

# Modelling the wear processes of the automotive brake pad and disc

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**Abstract:** *The automotive disc braking system is based on the friction between a disc and two pads and this friction determines their wear. The theoretical study proposed in this paper aims to model this wear process. It is based on a simplified geometrical model (approximation of the curvilinear trapezium with a rectangle of the same area) and considering the brake pad as a viscoelastic material, while the disc is elastic. With the help of this model we can determine the relative displacements of the pad and of the disc during the braking process. Also, when the process is stabilized, the pressure distribution can be determined. In the paper, both the relative displacements and the pressure distribution are exemplified for the inner edge, the middle and the outer edge of the brake pad.*

**Key Words:** *wear, friction, automotive brake system, brake pad, brake disc*

## 1. INTRODUCTION

The braking system is one of the most important safety systems of the vehicle [1]-[3]. Automotive disc brakes are based on the friction between a stator (brake pad) and a rotor (brake disc) in order to transform the kinetic energy transmitted to the wheels in thermal energy. Thus, the tribological study of the brake systems is of utmost importance in their analysis [1], [4-6].

The brake pads are an essential element in the operation of the brake system. They are subjected to very high compressive and shear stresses and pressures of up to 10 MPa. Also, they are subjected to cyclical mechanical and thermal stresses, thus being exposed to fatigue. All these stresses that brake pads are subjected to determine the wear of the pad friction material.

Brake pads wear is normal because they are designed to wear out more quickly than discs. Pad wear is influenced by the contact pressure, the relative sliding speed between the disc and the pads and by the temperature of the surfaces in contact [7]. The wear mechanism of the brake pad friction material is a complex phenomenon, being composed of many types of wear: abrasive, adhesive, superficial fatigue and thermal fatigue [6], [8].

Like brake pads, the brake discs are designed to be taken out of use when they have reached the wear limit and will have a thickness below that prescribed by the manufacturers.

Taking these into consideration, for the automotive manufacturers it is very important to predict the wear behavior of the brake pads and discs [9]. The current paper aims to theoretically model the wear process of the automotive brake pad and disc.

### 2. GEOMETRIC MODEL

The geometry (Fig. 1), the angular speed ( $\omega$ ), the friction regime (the friction coefficient  $\mu$  and its variation with speed, contact pressure and temperature), the wear parameters of the brake pads ( $k_{w_1}$ ) and the brake disc ( $k_{w_2}$ ) for the contact between the brake disc and the two brake pads are considered to be known [9].

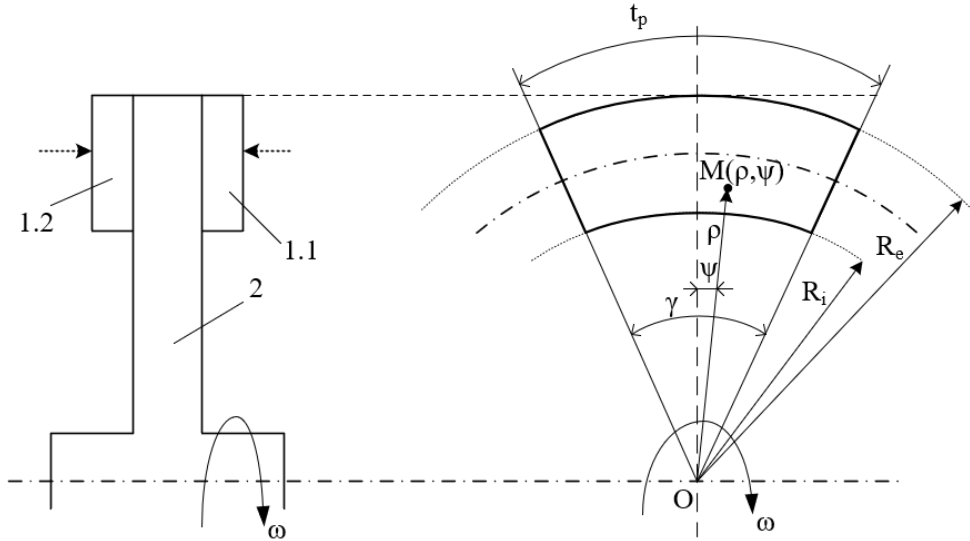


Fig. 1 – Brake system geometry at the contact area between the disc and the brake pads

The geometric particularities of the disc brake impose the following specific measures of the friction and wear processes:

- a) friction areas
- for the brake pad:

$$A_1 = \frac{1}{2} \gamma (R_i + R_e)(R_e - R_i) = \frac{1}{2} \gamma (R_e^2 - R_i^2), \tag{1}$$

where  $\gamma$  is the mutual coverage angle of the brake pad and disc and can be approximated based on the mean semi-width of the pad ( $a_p$ ),

$$\gamma \approx \frac{8a_p}{D_e(1 + r_a)} = \frac{8a_{ap}}{1 + r_a}, \tag{2}$$

where  $a_{ap} = a_p/D_e$  is the dimensionless semi-width of the pad and  $r_a = D_i/D_e$  is the radius.

Thus, the friction area for the pad ( $A_1$ ) coincides with the contact area,

$$A_1 \approx D_e^2 a_{ap} (1 - r_a), \tag{3.1}$$

- for the brake disc:

$$A_2 = \pi(R_e^2 - R_i^2) = \frac{\pi}{4} D_e^2 (1 - r^2). \quad (3.2)$$

b) the wear time for a work period  $t$ :

- for the brake pad:

$$t_1 = t; \quad (4.1)$$

- for the brake disc:

$$t_2 = \frac{\frac{1}{2}(R_i + R_e) \cdot \gamma}{\frac{1}{2} \pi (R_i + R_e)} t = \frac{\gamma}{\pi} \approx \frac{8 a_{ap}}{\pi(1 + r_a)}. \quad (4.2)$$

The wear rates for the brake pad ( $v_{u_1}$ ) and disc ( $v_{u_2}$ ) materials are defined:

$$v_{u_1} = \left(\frac{du}{dt}\right)_1 = \frac{\Delta v_{u_1}}{A_1 \cdot t_1} = \frac{2 \cdot \Delta v_{u_1}}{\gamma \cdot t (R_e^2 - R_i^2)}; \quad (5.1)$$

$$v_{u_2} = \left(\frac{du}{dt}\right)_2 = \frac{\Delta v_{u_2}}{A_2 \cdot t_2} = \frac{\Delta v_{u_2} \cdot \pi}{\pi (R_e^2 - R_i^2) \cdot \gamma t} = \frac{\Delta v_{u_2}}{\gamma \cdot t (R_e^2 - R_i^2)}. \quad (5.2)$$

where  $\Delta v_{u_1}$  is the volume of the worn and removed material from the pad and  $\Delta v_{u_2}$  is the volume of the worn and removed material from the disc.

The wear rates can be determined from the analysis of the experimental results directly on the brake or from the laboratory tests.

For the material pair used for the modern disc brakes, for the movement type between the disc and pads (variable speed sliding) and for the initial surface contact form (plane-plane), the wear is complex (adhesion and abrasion). The friction material of the pad is a composite material with a viscoelastic matrix and elastic components. The disc material is gray cast iron, considered as an elastic material.

For these conditions, the wear rates of the two brake materials are considered to have dependences as following:

$$v_{u_{1,2}} = k_{u_{1,2}} \left(\frac{p}{H}\right)^\alpha \cdot \left(\frac{v}{v_{cr}}\right)^\beta = k_{u_{1,2}} p_a^\alpha \cdot v_a^\beta, \quad (6)$$

where  $k_{u_{1,2}}$  are the wear coefficients of the pad (1) and the disc (2);  $p$  – contact pressure;  $H$  – the parameter that characterizes the wear penetration (hardness for the disc material, compliance for the pad material),  $v$  – sliding speed;  $v_{cr}$  – critical speed – the speed at which the pad material has maximum damping;  $p_a = p/H$ ;  $v_a = v/v_{cr}$ ;  $\alpha$  and  $\beta$  – coefficients which depend on material properties, friction conditions, temperature etc. (in many situations  $\alpha = \beta \approx 1$ ). The product  $(p \cdot v)$  is a parameter of the energy flux generated by the friction process and characterizes the thermic regime of the brake.

### 3. THE CONTACT CONDITION IN THE WEAR PROCESS

In the wear process the form of the contact surfaces modifies continuously, thus the aspects regarding the mathematical formulation of the contact pressure and of the displacements become important. It is considered that the dimensions of the pad and the disc modify only in the direction perpendicular to the contact surface.

In each point  $M(\rho, \psi)$  (Fig. 1.) on the contact surface the linear wear is  $u_{1,2}(\rho, \psi)$  and it depends on the contact pressure  $p(\rho, \psi)$  and on the sliding speed  $v(\rho)$ . The system of cylindrical coordinates (Fig. 1) has the origin in the center of the disc ( $O$ ), current radius  $\rho$  and angle  $\psi$ , measured from the vertical axis of the disc in the direction of angular speed. To find the correlation between the contact pressure  $p$  and the viscoelastic displacement of the pad ( $w_1$ ) and elastic displacement of the disc ( $w_2$ ), in the terms of the linear wear  $u_{1,2}$ , Goryacheva's [10] solution is adapted for the case of a rectangular plane surface with pure elastic deformation.

In the hypothesis that the wear ( $u$ ) is small as compared to the radius ( $\rho$ ), but comparable to the displacement ( $w$ ) at the same radius ( $\rho$ ), the limit conditions are those for the undistorted surfaces, neglecting the elastic or viscoelastic displacement ( $w(\rho, \psi)$ ) and the wear  $u(\rho, \psi)$ . The contact condition in the wear process of the pad at a given moment is:

$$w_1(\rho, \psi, t) + u_1(\rho, \psi, t) + w_2(\rho, \psi, t) + u_2(\rho, \psi, t) = D(t), \quad (7.1)$$

where  $D(t)$  is the proximity (penetrations) of the pad to the disc.

The contact condition in the wear process of the disc is:

$$w_1(\rho, \psi, 0) + w_2(\rho, \psi, 0) = D(0), \quad (7.2)$$

because the relative displacement of the disc is allowed by the friction process at any moment ( $t$ ) and thus at the initial moment  $t = 0$ .

The wears  $u_1$  and  $u_2$  can be deduced from (5.1), (5.2) and (6):

$$u_{1,2} = k_{u_{1,2}} \int_0^t p_a^\alpha(\rho, \psi, t') \cdot v_a^\beta(\rho, \psi, t') dt'. \quad (8)$$

#### 4. THE CONTACT PRESSURE BETWEEN THE DISC AND THE PAD

To determine the pressure variation in time, it is necessary to know the dependence of the displacements ( $w_1$ ) and ( $w_2$ ) on the pressure.

Thus, for the brake pad material, considered a viscoelastic modified Voigt material, the pressure ( $p$ ) has the expression [11]:

$$p = E_a \varepsilon + i\omega \eta_a \cdot \varepsilon = E_a (1 + i \tan \gamma_a) \frac{\delta}{s} = \frac{w_1}{s} E_a (1 + i \tan \gamma_a), \quad (9)$$

where  $E_a$  is the apparent elasticity modulus;  $\varepsilon$  – the specific deformation;  $\delta$  – the displacement of the pad in the direction of the normal load;  $s$  – the pad thickness;  $\eta_a$  – apparent viscosity (characterizes the internal friction of the material);  $\gamma_a$  – the angle of the hysteretic losses.

$$\tan \gamma_a = \frac{\eta_a}{E_a} \cdot \omega, \quad (10)$$

where  $\omega$  is the pd pulsation to the direction of the normal force.

The apparent elasticity modulus ( $E_a$ ) takes into consideration that the pad material deforms sideways when compressed. Moore's [11] relation is considered:

$$E_a = E(1 + 2S_f^2), \quad (11)$$

in which  $E$  is the elasticity modulus and  $S_f$  is form factor of the section, defined as the ratio between the loaded area ( $A_p$ ) and the unloaded area ( $A_e$ ) (the free surface).

$$S_f = \frac{A_p}{A_e} \tag{12}$$

For the brake pad,

$$A_p = A_1 = \frac{\gamma}{2} (R_e^2 - R_i^2) = D_e^2 a_{ap} (1 - r_a), \tag{13}$$

$$A_e = s[R_e\gamma + R_i\gamma + 2(R_e - R_i)] \approx D_e^2 s_a (4 a_{ap} + 1 - r_a), \tag{14}$$

where  $s_a = s/D_e$  is the dimensionless thickness of the pad.

Thus, the form factor has the expression:

$$\begin{aligned} S_f &= \frac{0.5 \gamma R_e (1 - r_a^2)}{s(\gamma + \gamma r_a + 2 - 2r_a)} = \frac{R_e}{s} \cdot \frac{0.5 \gamma (1 - r_a^2)}{\gamma(1 + r_a) + 2(1 - r_a)} = \\ &= \frac{\gamma}{4s_a} \cdot \frac{1 - r_a^2}{\gamma(1 + r_a) + 2(1 - r_a)} \approx \frac{a_{ap}^2}{s_a} \cdot \frac{1 - r_a}{4a_{ap} + 1 - r_a}. \end{aligned} \tag{15}$$

The pressure  $p$  is nondimensionalized from expression (9) in relation to the hardness of the disc ( $H_2$ ) ( $p_a = p/H_2$ ), the disc thickness ( $s$ ) in relation to the exterior diameter ( $s_a = s/D_e$ ) and the displacement  $w_1$  in relation to the exterior diameter ( $w_{1a} = w_1/D_e$ ). Thus, the dimensionless contact pressure between the pad and the disc is:

$$p_a = \frac{p}{H_2} = \frac{w_{1a}}{s_a} \cdot \frac{E_1}{H_1} (1 + 2S_f^2) |1 + i \tan \gamma_a|. \tag{16}$$

The displacement ( $w_{1a}$ ) of any point of the viscoelastic surface is determined from (16):

$$w_{1a} = \frac{H_1 \cdot s_a}{E_1 (1 + 2S_f^2) |1 + i \tan \gamma_a|} p_a. \tag{17}$$

The case concerning the constant pressure on the contact surface is analyzed.

If it is considered that at a given time the contact pressure is constant on the pad, the different points of the perfect elastic disc will displace differently, depending on the position of the points related to the loaded area (Fig. 2).

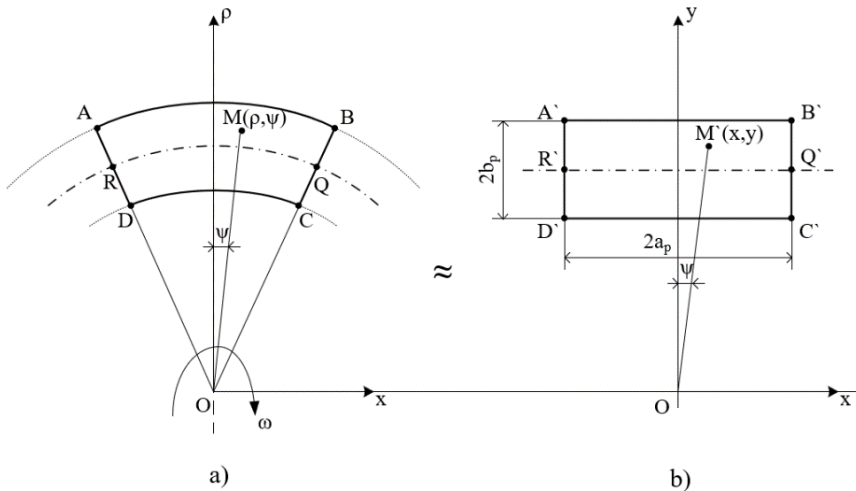


Fig. 2 – Approximation of the curvilinear trapezium with a rectangle of the same area

The curvilinear trapezium  $ABCD$  (exterior radius  $R_e$ , interior radius  $R_i$ , center angle  $\gamma$  (Fig. 1)) is approximated with rectangle  $A'B'C'D'$  with the sides  $2a_p = \gamma D_e (1 + r_a)/2$  and  $2b_p = R_e - R_i = D_e(1 - r_a)/2$  or by nondimensionalizing in relation to the exterior diameter  $D_e$ ,  $a_{ap} = \gamma (1 + r_a)/4$ ;  $b_{ap} = (1 - r_a)/4$ .

The correlation of the coordinates of points  $A, B, C, D$  in the system of cylindrical coordinates ( $O\psi\rho$ ) with the coordinates of points  $A', B', C', D'$  in the cartesian system  $x \circ y$  (Fig. 2.a, Fig. 2.b) is:  $A(R_e, -\gamma/2)$ ;  $B(R_e, \gamma/2)$ ;  $C(R_i, \gamma/2)$ ;  $D(R_i, -\gamma/2)$ ;  $A'(-a_p, R_e)$ ;  $B'(a_p, R_e)$ ;  $C'(a_p, R_i)$ ;  $D'(-a_p, R_i)$ .

A current point  $M(x, y)$  is placed in the system  $x \circ y$  with  $x = \rho \sin \psi$ ,  $y = \rho \cos \psi$ ,  $\rho = \sqrt{x^2 + y^2}$ .

The dimensionless coordinates of points  $A'B'C'D'$  are:

$$A' \left( -1, \frac{0.5}{a_{ap}} \right); B' \left( 1, \frac{0.5}{a_{ap}} \right); C' \left( 1, \frac{0.5r_a}{a_{ap}} \right); D' \left( -1, \frac{0.5r_a}{a_{ap}} \right).$$

The displacement in direction  $z$  (direction of the pressure) ( $w_2$ ), determined by Love in 1929, for a constant pressure ( $p$ ) which acts in the rectangle ( $2a \times 2b$ ) is:

$$\begin{aligned} \phi(x, y) = \frac{\pi E_r}{1 - \nu_2^2} \cdot \frac{w_2}{p} = (x + a) \ln \left[ \frac{(y + b) + \{(y + b)^2 + (x + a)^2\}^{1/2}}{(y - b) + \{(y - b)^2 + (x + a)^2\}^{1/2}} \right] + \\ + (y + b) \ln \left[ \frac{(x + a) + \{(y + b)^2 + (x + a)^2\}^{1/2}}{(x - a) + \{(y + b)^2 + (x - a)^2\}^{1/2}} \right] + \\ + (x - a) \ln \left[ \frac{(y - b) + \{(y - b)^2 + (x - a)^2\}^{1/2}}{(y + b) + \{(y + b)^2 + (x - a)^2\}^{1/2}} \right] + \\ + (y - b) \ln \left[ \frac{(x - a) + \{(y - b)^2 + (x - a)^2\}^{1/2}}{(x + a) + \{(y - b)^2 + (x + a)^2\}^{1/2}} \right]. \end{aligned} \tag{18}$$

The dimensionless pressure will be noted as  $p_a = p/H_2$ , while  $w_{2a} = w_2/D_2$ ,  $a_{ap} = a/D_e$  and  $b_a = b/a$ .

Thus, we will get the contact pressure with which the pad acts on the disc:

$$p_a = \frac{p}{H_2} = \frac{\pi E_2}{(1 - \nu_2^2)H_2} \cdot \frac{w_{2a}}{a_{ap}} \cdot \frac{1}{\phi_w(x_a, y_a, b_a)}, \tag{19}$$

where

$$\phi_w(x_a, y_a, b_a) = \frac{\phi(x, y)}{a_p} = \frac{\phi(x, y)}{a_p \cdot D_e}.$$

In the case of the brake pad, if the pressure is constant, then we can apply Love's relation for a viscoelastic material, characterized by the apparent elasticity modulus  $E_{a1}$ .

$$E_{a1} = E_1(1 + 2S_f^2) |1 + i \tan \gamma_a| \tag{20}$$

In this case, the contact pressure  $p_a$  is the same, while displacements  $w_1$  are different and depend on the coordinates of the contact points:

$$w_{1a}(x_a, y_a) = \frac{w_1}{D_e} = \frac{H_2(1 - v_1^2)}{\pi E_{a1}} \cdot a_{ap} \cdot \phi_z(x_a, y_a) \cdot p_a = k_1 \phi_w(x_a, y_a) a_{ap} \quad (21)$$

The expression of the dimensionless displacement  $w_{2a}$  in any point on the disc (Fig. 2.b) at constant pressure is:

$$w_{2a}(x_a, y_a) = \frac{H_2(1 - v_2^2)}{\pi E_2} \cdot a_{ap} \cdot \phi_w(x_a, y_a) \cdot p_a = k_2 \phi_w(x_a, y_a) p_a \quad (22)$$

The total displacement ( $w_t$ ) of the points from the contact area is:

$$w_t = w_1 + w_2 \text{ or } w_{ta} = w_{1a} + w_{2a} \quad (23)$$

In Fig. 3, Fig. 4 and Fig. 5, the variation of the dimensionless displacements  $w_{1a}$ ,  $w_{2a}$  and  $w_{ta}$  is exemplified for sides  $A'B'$ ,  $R'Q'$  and  $D'C'$  of rectangle  $A', B', C', D'$  (Fig. 2.b) dimensionless pressure  $p_a = 10^{-5}$  and loss coefficients  $\gamma_{a1} = 0.1$  and  $\gamma_{a2} = 0.15$ . It can be observed that, the relative displacement rises in the radial direction from exterior to interior, both for the pad and the disc. Also, it can be observed that the relative displacement is influenced by the hysteretic loss coefficient  $\gamma_a$ , an increase in its value determining a slight increase of the relative displacements.

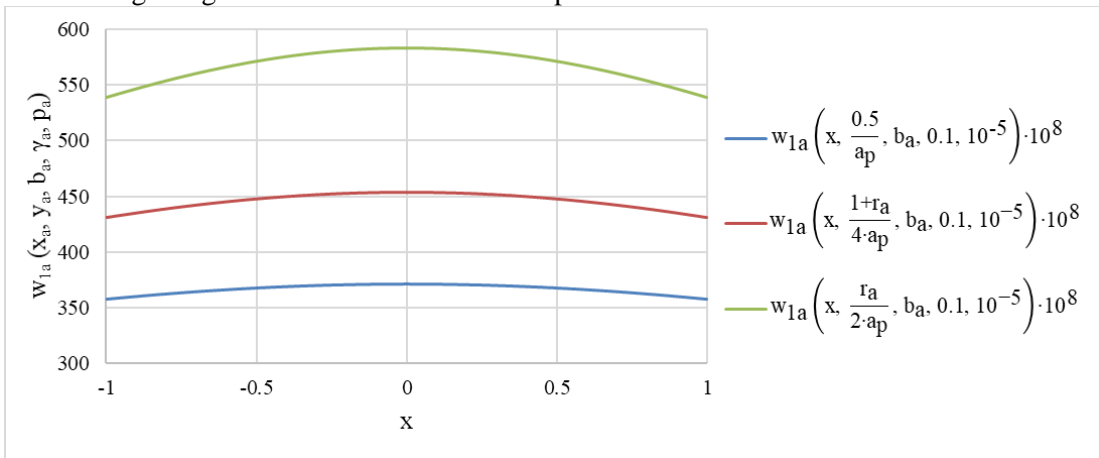


Fig. 3 – The relative displacement of the viscoelastic brake pad

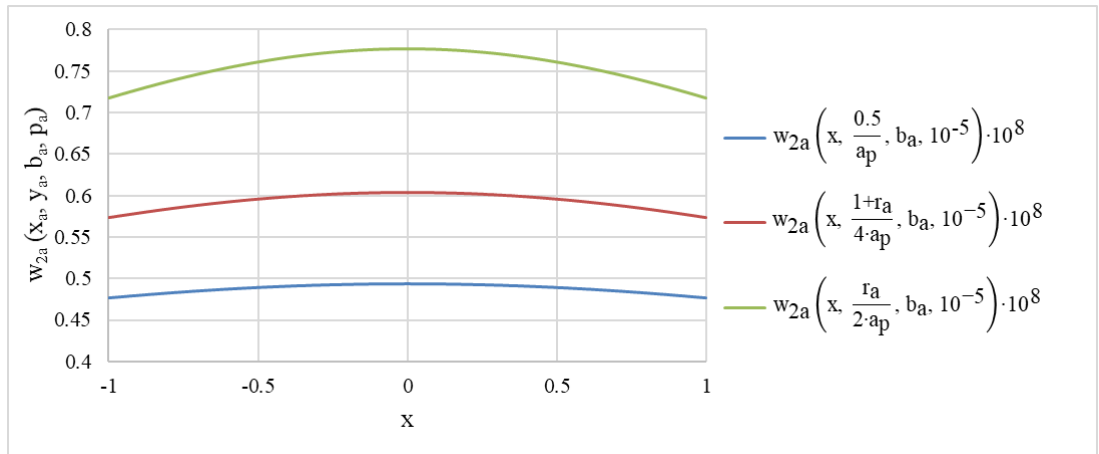


Fig. 4 – The relative displacement of the elastic brake disc

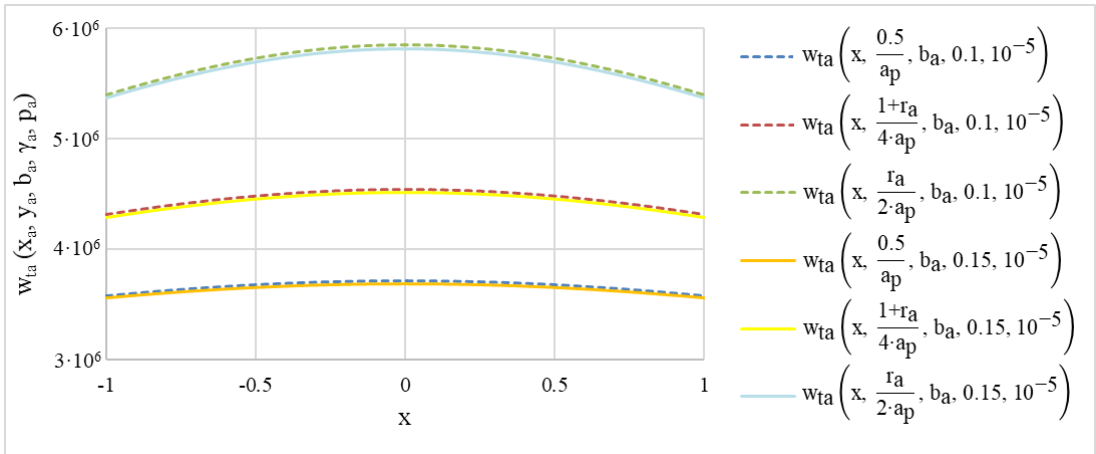


Fig. 5 – The total relative displacement of the brake pad and disc

### 5. THE WEAR OF THE PAD AND THE DISC

The contact condition in the wear process of the brake pad (7.1) in the  $x \circ y$  coordinate system (Fig. 2.b) and for the elastic displacements form expressions (21) and (22) is:

$$(k_1 + k_2)\phi_w(x_a, y_a)p_a(x_a, y_a, t) + (k_{u_1} + k_{u_2}) \int_0^t p_a^\alpha(x_a, y_a, t')v_a^\beta(x_a, y_a, t') dt' = D_a(t) . \tag{24}$$

where  $D_a(t) = D(t)/D_e$  is the dimensionless penetration;

$$k_1 = \frac{H_2(1 - v_1^2)}{\pi E_{a1}} ; \quad k_2 = \frac{H_2(1 - v_1^2)}{\pi E_2} .$$

The expression of the contact condition (24) is derived in relation to time  $t$  and the differential equation of dimensionless pressure distribution will result:

$$a_p(k_1 + k_2)\phi_w \dot{p}_a(t) + (k_{u_1} + k_{u_2}) p_a^\alpha(t) v_a^\beta = \dot{D}_a(t) . \tag{25}$$

In a stable wear regime, the wear rate (surface proximity) is:

$$v_{u_1} + v_{u_2} = \left(\frac{du}{dt}\right)_1 + \left(\frac{du}{dt}\right)_2 = \frac{dD(t)}{dt} = D_s = \text{constant} . \tag{26}$$

Thus, equation (25) becomes a differential equation with separable variables ( $p_a, t$ ):

$$\frac{dp_a}{Ap_a^\alpha + B} = dt , \tag{27}$$

where  $A$  and  $B$  are functions only of  $x_a, y_a$  and are independent of time  $t$ .

$$A = \frac{-(k_{u_1} + k_{u_2})v_a^\beta}{a_{ap}(k_1 + k_2)\phi_w} ; \quad B = \frac{D_{oa}}{a_{ap}(k_1 + k_2)\phi_w} ; \quad D_{oa} = D_o/D_e . \tag{28}$$

The speed parameter  $v_a$  varies in the contact zone:



$$v_a = \frac{v}{v_{cr}} = \frac{\omega_m r}{v_{cr}} = \frac{\omega_m \sqrt{x^2 + y^2}}{v_{cr}} = \frac{\omega_m \sqrt{(x_c D_e a_{ap})^2 + (y_c D_e a_{ap})^2}}{v_{cr}} = \tag{29}$$

$$= \frac{(v_0 + v_f) \cdot D_e \cdot a_{ap}}{2 R_{pn} \cdot 3.6 \cdot v_{cr}} \sqrt{x_c^2 + y_c^2}.$$

where  $v_0$  and  $v_f$  are the vehicle’s speeds before and after the braking (km/h);  $R_{pn}$  – wheel radius with the tire inflated normally (m);  $\omega_m$  – average angular speed of the disc while braking (rad/s);  $a_{ap}, r_a, b_a$  – notations corresponding to Fig. 2.

$$x_c = \frac{x}{D_e \cdot a_{ap}}; \quad y_c = \frac{y}{D_e \cdot a_{ap}}.$$

When the wear process is stabilized, the contact pressure will not vary with time anymore, thus, the stationary pressure  $p_{as}(x_a, y_a)$  will be determined from (25):

$$p_{as}(x, y, D_s) = \left[ \frac{D_s}{(k_{u1} + k_{u2}) v_a(x, y)^\beta} \right]^{\frac{1}{\alpha}}. \tag{30}$$

The displacement ( $D_s$ ) in stationary regime is determined form the mechanical equilibrium condition:

$$\int_{\frac{r_a}{2a_{ap}}}^{\frac{1}{2a_{ap}}} \int_{-1}^1 p_{as}(x_c y_c D_s) dx_c dy_c = 4 p_{a0} \cdot b_a, \tag{31}$$

where  $b_a = b/a$  – the ratio between the pad dimensions;  $p_{a0} = p_0/H_2$ , where  $p_0$  is the initial contact pressure considered constant.

Thus, Fig. 6 shows the dependence of this dimensionless displacement ( $D_s$ ) on the dimensionless contact pressure ( $p_{a0}$ ) numerically determined.

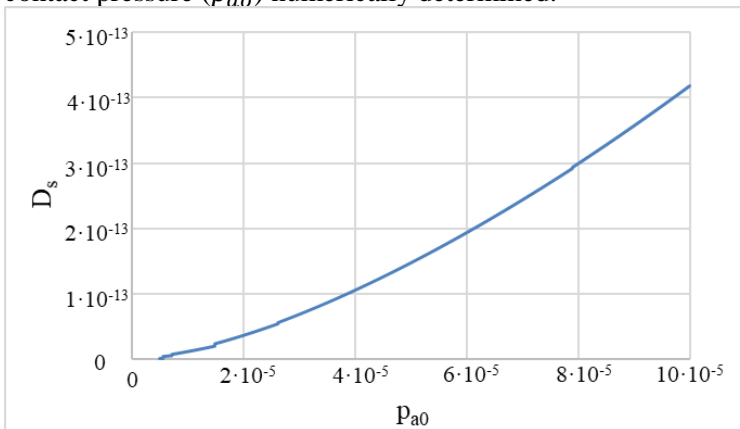


Fig. 6 – Pad displacement function of load in stationary regime

For a know initial pressure ( $p_{a0}$ ), when the process is stabilized the pressure distribution on the pad depends on the sliding speed spectrum  $v_a(x, y)$ .

Fig. 7 exemplifies the pressure distribution on the exterior, central and interior sides of the pad.

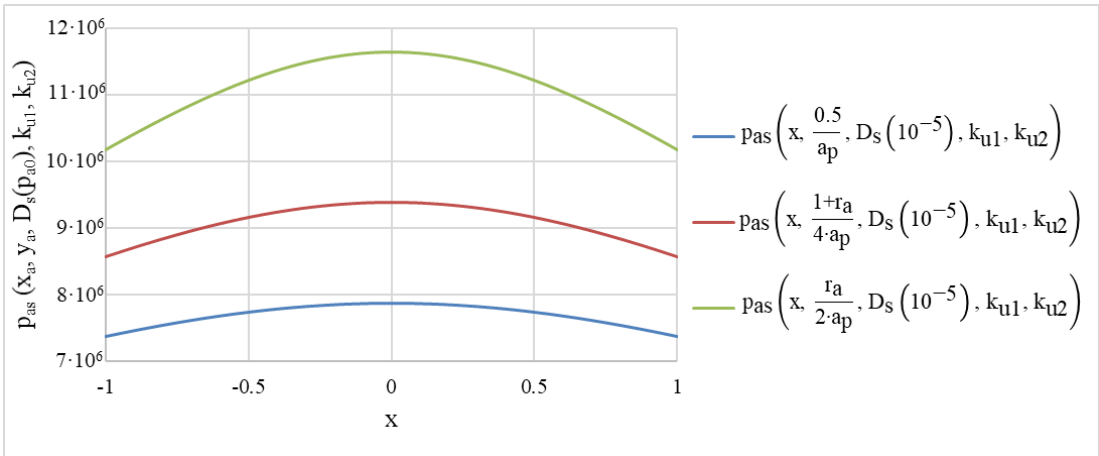


Fig. 7 – Circumferential pressure distribution on the pad surface

From Fig. 7 it can be observed that maximum value of the pressure is on the interior side of the pad  $y = r_a/(2a_{ap})$ , at its center  $x = 0$ .

The contact pressure in stationary regime of friction and wear decreases from the interior of the pad to its exterior, in the hypothesis that the initial pressure is constant.

The wear of the pad in stable regime (24) has a linear dependence of time:

$$uz_1(t) = k_{u1} \int_0^t (p_{as}^\alpha(t') \cdot v_a^\beta(t')) dt' = k_{u1} p_{as}^\alpha \cdot v_a^\beta \cdot t = k_{u1} \pi_{uz} \cdot t, \tag{32}$$

where  $\pi_{uz} = p_{as}^\alpha \cdot v_a^\beta$  is constant throughout the stationary friction and wear process (dimensionless wear parameter).

For example, for  $p_{as} = 10^{-5}$ ,  $k_{u1} = 5 \cdot 10^{-7}$  m/s,  $k_{u2} = 5 \cdot 10^{-8}$  m/s,  $\alpha = 1.5$ ,  $\beta = 1.5$ , results  $\pi_{uz} = 1.82 \cdot 10^{-9}$  and  $uz_1(t) = 0.91 \cdot 10^{-15} \cdot t$  [m], where  $t$  is the braking time in seconds.

The wear of the disc in stable regime is obtained similarly to that of the pad:

$$uz_2(t) = k_{u2} \int_0^t (p_{as}^\alpha(t') \cdot v_a^\beta(t')) dt' = k_{u2} p_{as}^\alpha \cdot v_a^\beta \cdot t = k_{u2} \pi_{uz} \cdot t. \tag{33}$$

For the given example,  $uz_2(t) = 0.091 \cdot 10^{-15} \cdot t$  [m].

## 6. CONCLUSIONS

The current theoretical study, that models the wear process of the brake pad friction material and the brake disc, allows the estimation of the relative deformations of the viscoelastic brake pad in the normal direction with respect to the contact surface and the brake disk.

Thus, it has been observed that the relative deformations of the brake pad and disc are higher around the inner diameter than around the outer diameter.

This can be explained by the fact that the pressure is higher on the inner edge than on the outer edge of the pad, its maximum value being in the central area of the inner edge.

It has been demonstrated that in a stable regime the wear of the pad and of the disc has a linear dependence of time and its value can be calculated when the friction coefficients of the pad and of the disc are known.

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