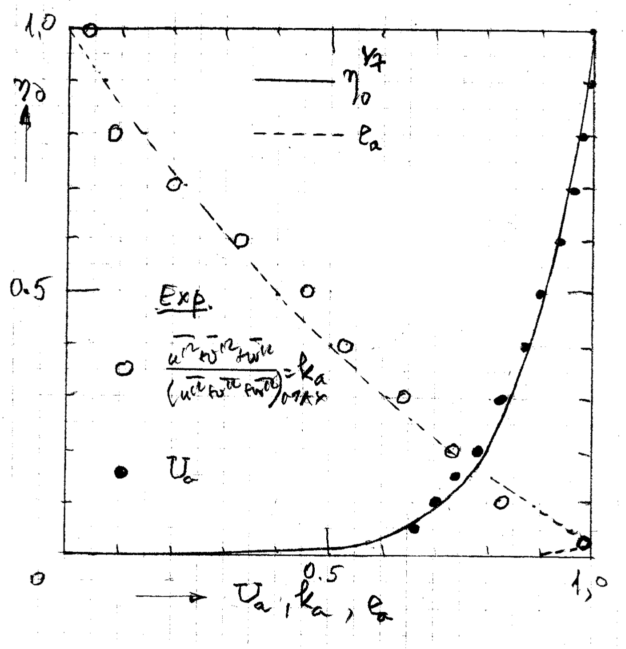

Fig. 4 – The quantity $k_a(\eta)$ given by the analytical formula and the corresponding experimental data for the fully developed turbulent boundary layer

We conclude that this theoretical approach provides useful qualitative and quantitative information in order to check the experimental data concerning the distribution and the intensity of fluctuations belonging to the equilibrium turbulent wall layers.

A final concluding remark refers to the extended applications of the IDF. By taking the general coordinate transformation in a finite space-time domain, we put the new variables τ, ξ, η, ζ as functions of t, x, y, z. In the addenda we present the expressions for the main operators ∇, ∇^2, $\frac{D}{Dt}$. We note the symmetry of their expressions which leads to the idea that the fluons activity is valid for any direction of a finite space-time domain, where the kinetical energy presents a strong variation. In this respect, we present in the figure 5 the case of the transitional boundary layer. The kinetic energy $E_C(x) = \int_0^{\delta(x)} U^2 dy$, given by experiments, manifest a variation along the normalized $x_a = x/x_L$, quoted as $E_{C,a} = E_C / E_{C.\max} = f(x_a)$. By the usual rule we get the transformed $(E_{C,a})^*$ distribution and the difference between them - the normalized $e_{a,x}$ distribution. The comparison of the $e_{a,x} = f(x_a)$ with the normalized experimental data $\int_0^{\delta(x)} \overline{u'^2} dy$ is quite satisfactory. We also state that the fluons activity in the finite space-time domain can be related to the existence of the turbulent spots and can explain their non-physical interaction.

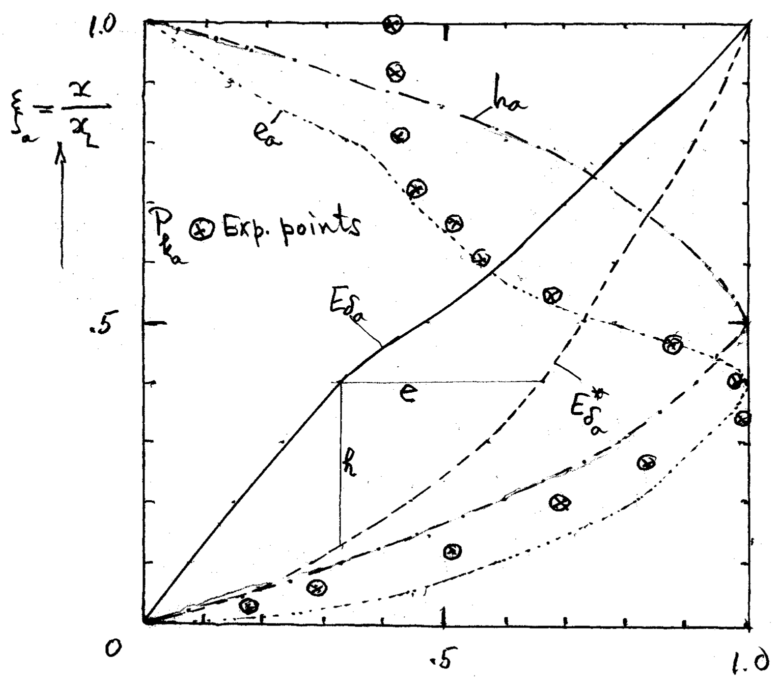

Fig. 5 – The fluons activity in the finite space-time domain for the transitional boundary layer.

As a general conclusion, we say that the role played by the IDF in shear flows is similar to the role played by the vorticity in potential flows.

APPENDIX

1. We consider the transformation of coordinates:

$$\left\{\begin{array}{l}\tau = t \\ \xi = x \\ \eta = \dfrac{1}{\psi_\delta}\displaystyle\int_0^y \varphi\, dy \\ \zeta = z\end{array}\right\} \leftrightarrow \left\{\begin{array}{l}t = \tau \\ x = \xi \\ y = \psi_\delta \displaystyle\int_0^\eta \dfrac{d\eta}{\varphi} \\ z = \zeta\end{array}\right\} \tag{A1}$$

In the new coordinates we get the mathematical operators:

$$\nabla = \frac{\partial}{\partial x}\vec{\boldsymbol{i}} + \frac{\partial}{\partial y}\vec{\boldsymbol{j}} + \frac{\partial}{\partial z}\vec{\boldsymbol{k}} = \frac{\partial}{\partial \xi}\vec{\boldsymbol{i}} + \frac{\partial}{\partial \zeta}\vec{\boldsymbol{k}} - \eta_y\left(y_\xi \vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + y_\zeta \vec{\boldsymbol{k}}\right)\frac{\partial}{\partial \eta} \tag{A2}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\vec{\boldsymbol{v}}\,\nabla\right) = \frac{\partial}{\partial \tau} + u\frac{\partial}{\partial \xi} + w\frac{\partial}{\partial \zeta} +$$
$$+\eta_y\left[v_0 + \psi_\delta\int_0^\eta \frac{\varepsilon}{\varphi}d\eta - y_\tau - \frac{\partial}{\partial \xi}\left(\psi_\delta\int_0^\eta \frac{u}{\varphi}d\eta\right) - \frac{\partial}{\partial \zeta}\left(\psi_\delta\int_0^\eta \frac{w}{\varphi}d\eta\right)\right]\frac{\partial}{\partial \eta} \tag{A3}$$

$$\nabla^2 = \eta_y\frac{\partial}{\partial \eta}\left[\left(1 + y_\xi^2 + y_\zeta^2\right)\eta_y\frac{\partial}{\partial \eta}\right] + \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \zeta^2} -$$
$$-\eta\eta_y\left(y_\xi\frac{\partial^2}{\partial \xi\,\partial \eta} + y_\zeta\frac{\partial^2}{\partial \zeta\,\partial \eta}\right) - \frac{y_{\xi\xi} + y_{\zeta\zeta}}{y_\eta}\frac{\partial}{\partial \eta} \tag{A4}$$

where:

$$y_\xi = \frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\psi_\delta\int_0^\eta \frac{d\eta}{\varphi}\right), \quad y_\eta = \frac{\psi_\delta}{\varphi}, \quad y_\zeta = \frac{\partial}{\partial \zeta}\left(\psi_\delta\int_0^\eta \frac{d\eta}{\varphi}\right) \tag{A5}$$

The continuity equation leads to the v expression:

$$v = v_0 + \psi_\delta\int_0^\eta \frac{1}{\varphi}\left(\varepsilon_V - \frac{\partial u}{\partial \xi} - \frac{\partial w}{\partial \zeta}\right)d\eta + \int_0^\eta\left(y_\xi\frac{\partial u}{\partial \eta} + y_\zeta\frac{\partial w}{\partial \eta}\right)d\eta \tag{A6}$$

where $v_0 = v\big|_{\eta=0}$.

The equation $\alpha_K\nabla^2 K + \Sigma = \dfrac{DK}{Dt}$ becomes:

$$\alpha_K \frac{\partial}{\partial\eta}\left[\left(1+y_\xi^2+y_\zeta^2\right)\varphi\right]+\frac{\psi_\delta^2}{\varphi}\Sigma=\left(\frac{\partial K}{\partial\tau}+u\frac{\partial K}{\partial\xi}+w\frac{\partial K}{\partial\xi}\right)\frac{\psi_\delta^2}{\varphi}+$$
$$+\psi_\delta\left[v_0+\psi_\delta\int_0^\eta\frac{\varepsilon_V}{\varphi}d\eta-y_\tau-\frac{\partial}{\partial\xi}\left(\psi_\delta\int_0^\eta\frac{u}{\varphi}d\eta\right)-\frac{\partial}{\partial\zeta}\left(\psi_\delta\int_0^\eta\frac{w}{\varphi}d\eta\right)\right]\frac{\partial K}{\partial\eta}- \tag{A7}$$
$$-\alpha_K\frac{\psi_\delta^2}{\varphi}\left(\frac{\partial^2K}{\partial\xi^2}+\frac{\partial^2K}{\partial\zeta^2}\right)+2\alpha_K\psi_\delta\left(y_\xi\frac{\partial^2K}{\partial\xi\partial\eta}+y_\zeta\frac{\partial^2K}{\partial\zeta\partial\eta}\right)+\alpha_K\psi_\delta\left(y_{\xi\xi}+y_{\zeta\zeta}\right)\frac{\partial K}{\partial\eta}$$

where:

$$y_{\xi\xi}=\psi_\delta\frac{\partial^2}{\partial\xi^2}\int_0^\eta\frac{d\eta}{\varphi}+2\frac{\partial\psi_\delta}{\partial\xi}\frac{\partial}{\partial\xi}\int_0^\eta\frac{d\eta}{\varphi}+\frac{\partial^2\psi_\delta}{\partial\xi^2}\int_0^\eta\frac{d\eta}{\varphi} \tag{A8}$$

2. The terms that appear in equation (5) are given by :

$$f_1=\psi_\delta,\ f_2=\psi_\delta^2,\ f_3=\frac{\partial}{\partial\tau}\left(\psi_\delta^2\right),\ f_4=\frac{\partial}{\partial\xi}\left(\psi_\delta^2\right),$$
$$f_5=\frac{\partial}{\partial\zeta}\left(\psi_\delta^2\right),\ f_6=\psi_\delta\left(\frac{\partial^2\psi_\delta}{\partial\xi^2}+\frac{\partial^2\psi_\delta}{\partial\zeta^2}\right) \tag{A9}$$

$$\sigma_1=\frac{v_0}{\alpha_K}\frac{\partial K}{\partial\eta} \tag{A10}$$

$$\sigma_2=\frac{1}{\alpha_K}\left[\frac{1}{\varphi}\left(\frac{\partial K}{\partial\tau}+u\frac{\partial K}{\partial\xi}+w\frac{\partial K}{\partial\zeta}\right)+\frac{\partial K}{\partial\eta}\int_0^\eta\frac{\varepsilon_V}{\varphi}d\eta+\frac{S}{\varphi}-\right.$$
$$\left.-\frac{\partial K}{\partial\eta}\left(\frac{\partial}{\partial\tau}\int_0^\eta\frac{d\eta}{\varphi}+\frac{\partial}{\partial\xi}\int_0^\eta\frac{u}{\varphi}d\eta+\frac{\partial}{\partial\zeta}\int_0^\eta\frac{w}{\varphi}d\eta\right)\right]+ \tag{A11}$$
$$+\left[\frac{\partial K}{\partial\eta}\left(\frac{\partial^2}{\partial\xi^2}\int_0^\eta\frac{d\eta}{\varphi}+\frac{\partial^2}{\partial\zeta^2}\int_0^\eta\frac{d\eta}{\varphi}\right)-\frac{1}{\varphi}\left(\frac{\partial^2K}{\partial\xi^2}+\frac{\partial^2K}{\partial\zeta^2}\right)-\right.$$
$$\left.-2\left(\frac{\partial^2K}{\partial\xi\partial\eta}\cdot\frac{\partial}{\partial\xi}\int_0^\eta\frac{d\eta}{\varphi}+\frac{\partial^2K}{\partial\zeta\partial\eta}\cdot\frac{\partial}{\partial\zeta}\int_0^\eta\frac{d\eta}{\varphi}\right)\right]$$

$$\sigma_3 = -\frac{1}{2\alpha_K}\frac{\partial K}{\partial \eta}\int_0^\eta \frac{d\eta}{\varphi} \tag{A12}$$

$$\sigma_4 = -\frac{1}{2\alpha_K}\frac{\partial K}{\partial \eta}\int_0^\eta \frac{u}{\varphi}d\eta + \frac{\partial K}{\partial \eta}\cdot\frac{\partial}{\partial \xi}\int_0^\eta \frac{d\eta}{\varphi} - \frac{\partial^2 K}{\partial \xi \partial \eta}\cdot\int_0^\eta \frac{d\eta}{\varphi} \tag{A13}$$

$$\sigma_5 = -\frac{1}{2\alpha_K}\frac{\partial K}{\partial \eta}\int_0^\eta \frac{w}{\varphi}d\eta + \frac{\partial K}{\partial \eta}\cdot\frac{\partial}{\partial \zeta}\int_0^\eta \frac{d\eta}{\varphi} - \frac{\partial^2 K}{\partial \zeta \partial \eta}\cdot\int_0^\eta \frac{d\eta}{\varphi} \tag{A14}$$

$$\sigma_6 = \frac{\partial K}{\partial \eta}\int_0^\eta \frac{d\eta}{\varphi} \tag{A15}$$

3. Unsteady 2-D Incompressible Boundary Layer Example
Basic equations:

$$\frac{1}{\rho}\frac{\partial \tau}{\partial y} = \frac{\partial U}{\partial t} - \frac{\partial U_e}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - U_e\frac{\partial U_e}{\partial x} \tag{A16}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{A17}$$

Initial and boundary conditions:

$$\begin{aligned}
&t = 0, && U = U(x),\ \delta = \delta(x) \\
&x = 0, && \delta = 0 \\
&y = 0, && U = 0,\ V = V_0 \\
&y = \delta(x,t) && U = U_e(x,t),\ \frac{\partial U}{\partial y} = 0
\end{aligned} \tag{A18}$$

Real flow model:

$$\tau = \mu(1+Kh)\frac{\partial U}{\partial y},\ \nu_a^* = 1 + Kh,\ \ h = \frac{y}{\delta} - \int_0^y U^2 dy \Big/ \int_0^\delta U^2 dy \tag{A19}$$

For $\varphi = U_a^m$, $K = U_a$ we get:

$$A_\tau = \frac{\partial U_a}{\partial \eta}\int_0^\eta U_a^{-m}\, d\eta,\ A_\xi = \frac{\partial U_a}{\partial \eta}\int_0^\eta U_a^{1-m}\, d\eta \tag{A20}$$

$$B_\tau = U_a^{1-m} - U_a^{-m} + m\frac{\partial U_a}{\partial \eta_m}\int_0^{\eta} U_a^{-m}\, d\eta\,,\; B_\xi = U_a^{2-m} - U_a^{-m} - (1-m)\frac{\partial U_a}{\partial \eta_m}\int_0^{\eta_m} U_a^{1-m}\, d\eta_m \tag{A21}$$

$$C_\tau = U_a^{-m}\frac{\partial U_a}{\partial \tau} - \frac{\partial U_a}{\partial \eta}\frac{\partial}{\partial \tau}\int_0^{\eta_m} U_a^{-m}\, d\eta_m\,,\; C_\xi = U_a^{1-m}\frac{\partial U_a}{\partial \xi} - \frac{\partial U_a}{\partial \eta}\int_0^{\eta} U_a\frac{\partial}{\partial \xi}\left(U_a^m\right) d\eta \tag{A22}$$

The IDF for U_a and $\psi^2_{m,\delta}$ becomes:

$$\begin{aligned}\frac{\nu U_e^m}{1+m}\frac{\partial}{\partial \eta}\left[(1+Kh)\frac{\partial}{\partial \eta}\left(U_a^{1+m}\right)\right] = V_0\frac{\partial U_a}{\partial \eta} + \psi_{m,\delta}U_e^{-m}\left[C_\tau + U_e C_\xi + \frac{U_{e,\tau}}{U_e}B_\tau + U_{e,\xi}B_\xi\right] - \\ -\frac{1}{2}\frac{\partial}{\partial \xi}\left(\psi^2_{m,\delta}\right)U_e^{1-m}A_\xi - \frac{1}{2}\frac{\partial}{\partial \tau}\left(\psi^2_{m,\delta}\right)U_e^{-m}A_\tau\end{aligned} \tag{A23}$$

and:

$$\begin{aligned}\frac{\partial}{\partial \xi}\left(\psi^2_{m,\delta}\right) + \frac{1}{U_e}\frac{A^\oplus_{\tau,1}}{A^\oplus_{\xi,1}}\frac{\partial}{\partial \tau}\left(\psi^2_{m,\delta}\right) - \frac{2}{U_e A^\oplus_{\xi,1}}\left[U_{e,\xi}B^\oplus_{e,1} + \frac{U_{e,\tau}}{U_e}B^\oplus_{\tau,1} + U_e C^\oplus_{\xi,1} + C^\oplus_{\tau,1}\right]\psi^2_{m,\delta} - \\ -2U_e^m\frac{V_0}{U_e}\frac{\psi_{m,\delta}}{A^\oplus_{\xi,1}}\int_0^1 (U_a - 1)\,d\eta + \frac{2}{1+m}\frac{\nu U_e^{2m-1}}{A^\oplus_{\xi,1}}\int_0^1 (1+Kh)\,d\left(U_a^{1+m}\right) = 0\end{aligned} \tag{A24}$$

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