# Minisatellite Attitude Guidance Using Reaction Wheels

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**Abstract:** *In a previous paper [2], the active torques needed for the minisatellite attitude guidance from one fixed attitude posture to another fixed attitude posture were determined using an inverse dynamics method. But when considering reaction/momentum wheels, instead of this active torques computation, the purpose is to compute the angular velocities of the three reaction wheels which ensure the minisatellite to rotate from the initial to the final attitude. This paper presents this computation of reaction wheels angular velocities using a similar inverse dynamics method based on inverting Euler's equations of motion for a rigid body with one fixed point, written in the framework of the x-y-z sequence of rotations parameterization. For the particular case $A=B\neq C$ of an axisymmetric minisatellite, the two computations are compared: the active torques computation versus the computation of reaction wheels angular velocities $\omega_x$ , $\omega_y$ and $\omega_z$ . An interesting observation comes out from this numerical study: if the three reaction wheels are identical (with $I_w$ the moment of inertia of one reaction wheel with respect to its central axis), then the evolutions in time of the products between $I_w$ and the derivatives of the reaction wheels angular velocities, i.e. $I_w\dot{\omega}_x$ , $I_w\dot{\omega}_y$ and $I_w\dot{\omega}_z$ remain the same and do not depend on the moment of inertia $I_w$.*

**Key Words:** *minisatellite, attitude guidance, reaction/ momentum wheels, angular velocity, inverse dynamics*

## 1. INTRODUCTION

This paper concerns the minisatellite attitude guidance, providing in open loop the evolutions of the angular velocities to be applied on three reaction/momentum wheels in order to guide the satellite from one fixed attitude posture to another fixed attitude posture. The advantage of reaction wheels is their precision, while the disadvantages are their expensive cost and the weight [1].

The alternatives to reaction wheels are propulsion thrusters (they are fast, but fuel consumption increases) or magnetic actuators (they are cheap, but slow, appropriate for low altitude and presenting structural singularity) [1].

Previous work has shown the computation of the external active torques for satellite attitude guidance, using an inverse dynamics method [2]. Using a similar formulation for the satellite attitude guidance based on the same inverse dynamics method, but this time for the case where reaction wheels are considered instead of directly applying external torques, this paper shows how to compute the angular velocities of the three reaction wheels which ensure the minisatellite to rotate from the initial to the final attitude.

The parameterization using the *x-y-z sequence of rotations*, called also Tait-Bryan angles or Cardan angles, is considered here for satellite/rigid body attitude dynamics formulation [2]-[5].

Thus, $\varphi_1$, $\varphi_2$ and $\varphi_3$ are the angles of the *x-y-z* sequence of right hand positive rotations, used to parameterize the orientation/attitude of the reference frame ($\mathrm{O} \equiv \mathrm{G}$, $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$) attached to the rigid body with respect to the inertial reference frame ($\mathrm{O}$, $\overrightarrow{\boldsymbol{x}_0}, \overrightarrow{\boldsymbol{y}_0}, \overrightarrow{\boldsymbol{z}_0}$). For a satellite in orbit around the Earth, the so-called Local Vertical Local Horizon (LVLH) reference frame can be used as inertial reference frame [6]: the LVLH origin is located on the reference orbit, with its *z*-axis pointing in the nadir direction, its *y*-axis being normal to the orbital plane, opposite the angular momentum vector of the reference orbit and the *x*-axis completing the right-hand orthogonal frame.

The *x-y-z sequence of rotations* is considered as follows: $\varphi_1$ corresponds to a first rotation around *x* axis, the pitch angle $\varphi_2$ corresponds to a second *y*-axis elementary rotation, finally the roll angle $\varphi_3$ corresponds to a third *z*-axis elementary rotation. The singularity ("gimbal lock") associated to the considered *x-y-z* sequence of rotations is encountered for $\beta = \pm\frac{\pi}{2}$, it will be avoided in our numerical example; this singularity can be easily solved by alternate orientation representation in the proximity of singularities or by using a parameterization without singularities such as the 3×3 full rotation matrix representation of a 3D rotation.

## 2. DYNAMICS FORMULATION FOR THE SATELLITE ATTITUDE GUIDANCE, WHEN DIRECTLY APPLYING ACTIVE EXTERNAL TORQUES

Satellite attitude dynamics/guidance is the classical problem of a rigid body with one fixed point. Thus, the satellite can be considered as a rigid body rotating about its center of mass, with $A = J_{xx}$, $B = J_{yy}$ and $C = J_{zz}$ the principal moments of inertia of the satellite with respect to the body-attached reference frame ($\mathrm{O} \equiv \mathrm{G}$, $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$), placed in $\mathrm{O} \equiv \mathrm{G}$ (center of mass). Denoting as usual by $\omega_x$, $\omega_y$ and $\omega_z$ the components of the satellite/rigid body angular velocity vector in the body-attached reference frame ($\mathrm{O} \equiv \mathrm{G}$, $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$), for the considered *x-y-z sequence of rotations* parameterization their expression is [2]-[4]:

$$\begin{cases} \omega_x = \dot{\varphi}_1 \cos\varphi_2 \cos\varphi_3 + \dot{\varphi}_2 \sin\varphi_3 \\ \omega_y = -\dot{\varphi}_1 \cos\varphi_2 \sin\varphi_3 + \dot{\varphi}_2 \cos\varphi_3 \\ \omega_z = \dot{\varphi}_1 \sin\varphi_2 + \dot{\varphi}_3 \end{cases} \tag{1}$$

When directly applying active external torques, i.e., when the external torques are punctually applied following $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$ axes, the dynamics of a satellite is expressed in the body-attached reference frame ($\mathrm{O} \equiv \mathrm{G}$, $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$) using the following Euler's equations of motion [2]-[6]:

$$\begin{cases} A\dot{\omega}_x - (B - C)\omega_y\omega_z = M_x^{\mathrm{act}} + M_x^{\mathrm{pert}} \\ B\dot{\omega}_y - (C - A)\omega_z\omega_x = M_y^{\mathrm{act}} + M_y^{\mathrm{pert}} \\ C\dot{\omega}_z - (A - B)\omega_x\omega_y = M_z^{\mathrm{act}} + M_z^{\mathrm{pert}} \end{cases} \tag{2}$$

where $(M_x^{\mathrm{act}}, M_y^{\mathrm{act}}, M_z^{\mathrm{act}})$ is the active external torque vector and $(M_x^{\mathrm{pert}}, M_y^{\mathrm{pert}}, M_z^{\mathrm{pert}})$ the undesired perturbations torque vector (including perturbations such as solar radiation pressure, micrometeoroids, atmospheric drag, higher order of Earth gravity field, depending of the orbit). Both torque vectors are expressed in the body-attached reference frame $(\mathrm{O} \equiv \mathrm{G},\ \overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1})$.

## 3. DYNAMICS FORMULATION FOR THE SATELLITE ATTITUDE GUIDANCE, WHEN USING REACTION WHEELS

Instead of directly/punctually applying active external torques, let us consider that the satellite is actuated on its $\overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1}$ axes by using three identical reaction wheels. Let us denote by $I_{w_1}$ the moment of inertia of the reaction wheel actuating $\overrightarrow{\boldsymbol{x}_1}$ axis with respect to this $\overrightarrow{\boldsymbol{x}_1}$ axis, while the moments of inertia $J_{w_{1},y}$ and $J_{w_{1},z}$ of the same reaction wheel with respect to $\overrightarrow{\boldsymbol{y}_1}$ and $\overrightarrow{\boldsymbol{z}_1}$ will be considered as already included in $B$ and $C$, respectively.

Similarly, $I_{w_2}$ denotes the moment of inertia of the reaction wheel actuating $\overrightarrow{\boldsymbol{y}_1}$ axis with respect to this $\overrightarrow{\boldsymbol{y}_1}$ axis, while the moments of inertia $J_{w_{2},x}$ and $J_{w_{2},z}$ of the same reaction wheel with respect to $\overrightarrow{\boldsymbol{x}_1}$ and $\overrightarrow{\boldsymbol{z}_1}$ will be considered as already included in $A$ and $C$, while $I_{w_3}$ denotes the moment of inertia of the reaction wheel actuating $\overrightarrow{\boldsymbol{z}_1}$ axis with respect to this $\overrightarrow{\boldsymbol{z}_1}$ axis, while the moments of inertia $J_{w_{3},x}$ and $J_{w_{3},y}$ of the same reaction wheel with respect to $\overrightarrow{\boldsymbol{x}_1}$ and $\overrightarrow{\boldsymbol{y}_1}$ will be considered as already included in $A$ and $B$. More precisely, we will have:

$$A = J_{xx} + J_{w_{2},x} + J_{w_{3},x},\ B = J_{yy} + J_{w_{1},y} + J_{w_{3},y} \text{ and } C = J_{zz} + J_{w_{1},z} + J_{w_{2},z}.$$

For this case of using reaction wheels, instead of directly/punctually applying active external torques, the Euler's equations of motion take the following form [7], different compared with (2):

$$\begin{cases}(A+I_{w_1})\dot{\omega}_x - \left[(B+I_{w_2}) - (C+I_{w_3})\right]\omega_y\omega_z + (I_{w_3}\omega_{w_3}\omega_y - I_{w_2}\omega_{w_2}\omega_z) = -I_{w_1}\dot{\omega}_{w_1} + M_x^{\mathrm{pert}} \\ (B+I_{w_2})\dot{\omega}_y - \left[(C+I_{w_3}) - (A+I_{w_1})\right]\omega_z\omega_x + (I_{w_1}\omega_{w_1}\omega_z - I_{w_3}\omega_{w_3}\omega_x) = -I_{w_2}\dot{\omega}_{w_2} + M_y^{\mathrm{pert}} \\ (C+I_{w_3})\dot{\omega}_z - \left[(A+I_{w_1}) - (B+I_{w_2})\right]\omega_x\omega_y + (I_{w_2}\omega_{w_2}\omega_x - I_{w_1}\omega_{w_1}\omega_y) = -I_{w_3}\dot{\omega}_{w_3} + M_z^{\mathrm{pert}}\end{cases} \quad (3)$$

So, in this case of using reaction wheels, we have considered that no active external torque was directly/punctually applied as in (2), i.e. $M_x^{\mathrm{act}} = M_y^{\mathrm{act}} = M_z^{\mathrm{act}} = 0$.

But the angular velocities $\omega_{w_1}$, $\omega_{w_2}$ and $\omega_{w_3}$ of the three reaction wheels have been considered instead in (3), of course expressed with respect to the body-attached reference frame $(\mathrm{O} \equiv \mathrm{G},\ \overrightarrow{\boldsymbol{x}_1}, \overrightarrow{\boldsymbol{y}_1}, \overrightarrow{\boldsymbol{z}_1})$.

Obviously, $\omega_{w_1}$ is oriented following $\overrightarrow{\boldsymbol{x}_1}$ axis, $\omega_{w_2}$ following $\overrightarrow{\boldsymbol{y}_1}$ axis and $\omega_{w_3}$ is oriented following $\overrightarrow{\boldsymbol{z}_1}$ axis.

## 4. INVERSE DYNAMICS METHOD FOR COMPUTING THE ANGULAR VELOCITIES OF THE THREE REACTION WHEELS USED FOR ACTUATION

Let us consider the following attitude guidance maneuver: from its fixed initial attitude posture (given by $\varphi_1(0)$, $\varphi_2(0)$ and $\varphi_3(0)$, with null initial angular velocities $\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = \dot{\varphi}_3(0) = 0$), the satellite must rotate to another fixed final attitude posture (given by $\varphi_1(t_f)$, $\varphi_2(t_f)$, $\varphi_3(t_f)$, also with null final angular velocities $\dot{\varphi}_1(t_f) = \dot{\varphi}_2(t_f) = \dot{\varphi}_3(t_f) =$

0). In order to perform this attitude guidance maneuver, the following simple inverse dynamics method is proposed:

→ *Step 1*: The attitude motion to be reproduced is known. Thus, knowing the evolutions of $\varphi_1$, $\varphi_2$, $\varphi_3$ and of their derivatives during the motion interval $[0, t_f]$, the components of the angular velocity vector $\omega_x$, $\omega_y$ and $\omega_z$ and their derivatives are easily computed from equations (1) (they can be numerically computed by finite differentiation).

The undesired torque $(M_x^{\text{pert}}, M_y^{\text{pert}}, M_z^{\text{pert}})$ from possible perturbations is also considered as known.

→ *Step 2*: • *Case of active external torques directly/punctually applied*: the active torques necessary to perform the required attitude guidance maneuver can be easily computed from (2), at each moment $t \in [0, t_f]$:

$$\begin{cases} M_x^{\text{act}} = A\dot{\omega}_x - (B-C)\omega_y\omega_z - M_x^{\text{pert}} \\ M_y^{\text{act}} = B\dot{\omega}_y - (C-A)\omega_z\omega_x - M_y^{\text{pert}} \\ M_z^{\text{act}} = C\dot{\omega}_z - (A-B)\omega_x\omega_y - M_z^{\text{pert}} \end{cases} \tag{4}$$

• *Case of using reaction wheels*: the angular velocities $\omega_{w_1}$, $\omega_{w_2}$ and $\omega_{w_3}$ of the three reaction wheels used to perform the required attitude guidance maneuver can be computed from (3), at each moment $t \in [0, t_f]$, as follows:

$$\begin{cases} \dot{\omega}_{w_1} + \dfrac{(I_{w_3}\omega_y)\omega_{w_3} - (I_{w_2}\omega_z)\omega_{w_2}}{I_{w_1}} = \dfrac{(B+I_{w_2}) - (C+I_{w_3})}{I_{w_1}}\omega_y\omega_z - \left(1 + \dfrac{A}{I_{w_1}}\right)\dot{\omega}_x + \dfrac{M_x^{\text{pert}}}{I_{w_1}} \\ \dot{\omega}_{w_2} + \dfrac{(I_{w_1}\omega_z)\omega_{w_1} - (I_{w_3}\omega_x)\omega_{w_3}}{I_{w_2}} = \dfrac{(C+I_{w_3}) - (A+I_{w_1})}{I_{w_2}}\omega_z\omega_x - \left(1 + \dfrac{B}{I_{w_2}}\right)\dot{\omega}_y + \dfrac{M_y^{\text{pert}}}{I_{w_2}} \\ \dot{\omega}_{w_3} + \dfrac{(I_{w_2}\omega_x)\omega_{w_2} - (I_{w_1}\omega_y)\omega_{w_1}}{I_{w_3}} = \dfrac{(A+I_{w_1}) - (B+I_{w_2})}{I_{w_3}}\omega_x\omega_y - \left(1 + \dfrac{C}{I_{w_3}}\right)\dot{\omega}_z + \dfrac{M_z^{\text{pert}}}{I_{w_3}} \end{cases} \tag{5}$$

If expressions (4) are explicit algebraic equations and the unknown active external torques can be directly computed, for the case of using the reaction wheels equations (5) form a linear differential equations system, which can be easily numerically solved using the classical Runge-Kutta methods, thus obtaining the unknown $\omega_{w_1}$, $\omega_{w_2}$ and $\omega_{w_3}$.

## 5. NUMERICAL CASE STUDY OF SATELLITE ATTITUDE MOTION MANEUVER, USING THREE IDENTICAL REACTION WHEELS

The problem is to determine the angular velocities $\omega_{w_1}$, $\omega_{w_2}$ and $\omega_{w_3}$ of the three reaction wheels, so that the satellite to perform the motion from the considered fixed initial attitude posture of the satellite to the required fixed final attitude posture.

The considered case study assumes the following simplifications:

• the undesired torque from possible perturbations is considered negligible, i.e.: $M_x^{\text{pert}} \cong 0$, $M_y^{\text{pert}} \cong 0$, $M_z^{\text{pert}} \cong 0$;

• the three reaction wheels are identical, thus:

$$I_{w_1} = I_{w_2} = I_{w_3} = I_w. \tag{6}$$

For this particular case with no undesired torque from possible perturbations and with three identical reaction wheels involving (6), the inverse dynamics equations (5) of a satellite actuated by reaction wheels are simplified as follows:

$$\begin{cases} \dot{\omega}_{w_1} + (\omega_y \omega_{w_3} - \omega_z \omega_{w_2}) = \dfrac{1}{I_w}\left[(B-C)\omega_y\omega_z - (A+I_w)\dot{\omega}_x\right] \\ \dot{\omega}_{w_2} + (\omega_z \omega_{w_1} - \omega_x \omega_{w_3}) = \dfrac{1}{I_w}\left[(C-A)\omega_z\omega_x - (B+I_w)\dot{\omega}_y\right] \\ \dot{\omega}_{w_3} + (\omega_x \omega_{w_2} - \omega_y \omega_{w_1}) = \dfrac{1}{I_w}\left[(A-B)\omega_x\omega_y - (C+I_w)\dot{\omega}_z\right] \end{cases} \tag{7}$$

or

$$\begin{cases} I_w\left[\dot{\omega}_{w_1} + (\omega_y \omega_{w_3} - \omega_z \omega_{w_2})\right] = (B-C)\omega_y\omega_z - (A+I_w)\dot{\omega}_x \\ I_w\left[\dot{\omega}_{w_2} + (\omega_z \omega_{w_1} - \omega_x \omega_{w_3})\right] = (C-A)\omega_z\omega_x - (B+I_w)\dot{\omega}_y \\ I_w\left[\dot{\omega}_{w_3} + (\omega_x \omega_{w_2} - \omega_y \omega_{w_1})\right] = (A-B)\omega_x\omega_y - (C+I_w)\dot{\omega}_z \end{cases} \tag{8}$$

Besides the above simplifications, the considered case study concerns an axisymmetric minisatellite with 200 kg mass, with the following mass distribution: $A = B = 50\ \mathrm{kg \cdot m^2}$, $C = 35\ \mathrm{kg \cdot m^2}$, thus $A = B > C$.

In what concerns the three reaction wheels, the principal moments of inertia are: $I_{w_1} = I_{w_2} = I_{w_3} = I_w = 5\ \mathrm{kg \cdot m^2}$. As already mentioned in §3, $J_{w_2,x}$ and $J_{w_3,x}$ are already included in $A$ (being cumulated with $J_{xx}$), $J_{w_1,y}$ and $J_{w_3,y}$ are already included in $B$, while $J_{w_1,z}$ and $J_{w_2,z}$ are already included in $C$.

The requirement is to perform the rotation motion during 100 s, from the fixed initial attitude posture of the axisymmetric minisatellite given by $\varphi_1(0) = 0$, $\varphi_2(0) = \frac{\pi}{3}$ and $\varphi_3(0) = 0$, with null initial angular velocities $\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = \dot{\varphi}_3(0) = 0$, to the required fixed final attitude posture given by $\varphi_1(t_f = 100\mathrm{s}) = \frac{\pi}{2}$, $\varphi_2(100\mathrm{s}) = -\frac{\pi}{3}$, $\varphi_3(100\mathrm{s}) = \frac{\pi}{4}$, also with null final angular velocities $\dot{\varphi}_1(100\mathrm{s}) = \dot{\varphi}_2(100\mathrm{s}) = \dot{\varphi}_3(100\mathrm{s}) = 0$.

More precisely, between 0 to 100 seconds, the evolutions of $\varphi_1$, $\varphi_2$ and $\varphi_3$ will be as shown in Figure 1; in fact the evolutions were obtained by uniform acceleration of the angular variables from 0 to 50 sec, followed by uniform deceleration from 50 to 100 sec.

Figure 2 shows the evolutions of the angular velocity components $\omega_x$, $\omega_y$ and $\omega_z$ corresponding to the evolutions of $\varphi_1$, $\varphi_2$ and $\varphi_3$ angles in Figure 1, where $\omega_x$, $\omega_y$ and $\omega_z$ are obtained from equations (1) in *Step 1* of the inverse dynamics method proposed in §4.

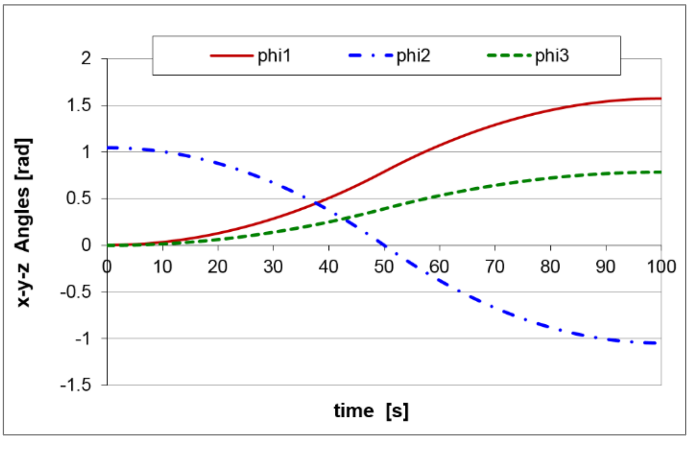


Fig. 1 – *x-y-z* sequence of rotations angles $\varphi_1$, $\varphi_2$, $\varphi_3$ corresponding to the required attitude guidance maneuver

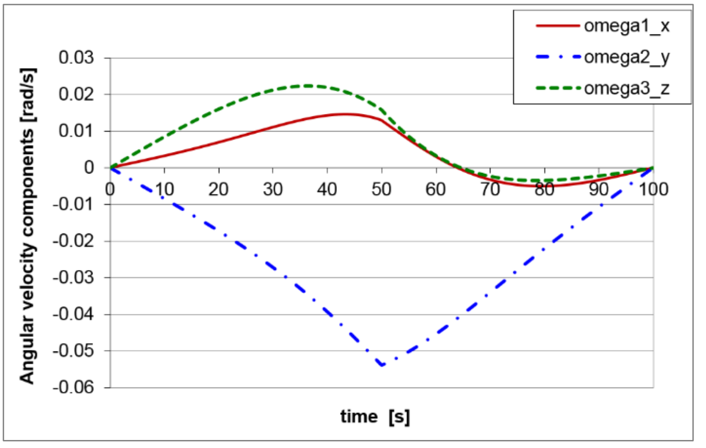


Fig. 2 – Angular velocity evolution for the desired minisatellite rotation motion from Figure 1

By applying *Step 2* of the inverse dynamics method proposed in §4, one obtains:

• *Case of using reaction wheels*: the angular velocities $\omega_{w_1}$, $\omega_{w_2}$ and $\omega_{w_3}$ of the three reaction wheels are computed by numerical integration of (7) using classical Runge-Kutta methods. Figure 3 presents the quantities $I_w\dot{\omega}_{w_1}$, $I_w\dot{\omega}_{w_2}$ and $I_w\dot{\omega}_{w_3}$, i.e., the angular accelerations of the reaction wheels multiplied by their principal moment of inertia $I_w$ .

• *Case of active external torques directly/punctually applied*: the active external torques ($M_x^{\text{act}}$, $M_y^{\text{act}}$, $M_z^{\text{act}}$), needed to perform the motion from the considered fixed initial attitude posture of the satellite to the required fixed final attitude posture, are computed directly from the explicit algebraic equations (4).

Figure 3 intends to compare $I_w\dot{\omega}_{w_1}$ with $M_x^{\text{act}}$, $I_w\dot{\omega}_{w_2}$ with $M_y^{\text{act}}$ and $I_w\dot{\omega}_{w_3}$ with $M_z^{\text{act}}$, with the goal of pointing out the differences in terms of motion generation between the case of using reaction wheels and the case of active external torques directly/punctually applied. These differences are shown in Figure 4, where the quantities $M_x^{\text{act}} - I_w\dot{\omega}_{w_1}$, $M_y^{\text{act}} - I_w\dot{\omega}_{w_2}$ and $M_z^{\text{act}} - I_w\dot{\omega}_{w_3}$. are presented.

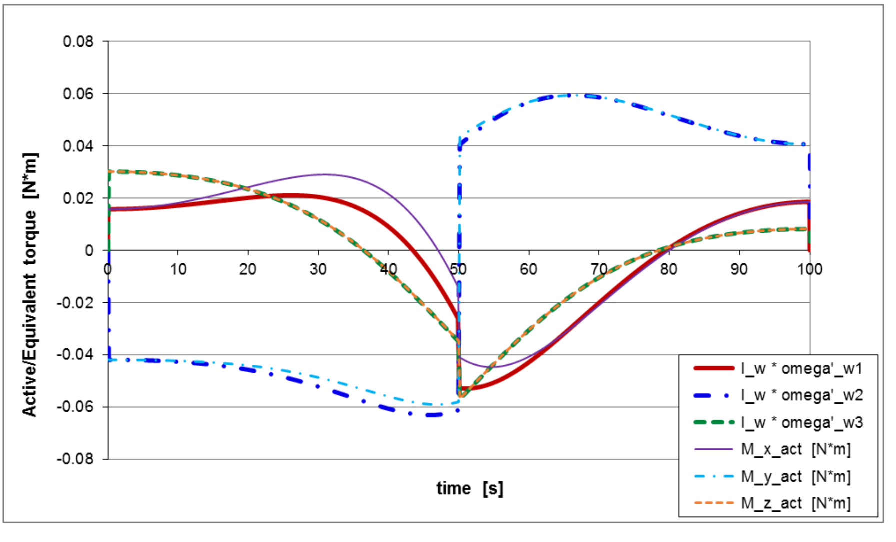


Fig. 3 – Angular accelerations of the reaction wheels multiplied by their principal moment of inertia, versus active external torques punctually applied, both computed by inverse dynamics.

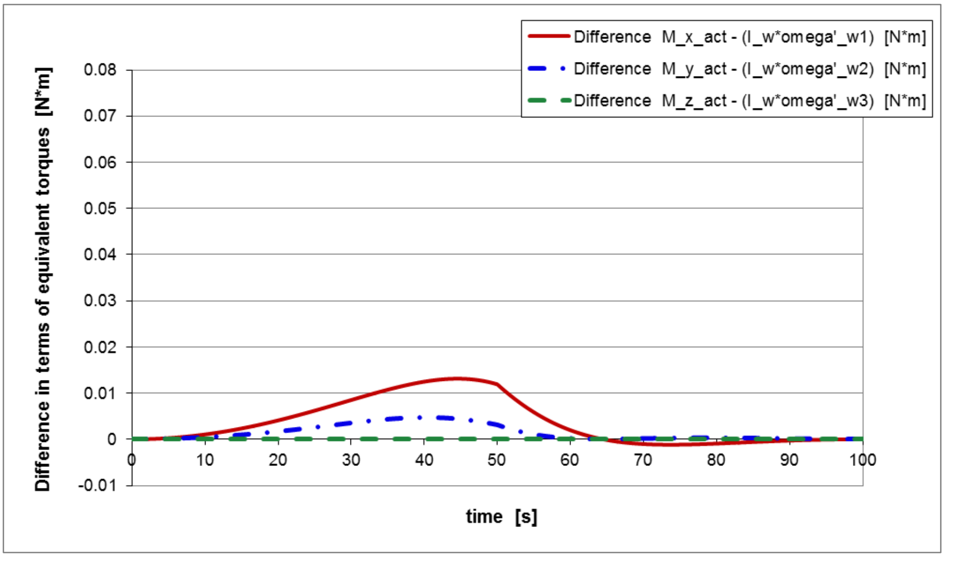


Fig. 4 – The differences $M_x^{\mathrm{act}} - I_w\dot{\omega}_{w_1}$, $M_y^{\mathrm{act}} - I_w\dot{\omega}_{w_2}$ and $M_z^{\mathrm{act}} - I_w\dot{\omega}_{w_3}$ between the case of active external torques directly/punctually applied and the case when using reaction wheels.

It was observed that the differences $M_x^{\mathrm{act}} - I_w\dot{\omega}_{w_1}$, $M_y^{\mathrm{act}} - I_w\dot{\omega}_{w_2}$ and $M_z^{\mathrm{act}} - I_w\dot{\omega}_{w_3}$ in Figure 4 do not depend on $I_w$.

So, if $I_w$ is either $1\ \mathrm{kg}\cdot\mathrm{m}^2$ or $5\ \mathrm{kg}\cdot\mathrm{m}^2$ or $10\ \mathrm{kg}\cdot\mathrm{m}^2$ or any other value, Figure 3 remains the same.

In fact, the differences shown in the figure are:

$$\begin{cases} M_x^{\mathrm{act}} - I_w\dot{\omega}_{w_1} = I_w(\omega_y\omega_{w_3} - \omega_z\omega_{w_2}) \\ M_y^{\mathrm{act}} - I_w\dot{\omega}_{w_2} = I_w(\omega_z\omega_{w_1} - \omega_x\omega_{w_3}) \\ M_z^{\mathrm{act}} - I_w\dot{\omega}_{w_3} = I_w(\omega_x\omega_{w_2} - \omega_y\omega_{w_1}) \end{cases} \tag{9}$$

## 6. CONCLUSION

For (mini)satellite attitude guidance maneuver, this paper illustrates the differences between the case when using reaction/momentum wheels and the case of active external torques directly/punctually applied. Both cases are solved numerically by an inverse dynamics method, derived from the direct dynamics equations of rotation motion.

When using three identical reaction wheels, an interesting observation came from the study: in what concerns the quantities $M_x^{\mathrm{act}} - I_w\dot{\omega}_{w_1}$, $M_y^{\mathrm{act}} - I_w\dot{\omega}_{w_2}$ and $M_z^{\mathrm{act}} - I_w\dot{\omega}_{w_3}$, between the active external torques for the case when they are punctually applied and the equivalent torque when using reaction wheels, these differences do not depend on $I_w$. So, one can extrapolate and say that the attitude guidance maneuver can be performed as well with "bigger" or with "smaller" reaction wheels.

Of course, if $I_w$ is small then $\dot{\omega}_{w_1}$, $\dot{\omega}_{w_2}$ and $\dot{\omega}_{w_3}$ must increase, while for bigger $I_w$ then $\dot{\omega}_{w_1}$, $\dot{\omega}_{w_2}$ and $\dot{\omega}_{w_3}$ will be smaller. Further work will try to explore this interesting observation.

Another further research direction will concern the attitude dynamics and guidance of (mini)satellites when parameterizing the 3D rotation using quaternions [8],[9].

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