STUDIES ON FLUTTER PREDICTION

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Abstract: *The purpose of this paper is to study the instability of the dynamic flutter. The justification is expressed by the fact that the occurrence of flutter within the aircraft's flight envelope results in irreversible structural deformation which consequently leads to serious damage. Therefore the mathematical modeling of this phenomenon and its validation are very important. The instability of the dynamic flutter is characterized by critical speed and critical pulsation of oscillatory movements. In this paper, the quasi-stationary model and the Theodorsen model have been analyzed for calculating the aerodynamic forces and torques, and a comparison of them has been carried out. The fluid-structure coupling is done by rewriting the equations, considering that the forces are given by closed formulas. For the mathematical modeling of the flutter there have been used the p-k and V-g methods based on the Theodorsen model and the quasi-stationary model. In order to modeling the free vortices aerodynamic forces and moments, the equations which describe both the motion of the structure and the fluid flow had to be integrated simultaneously in time. The fluid-structure coupling is considered as a combination of two systems that describe the aeroelastic behavior of the structure.*

Key Words: *typical section, quasi-stationary model, flutter prediction*

I. INTRODUCTION

Flutter's dynamic instability is characterized by critical speed and critical pulsation of oscillatory movements.

The occurrence of the phenomenon of flutter within the aircraft's flight envelope results in irreversible structural deformation and consequently to serious damage. Therefore the mathematical modeling of this phenomenon and its validation are very important. With the continuous increment of Mach number and flight incidence, the flow becomes more complex, e.g. for Mach numbers between 0.4 and 0.7, the flow becomes critical on the suction side of an airfoil, while for Mach numbers about 1.0 or higher the first shock wave occurs. The dynamic response is a transient response or movement of aircraft structural components produced as a result of gusts of air, sudden controls, shocks, etc. For flexible structures, the aeroelastic response of the structure interacts with the flow, resulting in complex situations. For example, structural vibrations cause alternating lift off and reattachment of the boundary layer.

Unsteady aerodynamic loads produce greater interaction with the structure causing unusual aeroelastic phenomena that can significantly change the flight envelope [1], [2], [3].

In order to describe these two phenomena, one needs to introduce the concept of the typical section. This is achieved by sectioning the wing with a plane parallel to the plane of

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symmetry, at the distance y. Since there are two types of movements, the wing is submitted to both bending movement and torsion (Figure 1) [1], [2].

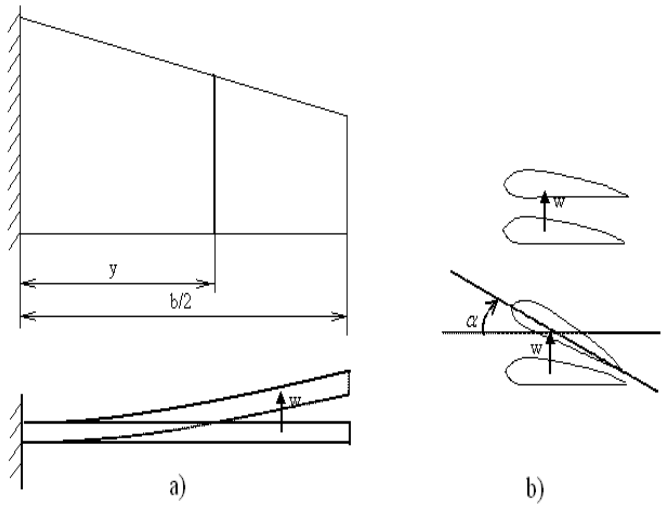


Fig. 1 Two types of movements

Common mathematical model describing these two phenomena is obtained from the Lagrange formalism which consists of the following equation:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial(T-U)}{\partial \dot{q}_i}\right)-\dfrac{\partial}{\partial q_i}(T-U)=Q_i \\ i=1,2,... \end{cases} \tag{1}$$

The significance of the parameters involved in eqn. (1) is as follows: T is the kinetic energy expressed in terms of the generalized coordinate q_i and the generalized speed $\dot{q}_i$; the potential energy U is expressed as a function of q_i and $\dot{q}_i$, terms of elastic deformation and the corresponding generalized coordinates and generalized forces Q_i, from the work of external forces on the nature of aerodynamics, mass, etc.

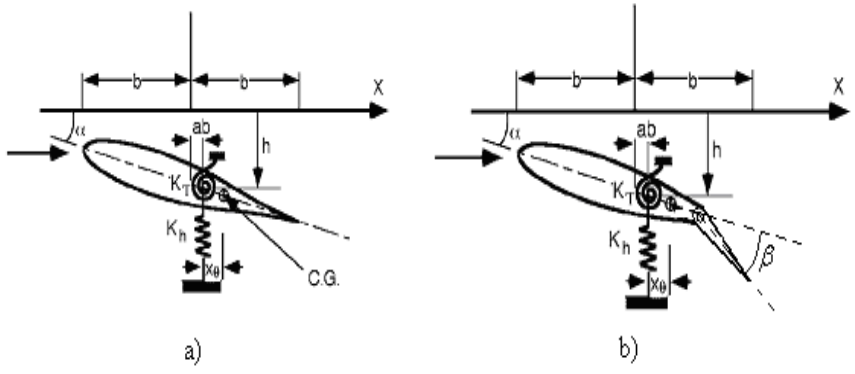


Fig. 2 Typical section

For the typical section shown in Figure 2, the kinetic energy and the potential energy are given by the following relations [1]:

$$U = \frac{1}{2}K_T\alpha^2 + \frac{1}{2}K_h h^2 \tag{2}$$

$$T = \frac{1}{2}\int \rho \dot{z}^2 dx = \frac{1}{2}\left(\dot{h}^2\int \rho dx + 2\dot{h}\dot{\alpha}\int \rho x dx + \dot{\alpha}^2\int \rho x^2 dx\right) = \frac{1}{2}m\dot{h}^2 + mx_\theta\dot{h}\dot{\alpha} + \frac{1}{2}I_\theta\dot{\alpha}^2 \tag{3}$$

For the model with two degrees of freedom

$$U = \frac{1}{2}\left(K_h h^2 + K_T\alpha^2 + K_\beta\beta^2\right) \tag{4}$$

$$T = \frac{1}{2}\int \dot{z}^2\rho dx = \frac{1}{2}m\dot{h}^2 + mx_\theta\dot{h}\dot{\alpha} + \frac{1}{2}I_\theta\dot{\alpha}^2 + \frac{1}{2}I_\beta\dot{\beta}^2 + mx_\beta\dot{h}\dot{\beta} + \left(I_\beta + mx_\beta\left(bc - ba\right)\right)\dot{\alpha}\dot{\beta} \tag{5}$$

For the model with three degrees of freedom, by substitution the Lagrange equations results in the form:

$$\left[M\right]\ddot{X} + \left[C\right]\dot{X} + \left[K\right]X = Q \tag{6}$$

where M is the mass matrix, C the damping matrix, K the stiffness matrix and the vector at right-hand side represents the aerodynamic forces, and it was obtained from the virtual mechanical work. Starting from this system one can express appropriate mathematical models for flutter and dynamic response.

For both the Theodorsen model and the model for calculating the quasi-aerodynamic forces and torques, the fluid-structure coupling is done by rewriting the equations, considering that the forces are given by closed formulas. For the mathematical modeling of the flutter, the ***p-k*** and ***V-g*** methods based on the Theodorsen model and the quasi-stationary model [1], [2] have been used.

In order to model the free vortices aerodynamic forces and moments, the equations which describe both the motion of the structure and the fluid flow must be integrated simultaneously in time. The numerical solving of the fluid-structure coupling raises some problems because the equations describing the behavior of the structure are expressed within a Lagrange reference system, while the equations describing the fluid flow are expressed in an Euler coordinate system. On the other hand, the deformation of the structure can unavoidable lead to the (partial or total) change of the border between fluid and structure, involving the control of the integration of the equations of fluid flow on cell volume, and therefore, a mobile computing network can be required.

The fluid-structure coupling is intended as a combination of two systems describing the aeroelastic behavior of the structure. The problem may be supplemented with the equations the motion equations of the network, a pseudo-structural system with its own dynamic [3].

If the aerodynamic forces are calculated with a model of free vortices, then an efficient method to calculate the flutter speed is the '*root locus design*' [1], [3].

Since the aerodynamic forces are those supposed to introduce energy into the system and their value depends on the speed for a given configuration (characteristic mass, elastic and geometric structure), then the accurate calculation of the critical flutter speed is very important, due to the fact that if the speed exceeds the critical value, then the system becomes unstable dynamic and can be severely irreversibly damaged, even destroyed.

Consequently, the critical wave speed is defined as the speed at which the motion is harmonic and the oscillation damping (structural and aerodynamic) is zero.

The determination of wave conditions (associated wave speed and frequency) is significantly dependent by the aerodynamic model considered; the harmonic oscillator

system (proposed by Theodorsen) approximates the reality better than a quasi-stationary model. In the following, this study will be expressed in terms of aerodynamics, for both simplified cases based on the study of the quasi-stationary aerodynamic forces as well as the periodic non-stationary.

II. DYNAMIC RESPONSE

Dynamic response is a transient response or movement of aircraft structural components produced as a result of the forces: burst data, sharp controls, different shocks, etc. There will be presented three methods for calculating the dynamic response (time integration methods of the motion equations) applied to an aeroelastic model.

The first method (called the Newmark method) is based on implicit discretization of the equations. The time constant is chosen and the periods of oscillation are known. A second method, called the HHT method (Hilbert, Hughes, Taylor) is dedicated for systems of second order differential equations and uses the physical meaning of terms such as displacements, velocities and accelerations. A third method, the Runge-Kutta, requires the transforming of the initial system into a first-order differential equations system, with unknowns either movements or speeds [1].

Runge-Kutta method

Supposing the first-order differential equations of the form:

$$\frac{dy_i(x)}{dx} = f_i(x, y_1, ..., y_n) \qquad i = 1,...,n \tag{7}$$

where the functions f_i describe usually nonlinear forms of independent variable x and the n dependent variables y_i.

Note: If the system of differential equations is of higher order, then one has to reduce the order.

The fourth order Runge-Kutta formula is most commonly used in practice and consists in:

$$\begin{aligned} K_1 &= h \cdot f(x_k, y_k) \\ K_2 &= h \cdot f(x_k + \frac{h}{2}, y_k + \frac{K_1}{2}) \\ K_3 &= h \cdot f(x_k + \frac{h}{2}, y_k + \frac{K_2}{2}) \\ K_4 &= h \cdot f(x_k + h, y_k + K_3) \\ y_{k+1} &= y_k + \frac{1}{6}(K_1 + 2 \cdot K_2 + 2 \cdot K_3 + K_4) \end{aligned} \tag{8}$$

The fourth order Runge-Kutta method has a truncation error of order h^5.

The appropriate system for typical section for the model with two degrees of freedom (the method extended for the model with three degrees of freedom comes out easily) gives [1]:

$$\left[\bar{M}\right]\begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \left[\bar{C}\right]\begin{Bmatrix} \dot{h} \\ \dot{\alpha} \end{Bmatrix} + \left[\bar{K}\right]\begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \{Q_0\} \tag{9}$$

where

$$x = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}.$$

The application of the Runge Kutta method involves the following algorithm:

The vector of unknowns is $W = \begin{Bmatrix} x \\ y \end{Bmatrix}$, where $y = \dot{x}$.

$$\left[\bar{M}\right]\dot{y} + \left[\bar{C}\right]y + \left[\bar{K}\right]x = \{Q_0\} \tag{10}$$

$$\dot{y} = \left[\bar{M}\right]^{-1}\left(\{Q_0\} - \left[\bar{C}\right]y - \left[\bar{K}\right]x\right) \tag{11}$$

$$\dot{W} = \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} y \\ \left[\bar{M}\right]^{-1}\left(\{Q_0\} - \left[\bar{C}\right]y - \left[\bar{K}\right]x\right) \end{Bmatrix}$$

$$\frac{dW}{dt} = \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\left[\bar{M}\right]^{-1}\left[\bar{K}\right] & -\left[\bar{M}\right]^{-1}\left[\bar{C}\right] \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} 0 \\ \left[\bar{M}\right]^{-1}\{Q_0\} \end{Bmatrix} \tag{12}$$

One further denotes:

$$R = \begin{bmatrix} 0 & I \\ -\left[\bar{M}\right]^{-1}\left[\bar{K}\right] & -\left[\bar{M}\right]^{-1}\left[\bar{C}\right] \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} 0 \\ \left[\bar{M}\right]^{-1}\{Q_0\} \end{Bmatrix} \tag{13}$$

Being given the system and the initial conditions [1], [2], then:

$$\begin{cases} \dfrac{dW}{dt} = R \\ W(0) = W_0 \end{cases} \tag{14}$$

III. NUMERICAL

The following data (i.e. velocity, flight incidence, center coordinate as shown in Fig.2, lift coefficient, aerodynamic moment coefficient) listed in Table 1 have been obtained for the equilibrium position.

Table 1. Conditions for the equilibrium position

$V_\infty = 11\,\mathrm{m/s}$	$V_\infty = 15\,\mathrm{m/s}$
$\alpha_0 = 1.85379$ deg	$\alpha_0 = 16.92$ deg
$h_0 = -2.9719 \cdot 10^{-3}$	$h_0 = -0.2713 \cdot 10^{-2}$
$Cz = 0.4224$	$Cz = 2.07$
$Cm = -0.2107 \cdot 10^{-1}$	$Cm = -0.9811 \cdot 10^{-1}$

A comparison of two models of aerodynamic forces (i.e. the Theodorsen model and the quasi-stationary model) has been carried on, based on the calculation of velocity, frequency and damping and expressed by the numerical results plotted in the following.

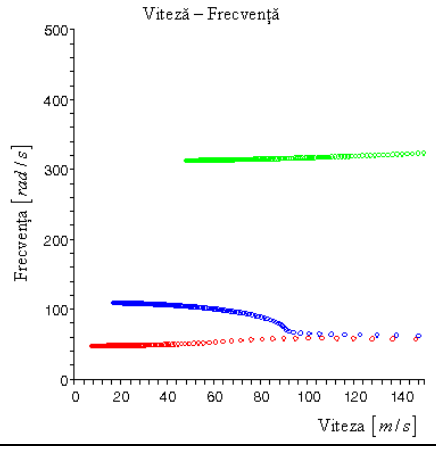
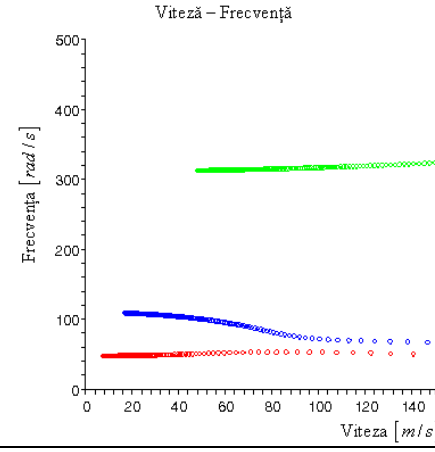
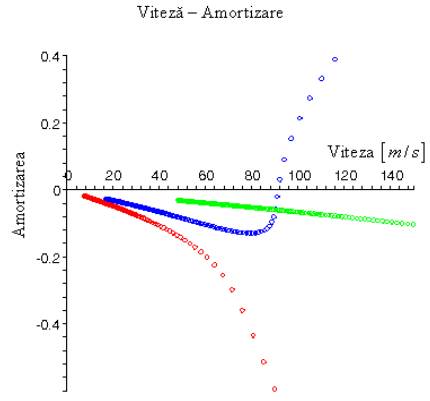
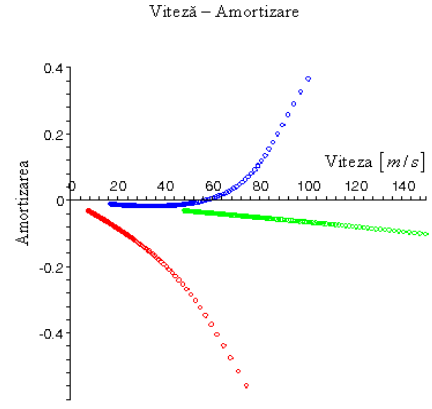
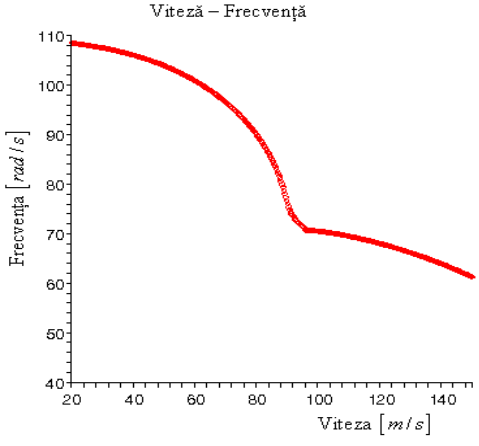
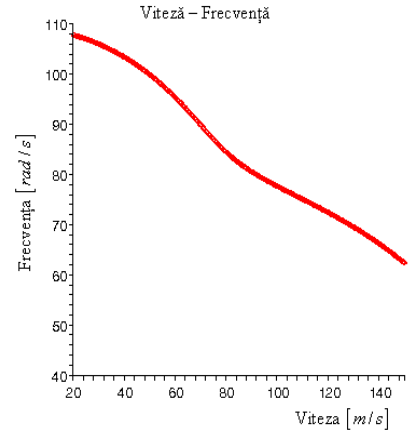
The harmonic oscillator (Theodorsen model)	The quasi-stationary model
	
	

Fig. 3 Variation of frequency and damping versus velocity

The harmonic oscillator (Theodorsen model)	The quasi-stationary model
	

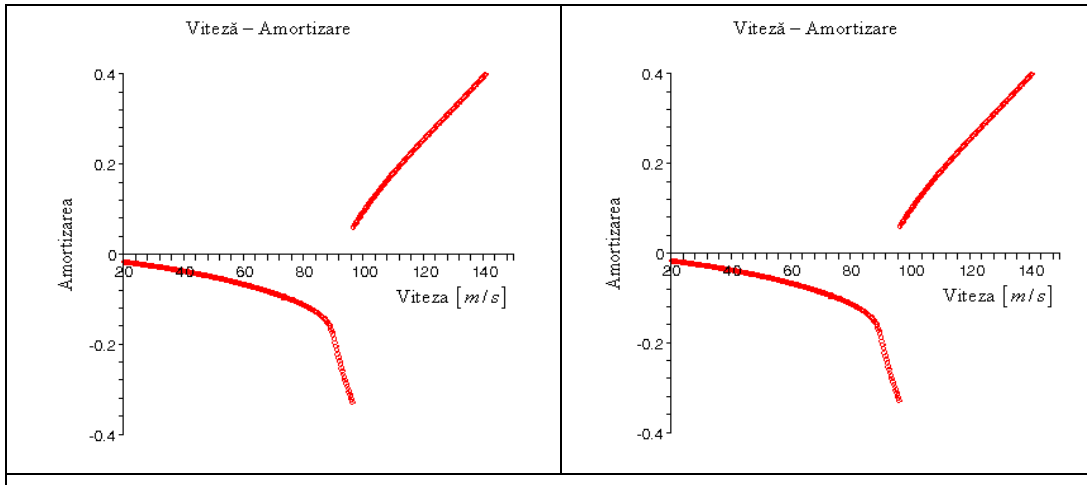
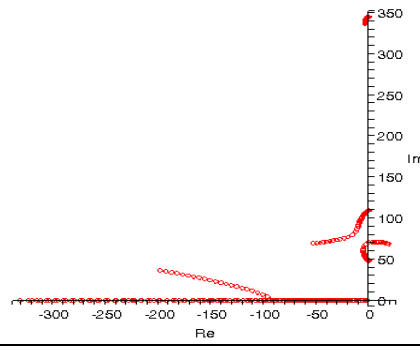
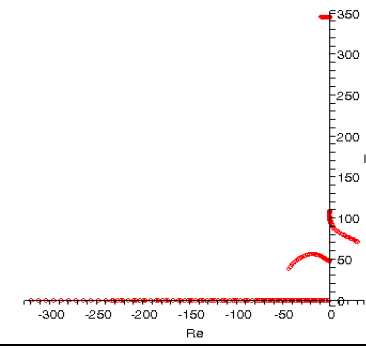
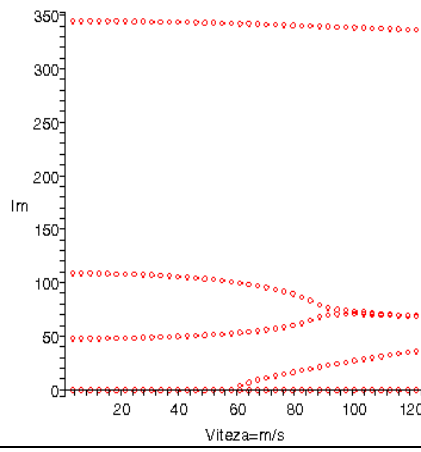
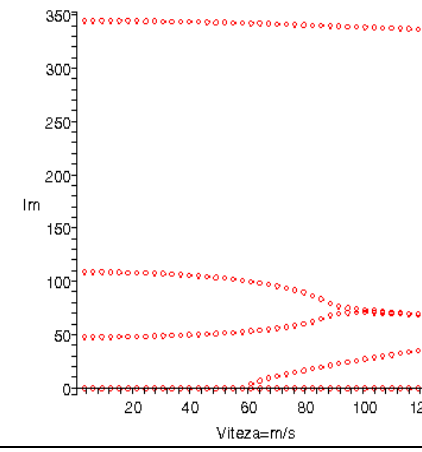


Fig. 4 Variation of frequency and damping versus velocity

The harmonic oscillator (Theodorsen model)	The quasi-stationary model
Real versus imaginar	**Real versus imaginar**
	
Viteza versus imaginar	**Viteza versus imaginar**
	

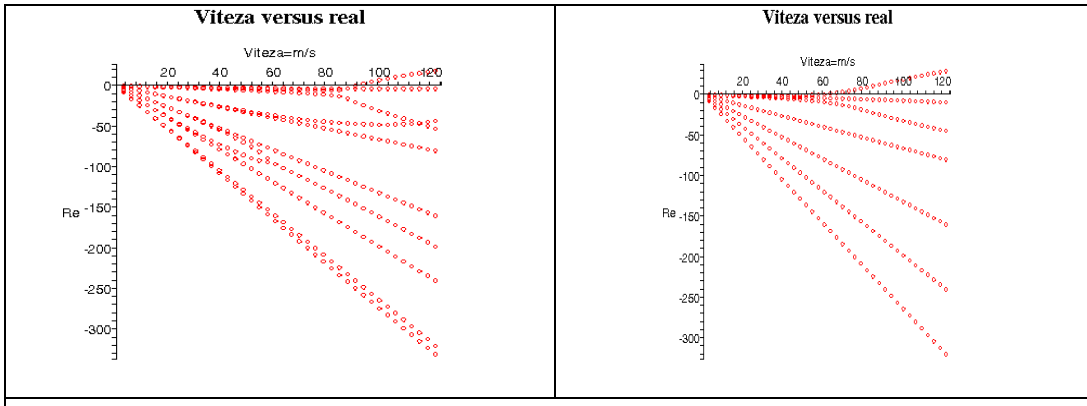


Fig. 5 Root locus

IV. CONCLUSIONS AND REMARKS

In this paper, the quasi-stationary model and the Theodorsen model have been analyzed for calculating the aerodynamic forces and torques, and a comparison of them was has been carried out The numerical results have been plotted as variation of frequency and damping versus velocity, see Fig. 3 and Fig. 4. The stability analysis is outlined in Fig.5. The fluid-structure coupling is done by rewriting the equations, considering that the forces are given by closed formulas. For the mathematical modeling of the flutter there have been used the *p-k* and *V-g* methods based on the Theodorsen model and the quasi-stationary model. In order to modeling the free vortices aerodynamic forces and moments, the equations which describe both the motion of the structure and the fluid flow had been simultaneously integrated in time. The fluid-structure coupling is considered as a combination of two systems that describe the aeroelastic behavior of the structure. Difficulties occur when solving numerically the fluid-structure coupling since the equations that describe the behavior of the structure are expressed within a Lagrange reference system, while the equations that describe fluid flow are written within Euler coordinate system. Also a mobile computing network is necessary due to the fact that the deformation of the structure unavoidable leads to a (partial or total) change of the border between fluid and structure, which involves to control the integrating of the fluid flow equations of fluid flow on cell volumes.

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