Automated Conflict Resolution in Air Traffic Management

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Abstract: *Collision prevention strategy in ATM is not only a short-term coordination between safety and efficiency, but also a long-term planning for national policy of airspace and air transportation system. The optimization of system should be based on good command of equipment, staff, procedure and operation restriction to meet the real-time requirement and integrity. Essentially, collision prevention strategy in ATM consists in finding synthetic and effective automatized avoidance methods in order to reduce the possibility of dangerous approach or collision, of which the conflict detection, conflict resolution and resolution trajectory optimization are the key technology. This paper proposes a research on the standard of intelligent prevention of collision, and theory and methodology of its optimization from the systems engineering perspective. This paper describes thoroughly the decision-making procedure of ATC, establishes the optimized target for ATC's decision-making, and puts forward an optimization of conflict detection and conflict resolution between several aircrafts in 4D space. The medium and long term collision prevention strategy through adjusting speed or altitude and short-term collision prevention strategy through adjusting heading makes the intelligent ATC system a perfect one.*

Key Words: *ATM, ATC, Automated Conflict Resolution*

1. INTRODUCTION

Air Traffic Conflict Detection and Resolution (CDR) involves multiple domains, the modeling of physical systems such as aircraft, encoding conflict detection algorithms as well as the procedures (tasks) for conflict resolution. In Air Traffic Management research there exists a multitude of conflict detection and resolution methods, each with its own specific modeling method. A common trait in most of these systems is that the various agents in the system exhibit hybrid behavior, continuous dynamics due to the physical systems such as aircraft dynamics, and discrete modes of operation such as the modes of the Flight Management System (FMS). [1], [3-4] A key aspect of landing multiple aircraft on a single runway is the Conflict Detection and Resolution. In the context of the aircraft landing, a conflict is defined as the situation of loss of minimum safe separation between two aircraft. The conflict detection and resolution process consists of predicting, communicating to the pilot, and resolving the conflict. Typically, evaluating the likelihood of a conflict is based on the current position and velocity of an aircraft. The conflict is then resolved by determining a maneuver required by one or more aircraft to avoid the predicted conflict. The required information is then provided to the Air Traffic Controller who communicates with the pilot to resolve the conflict. [1], [3-4]

In the current organization of the Air Traffic Management (ATM) system the centralized Air Traffic Control (ATC) is in complete control of the air traffic and ultimately responsible for safety. During the flight, ATC sends additional instructions to them, depending on the actual traffic, to improve the traffic flow and avoid dangerous encounters.

The primary concern of ATC is to maintain a safe separation between the aircraft. The level of accepted minimum safe separation may depend on the density of the air traffic and the region of the airspace. [3-4]

Uncertainty is introduced in air traffic by the action of wind, incomplete knowledge of the physical coefficients of the aircraft and unavoidable imprecision in the execution of ATC instructions. To perform conflict detection one has to evaluate the possibility of future conflicts given the current state of the airspace and taking into account uncertainty in the future position of the aircraft. For this task, one needs a model to predict the future.

In a probabilistic setting, the model could be either an empirical distribution of future aircraft positions, or a dynamical model, such as a stochastic differential equation, that describes the aircraft motion and defines implicitly a distribution for future aircraft positions. On the basis of the prediction model one can evaluate metrics related to safety. [3-4]

2. CONFLICT ENVELOPE MODELS

For the future ATM system, 4D trajectory based operation, defined as a precise description of an aircraft path in three-dimensional space and time, is an important concept to meet future air traffic growth. The primary concern of the ATM system is to guarantee safety, and one of the major safety critical situations is a conflict between aircraft, i.e., the situation where two or more aircraft experience a loss of the minimum allowed separation. All problems in the real world contain uncertainties which arise due to disturbances, modeling and estimation errors, and aircraft also fly under various uncertainties such as unpredicted weather and navigation errors. These uncertainties have effects on the aircraft motion and therefore conflict detection and resolution. [3-4]

The stochastic optimal control method is combined with the probabilistic conflict detection algorithm to guarantee the resolution of potential conflicts between aircraft under the wind uncertainty. By constructing the response surfaces, the optimal conflict-free trajectories starting from any given initial states under the wind uncertainty are generated in real time without actually solving the stochastic optimization problems and sacrificing accuracy. [3-4]

We consider the conflicts between aircraft in two-dimensional horizontal plane in which the aircraft coming from different directions merge to the waypoint. The aircraft dynamics is given by the following point mass model with three state variables $x = (x, y, \Psi)^T$ and one control variable - u: [3-4]

$$\dot{x} = v cos\Psi + \mathrm{w_x} \tag{1}$$

$$\dot{y} = v sin\Psi + \mathrm{w_y} \tag{2}$$

$$\dot{\Psi} = u \tag{3}$$

$$C\big((x,y),(x',y')\big) = \sigma_w^2 \exp\ (-\mu_x|x - x'|) exp\big(-\mu_y|y - y'|\big) \tag{4}$$

$$w_x(x,y) \approx \sum_{i=1}^{N_{KL}} \left(\sqrt{\lambda_i}\ g_i\ (x,y)\,\theta_{xi}\right) \tag{5}$$

$$w_y(x,y) \approx \sum_{i=1}^{N_{KL}} \left(\sqrt{\lambda_i}\ g_i\ (x,y)\,\theta_{yi}\right) \tag{6}$$

$$\lambda_i g_i(x,y) = \int_D C\big((x,y),(x',y')\big)\ g_i\ (x',y')\,dx'dy' \tag{7}$$

$$d_{H\ min} \leq L_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}\ \ (\forall\ i,j \in \{1,\ldots,s\}: i \neq j) \tag{8}$$

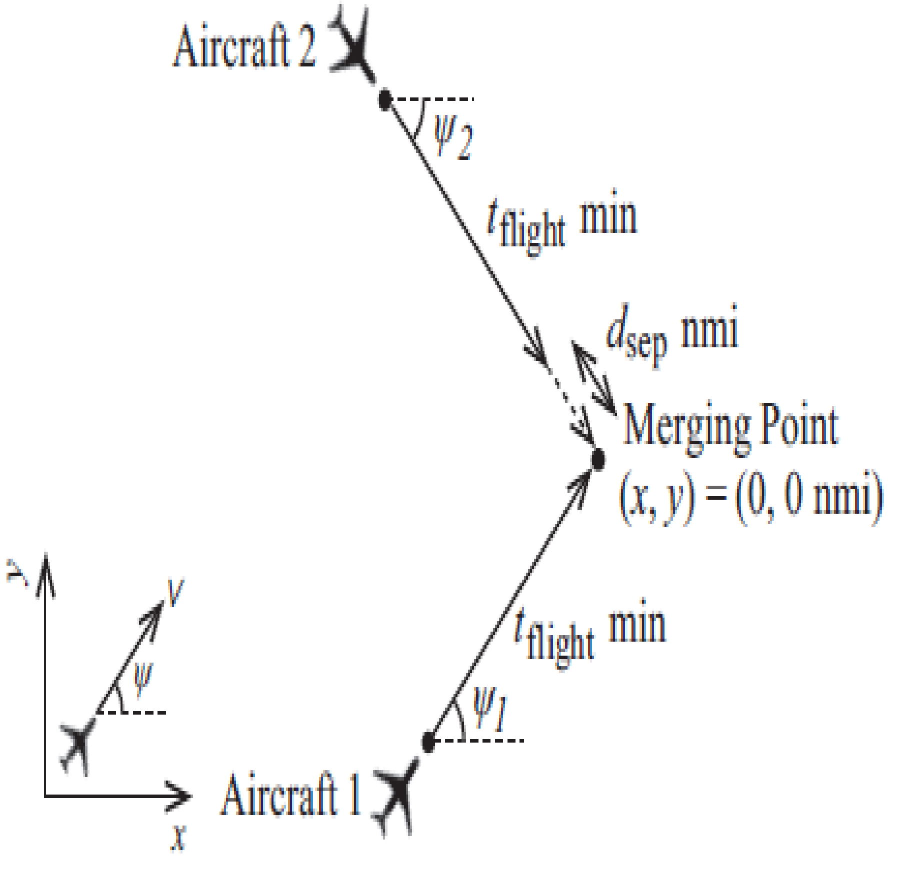

Fig. 1 Conflict scenario for CD-Conflict Detection Problem

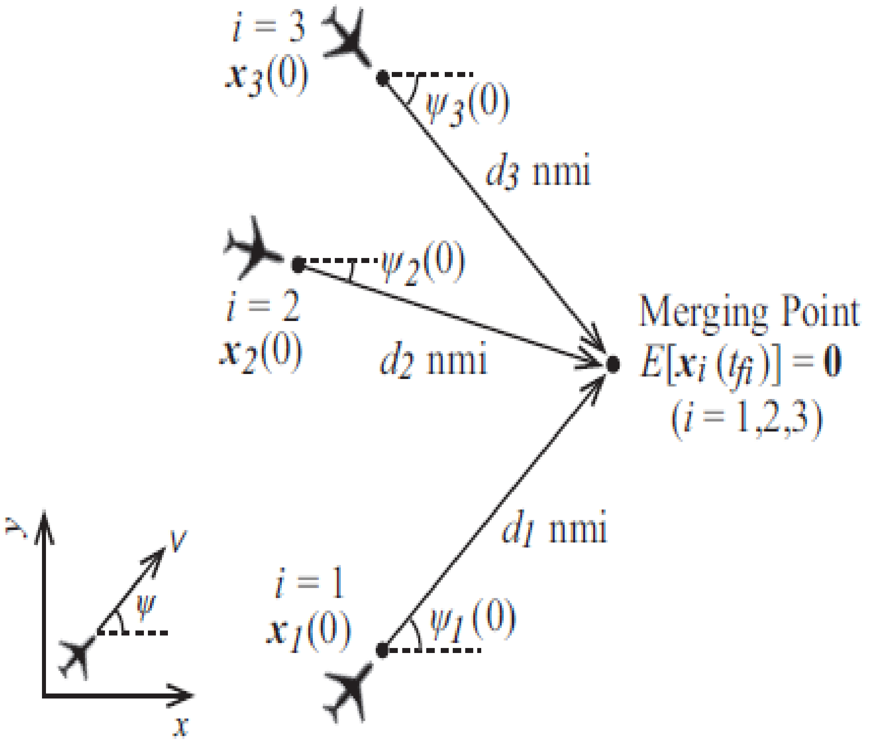

Fig. 2 Conflict scenario for CR-Conflict Resolution Problem

The wind model accounts for only the stochastic component, i.e., the wind prediction error representing the uncertainty in the deterministic meteorological prediction. Thus, the wind velocities w_x and w_y are referred to the wind prediction errors.

The random processes $w_x(x,y)$ and $w_y(x,y)$ are approximated as a linear combination of deterministic functions multiplied by independent random variables using the Karhunen-Loeve (KL) expansion, where θ_{xi} and θ_{yi} are the independent standard Gaussian random variables; N_{KL} is the number of independent random variables; and λ_i and $g_i(x,y)$ are the eigenvalue and eigenfunction of the integral equation in descending order of the magnitude of the eigenvalue λ_i, where x and y are defined over a given domain D. Thus, the wind error is represented as the spatially correlated wind error with the finite number of independent random variables by using the KL expansion. [5-6], [7-9]

Computing the distance between each pair of aircraft, we can identify the potential conflicts. The subscript i and j denote the i-th and j-th aircraft; s is the total number of aircraft; $d_{H\ min}$ is the horizontal separation requirement; and $L_{ij} = L_{ji}$ is the horizontal distance between the i-th and j-th aircraft. [5-6], [7-9]

The aircraft positions x and y become the random variables because the equations contain the stochastic terms w_x and w_y. Since x and y are the random variables, the horizontal distance between aircraft L also becomes a random variable. L cannot be determined analytically, and it needs to be calculated numerically. The stochastic optimal control method is combined with the proposed conflict detection algorithm to solve the conflict resolution problem [5-6], [7-9]. The following continuous-time stochastic optimal control problem is considered. Let's determine the state variables $x(t)$, the control variables $u(t)$ and the terminal time t_f on the time interval $t \in [0,\ t_f]$ that minimize the cost function: [10-14]

$$J = E\left[g_M\ \left(x(0),\ x(t_f)\right) + \int_0^{t_f} g_L(x(t), u(t), t)\ dt\right] \tag{9}$$

Subject to the dynamic constraints:

$$\dot{x}(t) = f(x(t), u(t), t) \tag{10}$$

The boundary conditions: [10-14]

$$b_{min} \le E\left[b\left(x(0), x(t_f)\right)\right] \le b_{max} \tag{11}$$

$$\eta_{min} \le P_r[c_{min} \le c(x(t), u(t), t) \le c_{max}] \le \eta_{max} \tag{12}$$

Where g_M and g_L define the Mayer and Lagrange terms in the cost function, respectively; *f* is the system dynamics; *b* expresses the boundary condition functions; *c* defines the path constraint functions; and η is the confidence level.

By solving the stochastic optimal control problem for conflict resolution, the optimal conflict-free trajectory under the wind uncertainty is generated. [3-4], [5-6]

We use the convolution integral to estimate the conflict probability. The conflict probability CP_{i-j} - between the aircraft *i* and *j* is given by the following equations. [3-4]

$$CP_{i-j} = \int_{-\Delta T_{sep}}^{\Delta T_{sep}} P_{i-j}\ (\tau)\ d\tau \tag{13}$$

$$P_{i-j}(\tau) = \int_{-\infty}^{\infty} P_i\ (t)\ \ P_j\ (t+\tau) d\tau \tag{14}$$

Where $P_{i-j}(\tau)$ expresses the conflict probability when the time separation (the time difference of the time of arrival) at the merging point between the aircraft i and j is τ. Therefore, by using $P_{i-j}(\tau)$ the conflict probability is CP_{i-j}, because the conflict occurs when τ satisfies the following condition: [3-4], [5-6]

$$-\Delta T_{sep} \le \tau \le \Delta T_{sep} \tag{15}$$

By the theory of the convolution integral, $P_{i-j}(\tau)$ is expressed as the Gaussian distribution.

Therefore, by using CP_{i-j} and $P_{i-j}(\tau)$ the conflict probability between any two aircraft can be estimated. [5-6]

3. AUTOMATED CONFLICT RESOLUTION

We would like to solve the problems of tracking multiple aircraft and managing their identities in a noisy environment for ATC, using the RMIMM-Residual-Mean Interacting Multiple Model - algorithm as a state estimator. [3-4], [5-6]

A noisy measurement is called *clutter*, from a radar surveillance system for ATC. Clutter is defined as measurements originated from non-targets, such as nearby objects, weather, and electromagnetic interference that are generally random in number, location and intensity. From the noisy measurements we cannot tell how many aircraft are in the surveillance region and how the trajectories of the aircraft look like.

Thus, the goal of multiple-aircraft tracking and identity management in a noisy environment is to keep track of the aircraft trajectories and of their identities simultaneously from the noisy measurements. In order to develop an algorithm for the conflict detection and resolution for ATC, we need a state estimation algorithm which can track the trajectories of all the aircraft in the surveillance region of the sensors used (radars in the case of the current ATC). [3-4], [5-6]

In fact, the multiple-target tracking and identity management (MTIM) problem is complicated by several problems related to the quality of available information about the targets. Firstly, the surveillance system makes measurement errors assumed to be Gaussian and may miss measurements entirely. In certain environments, the surveillance system may also measure extraneous signals, known as clutter.

The behavior of the targets also adds complexity to the problem: many targets may be interacting in a small spatial region, and these interactions increase the entropy (a measure of uncertainty) of the system.

These issues motivate the extension of the MTIM algorithm to cluttered environments and the solving of the problems that arise in these larger, more complex systems. One such algorithm is the Joint Probabilistic Data Association- JPDA algorithm in which target kinematic information (position and velocity) is used for associating measurements with targets. [3-4], [5-6]

The approximate JPDA is useful for tracking many aircraft, but unfortunately does not give a stochastic association matrix whose elements represent measurement-target association probabilities, thus losing some of the physical constraints imposed by the system.

Assignment algorithms have also been used to overcome the computational complexity of data association for multiple-target tracking problems. These assignment algorithms minimize the sum of all probabilistic distances between measurements and expected target positions. This means that the assignment algorithms select the measurement that is closest to the predicted measurement without considering measurement-target correlation like nearest neighbor data association. [3-4], [5-6]

For the data association, we develop a modified approximate JPDA- MAJPDA- which is more computationally efficient than JPDA and provides a stochastic association matrix as JPDA. The Joint Probabilistic Data Association- JPDA- algorithm is an extension of PDA- Probabilistic Data Association- to the situation in which there is a known number of targets in clutter.

The key to the JPDA algorithm is the evaluation of the conditional probabilities of the following joint events: [3-4], [5-6], [15]

$$\Theta = \bigcap_{j=1}^{m_k} \theta_{jt}, \; j = 1, \dots, m_k \quad ; \; t = 0,1, \dots, T \tag{16}$$

where θ_{jt} := {measurement *j* originated from target *t* }.

A joint event association matrix can be represented by the matrix

$$\widehat{\boldsymbol{\Omega}} = \left|\widehat{\omega_{Jt}}(\Theta)\right|; \;\; \widehat{\omega_{Jt}}(\Theta) \;\; \begin{cases} 1 \;\; if \;\;\;\; \theta_{jt} \in \Theta \\ 0 \;\; \text{otherwise} \end{cases} \tag{17}$$

Where $\widehat{\boldsymbol{\Omega}}$ is a $\left(m_k \times (T+1)\right)$ matrix. $\widehat{\omega_{Jt}} = 1$ represents an event that measurement *j* originates from target *t*. The interaction of multiple targets could make the problem complex. In the identity management algorithm, we assume that local information arrives in the form of a column vector whose elements represent the probabilities of identity belief of a target.

The MAJPDA algorithm cannot differentiate between the two measurements at time *k*; as a result, uncertainty in the belief matrix is essentially maximum. This uncertainty remains even after the aircraft separate. However, from analyzing the dynamics of the two aircraft, a belief matrix with lower entropy can be determined. If the aircraft are assumed to turn at $3^0/sec,$ a common turn rate for commercial jets, neither aircraft can execute a 90^0 turn in one time step. [3-4]

The swapping of aircraft-target association is physically impossible. Indeed, the only possible outcome is that Aircraft ***i(j)*** remains associated with Target 1(2). This should yield a belief matrix equal to the identity matrix, which is minimum entropy. [3-4]

The algorithm RMIMM is applicable to both ground and airborne control scenarios in ATC- to ground control using radar information, and to airborne control using information from Automatic Dependent Surveillance-Broadcast (ADS-B) data.

For the conflict detection, we use the aircraft model which has two flight modes: the constant velocity and the coordinated turn. ADS-B information is assumed to be used for measurements, and thus both aircraft's position and velocity information is available for the state estimation. However, there is no restriction in using only the aircraft's position information to estimate the aircraft's current states with RMIMM. [3-4]

Using Enhanced Traffic Management System- ETMS data, we show that even in cases in which the turn mode is a small portion of the whole flight trajectory, the accuracy of this hybrid conflict detection algorithm is better than that of continuous schemes, especially in the airspace around waypoints and airports where several airways converge. [3-6]

Optimized conflict resolution algorithms produce a resolution maneuver, which minimizes a cost function such as deviation from the original trajectory, flight time, fuel consumption, or energy. In general, the optimization process is computationally intensive and difficult to implement in real-time.

Resolution maneuvers are determined by solving a convex optimization problem to minimize the total length of trajectories (or energy) for the selected type. Due to the properties of convex optimization, the algorithms are numerically efficient. However, randomized algorithms could produce different solutions to the same conflict problems.

Conflict resolution maneuvers are obtained from a closed-form analytic solution, and thus can be applied in real-time to two-aircraft conflicts. In order to resolve multiple-aircraft conflicts, pairwise resolution algorithms should be executed successively, but it is fairly easy to come up with situations in which the successive application of pairwise resolution does not guarantee safety for multiple-aircraft conflicts. [3-6]

For example, a conflict situation may be classified as several predefined cases at the same time. It results from uncertainties in the aircraft's position and heading. The ambiguity about which rule should be chosen may lead to an unsafe resolution. The method may also require many rules to completely cover all possible conflicts. Aircraft are assumed to fly in the force field generated by a potential function; the forces induced by the potential function form a resolution maneuver.

Most importantly, safety cannot be proven about such multiple-aircraft maneuvers.

We present a conflict resolution algorithm, which is called Protocol-Based Conflict Resolution -PBCR, for multiple-aircraft conflicts. The PBCR algorithm is simple and easily understandable since the protocol is obtained from a closed-form analytic solution. Therefore, it can be implemented in airborne systems for real-time conflict resolution, as well as in ATC ground systems. [3-4]

Most importantly, the algorithm always guarantees safe conflict resolution within the limits of the model used. This is the main difference from many other currently available multiple aircraft resolution solutions. Since the aircraft fly in a vertically stratified airspace in the current ATC, they are assumed to cruise at the same altitude with varying velocities.

The position, velocity, and heading of an aircraft are assumed to be available to all aircraft which are involved in the conflict; this assumption can be justified with the proposed availability of the GPS and ADS-B. For the derivation of the protocol, the multiple-aircraft conflict is categorized into two cases: 1- *exact* and 2- *inexact conflict*. First, we derive a closed form analytic solution describing the resolution maneuver for the exact conflict case, which represents the situation in which all aircraft would come into a conflict at a single point in time and space. This result is then generalized to cover the inexact conflict case, in which conflict points of multiple aircraft do not coincide in time and space. [3-4]

A finite partition of the airspace around the conflict is constructed in real time according to the minimum relative angle between the aircraft. With the results from the exact and inexact conflict cases, the protocol for resolving the worst case conflict within each partition is derived. Heading change from the original path is the primary control input and is used exclusively when the conflict is exact. Velocity change is also used as a control input when an exact conflict assumption fails. [3-4]

The conflict prevention algorithm runs in real time, and detects all conflicts when an aircraft changes its heading and/or speed therefore it changes its flight-mode. By applying the conflict prevention algorithm, we find the smallest group of aircraft which will not cause another conflict. Computational complexity of the conflict prevention algorithm is polynomial in the number of aircraft, and thus the PBCR with the conflict prevention algorithm can be implemented with real time. [3-4]

Finally, we combine the Flight-Mode-Based Conflict Detection-FMBCD algorithm and the Protocol-Based Conflict Resolution algorithm and validate it with actual air traffic data. Multiple-aircraft conflict resolution is motivated by the fact that pairwise conflict resolution is not guaranteed to resolve multiple-aircraft conflicts. Since optimization procedures are computationally intensive, they are undesirable for real-time airborne applications. While the majority of conflicts occurring in the current airspace are pairwise conflicts, multiple-aircraft conflict resolution methods are important for two reasons. [3-4]

First, a method that evolves from treatment of two aircraft to treatment of multiple aircraft conflicts would be efficient in resolving even today's small number of multiple aircraft problems; secondly, as the airspace and air traffic system evolve to a stage in which aircraft are more often flying user preferred routes, one would expect more multiple aircraft conflicts. [3-4]

4. NUMERICAL SIMULATION AND CONCLUSIONS

When a conflict is detected, all aircraft involved in the conflict prepare to initiate a conflict resolution maneuver. The aircraft involved in the conflict are assumed to be flying at constant velocity during a resolution maneuver. [1], [3-4] Now, let's define for aircraft *i* and aircraft *j*: [2], [5-6], [7-9], [15]

$$\phi_i = \psi_i - u \,; \;\; \bar{a} = v_i \cos\phi_i - v_j \cos\psi_j \,; \; \bar{b} = r_i \cos\theta_i - r_j \cos\theta_j \tag{18}$$

$$\bar{c} = v_i \sin\phi_i - v_j \sin\psi_j \,; \; \bar{e} = r_i \sin\theta_i - r_j \sin\theta_j \,; \;\; i,j = 1,2,\dots,N \tag{19}$$

The distance squared between aircraft *i* and aircraft *j* is:

$$\begin{aligned} S_{ij}^1(t) &= \left(x_i(t) - x_j\,(t)\right)^2 + \left(y_i(t) - y_j\,(t)\right)^2 = \\ &= \left[\left(v_i \cos\phi_i - v_j \cos\psi_j\right)t + \left(r_i \cos\theta_i - r_j \cos\theta_j\right)\right]^2 + \\ &+ \left[\left(v_i \sin\phi_i - v_j \sin\psi_j\right)t + \left(r_i \sin\theta_i - r_j \sin\theta_j\right)\right]^2 = \\ &= \left(\bar{a}\,t + \bar{b}\right)^2 + \left(\bar{c}\,t + \bar{e}\right)^2 \end{aligned} \tag{20}$$

The safety condition between aircraft *i* and aircraft *j* becomes

$$\bar{b}^2 + \bar{e}^2 - \frac{\left(\bar{a}\,\bar{b} + \bar{c}\,\bar{e}\right)^2}{\bar{a}^2 + \bar{c}^2} \geq R^2 \tag{21}$$

$$w_{max} = \max_i \; |w_i| \cdot sign(w_i), \;\; i = 1,2,\dots,N. \tag{22}$$

Therefore, a protocol *u* for a general inexact conflict is:

$$u = \begin{cases} u_{exact\ conflict} \cdot sign\,(w_{max}) \;\; if \;\; u_{exact\ conflict} > |w_{max}| \\ w_{max} \;\; if \;\; u_{exact\ conflict} \leq |w_{max}| \end{cases} \tag{23}$$

Since this protocol *u* is computed in the transformed frame, aircraft *i* must actually change its heading by $u - w_i \;;\; i = 1,2,\dots,N$. The protocol for a general inexact conflict is as follows:

New protocol

For i=1,2,...,N:

1. Select the last conflict point among possible conflict points as the center of conflict resolution;

2. *If aircraft* ***i*** *is not involved in the conflict at the origin, adjust its velocity such that* $v_i = \frac{r_i}{T}$.

3. Aircraft ***i*** *computes its own heading change* $u_{exact\ conflict}$ *and computes* w_i, w_{max}, u

4. Aircraft ***i*** *changes its heading by* $u - w_i \;;\; i = 1,2,\dots,N$.

New protocol guarantees safety for the multiple-aircraft inexact conflict.

From the new protocol, a safe heading change required for the conflict resolution increases as θ_{min} decreases. If many aircraft are included in the resolution, it is more likely that θ_{min} would be small. This means that perturbation from the aircraft's original path will be large. Thus, it is desirable to keep a resolution group as small as possible.

The size of the resolution group can be reduced by applying the conflict prevention protocol. In the three aircraft case, only two aircraft take a resolution maneuver to resolve the conflict and thus, the perturbation from the desired paths is smaller than if all three aircraft take a resolution maneuver. In the four aircraft case, one aircraft is excluded from the resolution group and the other three aircraft are involved in the resolution maneuver. In the case of ten aircraft, two aircraft do not take a resolution maneuver and the resolution with only eight aircraft can avoid the conflicts. The PBCR algorithm resolves all conflicts safely, and it is noted that not all aircraft around conflict points join the conflict resolution due to the conflict prevention protocol. We consider non-cooperative conflict resolution between two aircraft, which we model as a dynamic game. In other words, we consider the worst case conflict scenario between two aircraft in which an aircraft (evader) tries to avoid a conflict for any maneuver of the other aircraft (pursuer). By solving this problem, safety for all possible conflict cases within the limits of the model used is guaranteed. We solve this problem through reachable set computation. Reach ability analysis for continuous and hybrid systems is important for the automatic verification of safety properties and for the synthesis of safe controllers for these systems. Using dynamic extension with σ_i as a new state variable, we can obtain a new nonlinear model which is feedback linearizable [10-14]

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\psi}_i \\ \dot{\sigma}_i \end{bmatrix} = \begin{bmatrix} \sigma_i \cos\psi_i \\ \sigma_i \sin\psi_i \\ \omega_i \\ a_i \end{bmatrix} \tag{24}$$

Where a_i is the acceleration of aircraft $\boldsymbol{i}$ and is a new control input. Thus, the new state and input variables are: $\xi_i = [\ x_i \quad y_i \quad \psi_i \quad \sigma_i\ ]^T$ and $\eta_i = [\ a_i \quad \omega_i\ ]^T$.

We introduce a change of the state variables and a change of the input variables

$$z_i = T(\xi_i)\ ; \quad \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}_i = \begin{bmatrix} x_i \\ y_i \\ \sigma_i \cos\psi_i \\ \sigma_i \sin\psi_i \end{bmatrix} \tag{25}$$

$$\eta_i = M(\xi_i)\, u_i\ ; \quad M(\xi_i) = \begin{bmatrix} \cos\psi_i & \sin\psi_i \\ -sin\,\psi_i/\sigma_i & \cos\psi_i\,/\sigma_i \end{bmatrix} \tag{26}$$

Where u_i is the control input for the linearized model.

$$\dot{z}_i = \frac{\partial T}{\partial \xi_i}\dot{\xi}_i \ \rightarrow \dot{z}_i = Az_i + Bu_i \tag{27}$$

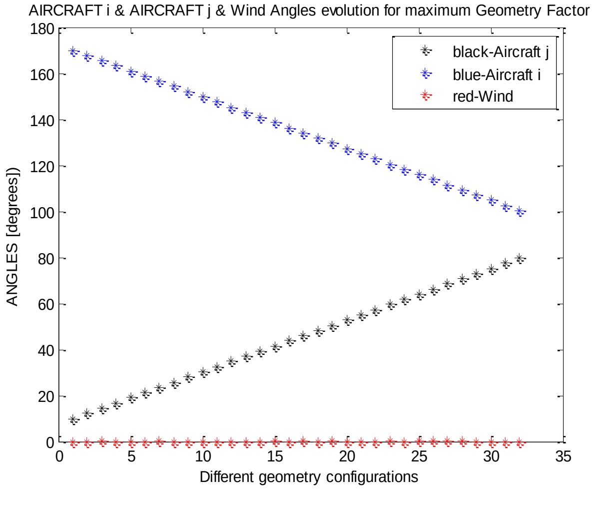

Fig. 3 Aircrafts *i* & *j* & Wind angles evolution for max. G.F.

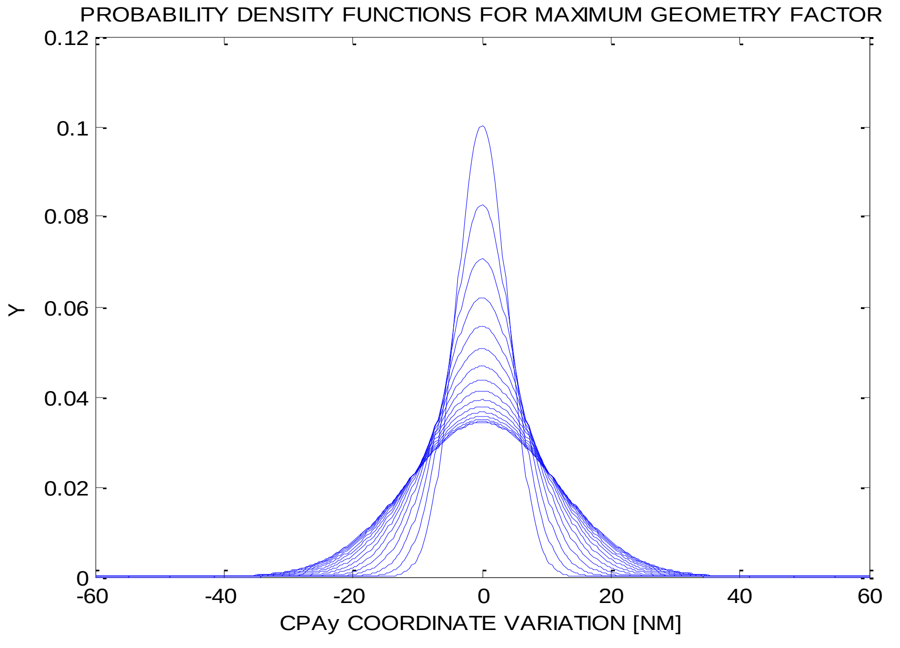

Fig. 4 Probability density function for maximum G.F.

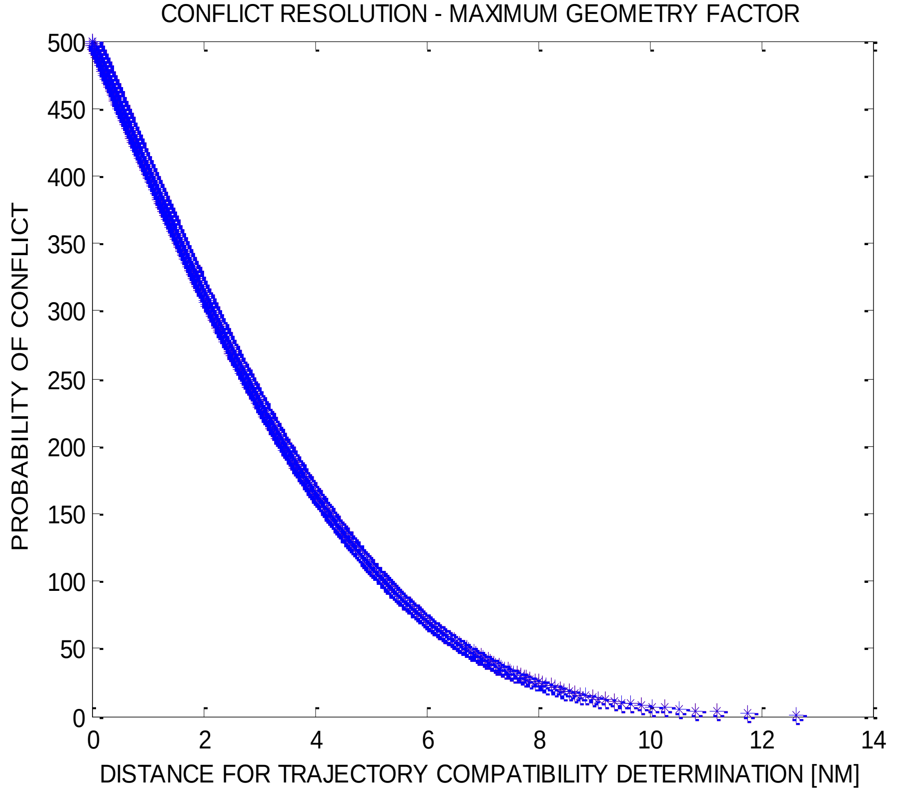

Fig. 5 Conflict Resolution - Scenario 1

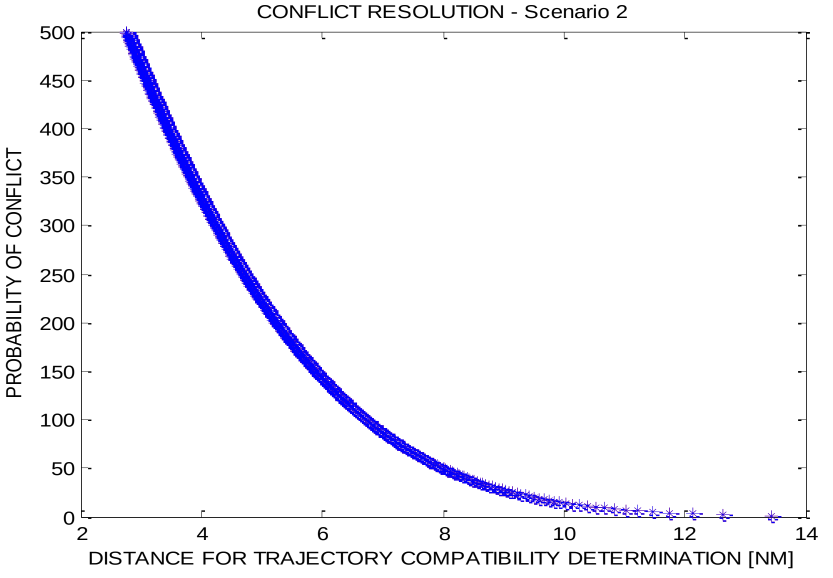

Fig. 6 Conflict Resolution - Scenario 2.

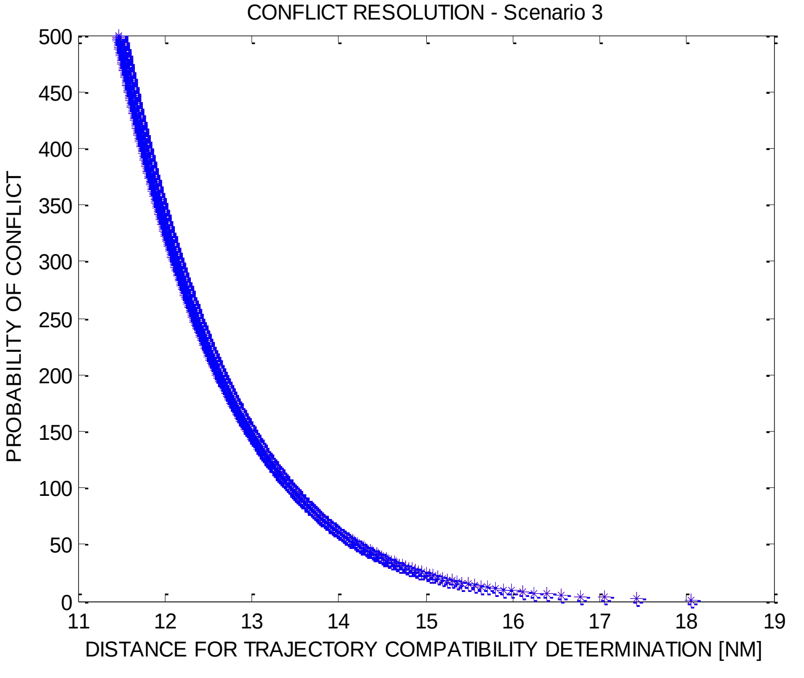

Fig. 7 Conflict Resolution - Scenario 3.

The relative kinematic aircraft model between two aircraft can be obtained by introducing new states $\xi_r = \xi_2 - \xi_1$ in the original nonlinear state space and $z_r = z_2 - z_1$ in the linearized state space.

Thus, a linearized relative kinematic aircraft model is

$$\dot{z_r} = Az_r + Bu_2 - Bu_1;\ \ u_2 \in U,\ \ u_1 \in D \tag{28}$$

This is a linear dynamic game since aircraft ***i*** (u_1) tries to keep aircraft ***j*** from entering into its protected zone (target set) to prevent a conflict, but aircraft ***j*** (u_2) tries to enter the protected zone of aircraft ***i***.

$$D_t\, v(x,t) + \min_{u_2 \in U} \max_{u_1 \in D} \{< D_x v(x,t),\ A(t)\, x(t) + B(t) u_2(t) - B(t) u_1(t) >\} = 0 \qquad (29)$$

Since both aircraft behave optimally, the relative position of aircraft ***j*** moves along the boundary of the unsafe set.

As expected, chattering occurs along the boundary. To avoid such a phenomenon, we could introduce a buffer zone around the boundary so that the control inputs change smoothly as aircraft ***j*** approaches the boundary.

The flight mode changes of an aircraft depend on the pilot's input, which is typically unknown to the surveillance system.

The Conflict Detection- CD system initialize CR- Conflict Resolution with:

1. d_{CPA}, t_{CPA} and Position of Closest Point of Approach (CPA).
2. The ownship trajectory segment.
3. All intruder trajectory segments which overlap regarding the time with ownships trajectory segment.
4. The applicable separation minima.

As soon as an intruder violates ownships Protected Airspace Zone, the CR- Conflict Resolution calculates a new heading in order to resolve the conflict. Upon re-establishment of the safe separation CR- Conflict Resolution is deactivated and the flight plan is being recaptured.

The automated Air Traffic Control Systems are necessary for facilitating and alleviating the work of Air Traffic Control. The primary concern of Air Traffic Control is to maintain a safe separation between the aircraft, and one of the major safety critical situations is a midair conflict. There are two interconnected procedures to predict a midair conflict, i.e., trajectory prediction and conflict detection.

Because all problems in the real world contain uncertainties which arise because of disturbances, modeling and estimation errors, we cannot predict the future position of the aircraft completely. Therefore, we consider the problem of probabilistic conflict detection and propose the novel stochastic conflict detection algorithm by considering various uncertainties during flight, which is the key element for the realization of the future Air Traffic Systems.

REFERENCES

[1] M. A. Patterson, A. V. Rao, Exploiting sparsity in direct collocation pseudospectral methods for solving optimal control problems, *Journal of Spacecraft and Rockets*, Vol. **49**, No. 2, pp 364–377, 2012.

[2]M. Grant and S. Boyd, *CVX: Matlab software for disciplined convex programming*, 2012, URL: http://cvxr.com/cvx/.

[3] W. Liu, I. Hwang, Probabilistic trajectory prediction and conflict detection for air traffic control, *Journal of Guidance, Control, and Dynamics*, Vol. **34**, No. 6, pp 1779–1789, 2011.

[4] W. Liu, I. Hwang, *Probabilistic aircraft midair conflict resolution using stochastic optimal control*, IEEE Transactions on Intelligent Transportation Systems, Vol. **15**, No. 1, pp 37–46, 2014.

[5] D. Xiu, *Numerical Methods for Stochastic Computations*, Princeton University Press, Princeton, 2010.

[6] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, New York, 2009.

[7] A. V. Rao, D. Benson, C. L. Darby, B. Mahon, C. Francolin, M. Patterson, I. Sanders, G.T. Huntington, *User's Manual for GPOPS Version 5.0: A MATLAB Software for Solving Multiple-Phase Optimal Control Problems Using hp-Adaptive Pseudospectral Methods*, University of Florida, Gainesville, 2011.

[8] M. Grant, S. Boyd, *CVX: Matlab software for disciplined convex programming, version 1.22*. URL: http://cvxr.com/cvx/, 2012.
[9] M. Grant, S. Boyd, *Graph implementations for nonsmooth convex programs*, Recent Advances in Learning and Control, Edited by V. Blondel, S. Boyd, H. Kimura, pp 95–110, 2008.
[10] M. Rauw, *FDC 1.2. - A SYMULINK Toolbox for Flight Dynamics and Control Analisys*, 2001.
[11] B. Etkin, *Dynamics of Flight-Stability and Control*, Wiley, New York, USA, 2-nd edition, 1982.
[12] D. McLean, *Automatic Flight Control System*, Prentice Hall, Hertfordshire, UK, 1990.
[13] D. McRuer, I. Ashkenas, D. Graham, *Aircraft Dynamics and Automatic Control*, Princeton University Press, July 1990.
[14] B. L. Stevens, F. L. Lewis, *Aircraft Control and Simulation*, John Wiley, USA, 1992.
[15] L. Moysis, M. Tsiaousis, N. Charalampidis, M. Eliadou, I. Kafetzis, *An Introduction to Control Theory Applications with MATLAB,* http://users.auth.gr/lazarosm/, 31 August 2015.