Equilibrium stability analysis by numerical simulations for a linear system with time-delayed control

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"As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality." Albert Einstein

Abstract: *The work starts from the linearization of a nonlinear mathematical model of the electrohydraulic servomechanism with structural switching and delayed control. The delay, as well as the switching structure, can lead to instability and to the deterioration of the dynamic system performance. Between the different approaches to the delay problem, this article considers the predictive feedback method. The question of switching structure is herein treated empirically, aiming in the numerical simulation not to impose on the system a magnitude of the disturbance of the zero equilibrium that leads to switching by changing the sign of the state variable which is the displacement of the servovalve spool. Numerical simulations allow to highlight a critical delay, which is attested by comparison to an analytical method.*

Key Words: *electrohydraulic servomechanism, delayed control, switching structure, state predictive feedback, discretization, numerical simulation*

1. INTRODUCTION

This paper focuses on some aspects associated with the treatment of numerical simulation by discretization for a system with delayed control for which the synthesis of control was made on the basis of state prediction to compensate for the delay. This results in a closed-loop system in which appear the delayed state and an integral term representing the control history. Therefore, the integration of the obtained system requires the substitution of the integral with a sum by dividing the length h of the integration interval - the length of the delay - into a convenient number of k sampling periods T. The dynamic system to which we refer in the paper is the linearized mathematical model of an electrohydraulic servomechanism with delay on control and with structural switching. The problem is formulated and solved in a wider context in the [1] paper, but the discretization procedure was summarized in only a few words. Further, this Chapter presents the mathematical model of servomechanism. In Chapter 2, a predictive feedback control problem is formulated and solved in order to assess the system equilibrium stability and the influence of the delay on this stability. Here, the discretization methodology is described in detail. Chapter 3 presents the results of the numerical simulations, highlighting a critical delay. The paper ends with a few conclusions.

The application refers to the mathematical model of an electrohydraulic servomechanism (EHS) which is a combination between an electrohydraulic servovalve and a hydrocylinder. EHS is an important part of an aircraft implied in the control of the primary flight surfaces. The principal performance of the EHS is to ensure the stability of aircraft. The mathematical model with five states and structural switching considered in [2-5] is completed by taking into account a leakage coefficient as in [6] and by introducing a time-delayed control. Due to the switching structure, the mathematical model of the EHS is split into two subsystems with respect to the sign of the state x_5:

$$\begin{aligned} &\dot{x}_1 = x_2;\ \dot{x}_2 = (-kx_1 - fx_2 + Sx_3 - Sx_4)/m \\ &\dot{x}_3 = E(-Cx_5\sqrt{p_s - x_3} - Sx_2 + k_l(p_s - 2x_3))/(V_0 + Sx_1) \qquad \text{for } x_5 > 0 \\ &\dot{x}_4 = E(-Cx_5\sqrt{x_4} + Sx_2 + k_l(p_s - 2x_4))/(V_0 - Sx_1) \\ &\dot{x}_5 = (-x_5 + k_{SV}u_1(x(t-h)))/\tau_{SV} \end{aligned} \tag{1}$$

$$\begin{aligned} &\dot{x}_1 = x_2;\ \dot{x}_2 = (-kx_1 - fx_2 + Sx_3 - Sx_4)/m \\ &\dot{x}_3 = E(-Cx_5\sqrt{x_3} - Sx_2 + k_l(p_s - 2x_3))/(V_0 + Sx_1) \\ &\dot{x}_4 = E(-Cx_5\sqrt{p_s - x_4} + Sx_2 + k_l(p_s - 2x_4))/(V_0 - Sx_1) \qquad \text{for } x_5 < 0 \\ &\dot{x}_5 = (-x_5 + k_{SV}u_2(x(t-h)))/\tau_{SV};\ C := c_d w\sqrt{2/\rho} \end{aligned} \tag{2}$$

The initial conditions are $u_i(t) = u_{0,i}(t), -h \le t \le 0, h > 0, \mathbf{x}_i(0) = \mathbf{x}_{0,i} \ne 0, i = 1,2$. The magnitude of the perturbation $x(t_0, \varphi_i) = \varphi_i(0) = x_{0,i} \ne 0$ is conditioned by the norm $\|\varphi_i\|_h = \sup_{t_0 - h \le \theta \le t_0} \|\varphi_i(\theta)\|$. Assume that $0 < x_i < p_s$, i = 1, 2, and $|x_1| < V_0 / S$. The leakage coefficient is a combination of the internal leakage coming from the spool valve and the external one which is characteristic to the hydrocylinder.

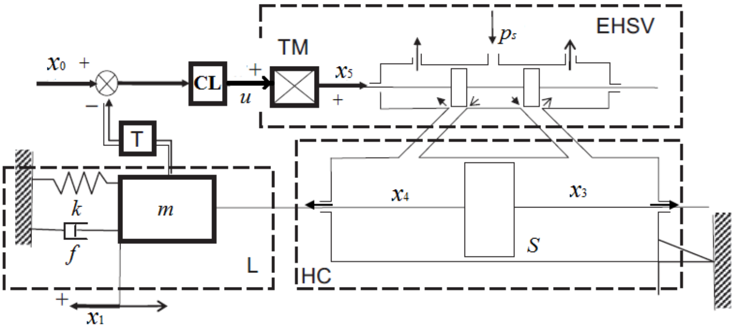


Fig. 1 Block diagram of the servovalve controlled EHS. HC: hydraulic cylinder with piston; L: load; CL: controller; T: transducer; TM: torque motor; EHSV: electrohydraulic servovalve

The following notations are used in equations (1)-(2) and in Fig. 1: x_1 - the load displacement, x_2 - the load velocity, x_3, x_4 - the pressures in the hydraulic cylinder chambers, x_5 - the EHSV valve opening, u - the control variable, p_s is the supply pressure, m - the inertial load of primary control surface, f - the combined coefficient of the damping and viscous friction forces on the load, k - an equivalent aerodynamic elastic force coefficient, k_l - the cumulative coefficient of leakages, S - the area of the piston, V_0 - the semivolume of the hydrocylinder; E- the bulk modulus of hydraulic oil, τ_{SV} - the servovalve time constant, k_{SV} - a coefficient of proportionality between the servovalve voltage and the displacement of the servovalve spool, c_d - the discharge coefficient in the servovalve spool, w - the valve port's width, ρ - the hydraulic oil density.

The Jacobian matrices of the two component systems (1), (2) are calculated. The equilibrium point for the subsystem (1) is given by $\hat{x}_{1,1} = x_{0,1}$, $\hat{x}_{2,1} = 0$, $\hat{x}_{3,1} = p_s / 2 + kx_{0,1} / (2S)$, $\hat{x}_{4,1} = p_s / 2 - kx_{0,1} / (2S)$, the fifth state equilibrium $\hat{x}_{5,1}$ will be a solution of the equation $Cx_5\sqrt{(p_s - kx_{0,1}/S)/2} - k_l kx_{0,1}/S = 0$. A similar equilibrium point is found for system (2). System (1)-(2) is transformed into a system with zero equilibria in the new coordinates system y

$$\begin{gathered} y_{1,i} = x_{1,i} - \hat{x}_{1,i},\, y_{2,i} = x_{2,i},\, y_{3,i} = x_{3,i} - \hat{x}_{3,i} \\ y_{4,i} = x_{4,i} - \hat{x}_{4,i},\, y_{5,i} = x_{5,i} - \hat{x}_{5,i}, U_i = u_i - u_{i,0} \end{gathered} \tag{3}$$

Let A be the Jacobian matrix calculated in zero in the cases $x_5 > 0$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{ES}{V_0 + S\hat{x}_{1,1}} & -\frac{E}{V_0 + S\hat{x}_{1,1}}\left(\frac{C\hat{x}_5}{2\sqrt{p_s - \hat{x}_{3,1}}} + 2k_l\right) & 0 & \frac{EC\sqrt{p_s - \hat{x}_{3,1}}}{V_0 + S\hat{x}_{1,1}} \\ 0 & \frac{ES}{V_0 - S\hat{x}_{1,1}} & 0 & -\frac{E}{V_0 - S\hat{x}_{1,1}}\left(\frac{C\hat{x}_{5,1}}{2\sqrt{\hat{x}_{4,1}}} + 2k_l\right) & -\frac{EC\sqrt{\hat{x}_{4,1}}}{V_0 - S\hat{x}_{1,1}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix} \tag{4}$$

The matrix of control influence B is a column vector with the first four elements zero and the fifth element equal with k_{SV} / τ_{SV} .

2. PROBLEM FORMULATION. SOLUTION AND DISCRETIZATION

2.1 Time domain synthesis and discretization

Consider the linear invariant system with input control delay

$$\dot{x}(t) = Ax(t) + Bu(t-h), x(0) = x_0 \tag{5}$$

and the control law

$$u(t) = -Kx(t). \tag{6}$$

The delay can have different sources, as follows:

1) in the case when the control law is numerically (discrete) implemented and/or synthesized, the time delay h is equal with the sampling period and it refers, in principle, at the necessary time to compute the control law

2) it is considered as a characteristic of the plant.

Suppose that the feedback matrix corresponding to the control law (6) was determined such that the matrix *A-BK* to be stable and the closed loop system (with no delay) satisfies some performances.

In this case, ignoring the time delay in the synthesis of the control law could have negative effects, as instability or performances degradation. Thus, the following study directions appear:

a) the analysis of the stability of system (5) with the control law (6), i.e. the determination of the critical delay h_c which destabilises the system (5)

b) the synthesis of a feedback control law which counteracts the effects of a given time delay *h*, i.e. for $t > h$ the closed loop system behaves like the following system

$$\dot{x} = (A - BK)x,\ x(h) = e^{Ah}x_0.$$

Proposition 1. *The feedback control law for the linear system (5) which achieves the objective b) stated above is the predictive feedback control law*

$$u(t) = -Kx(t+h) = -K\,e^{Ah}x(t) - K\int_{t-h}^{t} e^{A(t-\sigma)}Bu(\sigma)d\sigma \tag{7}$$

with matrix K mentioned above.

Proof. First, let's consider that $u(t-h) = 0$ on the time interval $[0, h),$ which is a certain simplification of the usual approach in the theory of systems with delay [7]. Thus the equation (5) becomes $\dot{x}(t) = Ax(t),\ x(0) = x_0.$ On the other hand, for $t \geq h$, the control law is given by $u(t) = -Kx(t+h)$ and the equation (5) becomes $\dot{x}(t) = (A - BK)x(t),\ x(h) = e^{Ah}x_0.$

The general solution of equation (5), for the initial time $t_0 \neq 0$ is expressed as

$$x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-\sigma)}Bu(\sigma)d\sigma. \tag{8}$$

To determine the state prediction $x(t+h),$ the following substitutions $t_0 \to t,\ \ t \to t+h,$ $x_0 \to x(t),\ \ u(\sigma) \to u(\sigma - h)$ are made in relation (8)

$$x(t+h) = e^{Ah}x(t) + \int_{t}^{t+h} e^{A(t+h-\sigma)}Bu(\sigma - h)d\sigma. \tag{9}$$

Using change of variable $\tau = \sigma - h$ it is obtained

$$x(t+h) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-\tau)}Bu(\tau)d\tau. \tag{10}$$

Thus, the feedback state predictive control law is

$$u(t) = -K\,e^{Ah}x(t) - K\int_{t-h}^{t} e^{A(t-\sigma)}Bu(\sigma)d\sigma \quad \text{q.e.d.} \ \blacksquare \tag{11}$$

As it could be seen in (7), the current state $x(t)$ is necessary for control law elaboration as well as the control "history" on the interval $[t-h, t)$. The following Proposition shows a discretization procedure for numerical calculation of the closed loop control, respectively, for the numerical calculation of the integral from (7).

Proposition 2. *The feedback control law* (7) *is computed according to the discretised form*

$$u(n) = -K\, A_D^k x(n) - K \sum_{i=n-k}^{n-1} A_D^{n-1-i}\, B_D\, u(i) \tag{12}$$

$$t := nT,\ n = 0, 1, 2, \ldots, h = kT, A_D := e^{AT},\ B_D := A^{-1}[e^{AT} - I]B$$

In the relation above, $u(n),\ x(n),\ u(i)$ represent the shorten expressions for $u(nT),\ x(nT),\ u(iT)$, with T being the sampling period.

Proof. We are interested to calculate the integral from (7)

$$I(t) = \int_{t-h}^{t} e^{A(t-\sigma)} Bu(\sigma) d\sigma. \tag{13}$$

For this purpose, the interval $[t-h, t)$ is divided in k equal intervals

$$[t - kT + jT, \quad t - kT + (j+1)T], \quad j = 0, 1, 2, \ldots, k-1 \tag{14}$$

where, obvious $h = kT$. On each interval (14) it is considered that $u(\sigma)$ is constant, that is $u(\sigma) = u(t - kT + jT)$. Thus we have

$$I(t) = \sum_{j=0}^{k-1} \int_{t-kT+jT}^{t-kT+(j+1)T} e^{A(t-\sigma)} Bu(\sigma) d\sigma. \tag{15}$$

Note with

$$J(j) = \int_{t-kT+jT}^{t-kT+(j+1)T} e^{A(t-\sigma)} d\sigma\ Bu(t - kT + jT) \tag{16}$$

and

$$\overline{J} = \int_{t-kT+jT}^{t-kT+(j+1)T} e^{A(t-\sigma)} d\sigma\ . \tag{17}$$

Changing the variable $t - \sigma = \lambda$, we have

$$\overline{J} = -\int_{(k-j)T}^{(k-j-1)T} e^{A\lambda} d\lambda\ ,\ \text{which is equivalent with}\ \overline{J} = \int_{(k-j-1)T}^{(k-j)T} e^{A\lambda} d\lambda. \tag{18}$$

Thus

$$\overline{J} = A^{-1} e^{A\lambda} \Big|_{(k-j-1)T}^{(k-j)T} = A^{-1} e^{A(k-j)T} - A^{-1} e^{A(k-j-1)T}.$$

Therefore, it results

$$J(j) = [^{-1} e^{A(k-j)T} - A^{-1} e^{A(k-j-1)T}] Bu(t - kT + jT). \tag{19}$$

Moreover, taking into account that the matrix A^{-1} commutes with the matrix exponentials, the following relations is obtained

$$J(j) = A^{(k-j-1)T} A^{-1} (e^{AT} - I) B\, u(t - kT + jT). \tag{20}$$

One could observe that (20) can be rewritten as

$$J(j) = A_D^{(k-j-1)} B_D\, u(t - kT + jT) \tag{21}$$

where $A_D = e^{AT}$ and $B_D = A^{-1}(e^{AT} - I)B$ are the matrices of the discrete system obtained by the discretization of the continuous system (A, B) with the sampling period T and zero order hold control. Further (see (15) and (16))

$$I(t) = \sum_{j=0}^{k-1} J(j), \text{ i.e. } I(t) = \sum_{j=0}^{k-1} A_D^{k-j-1}\, B_D\, u(t - kT + jT). \tag{22}$$

Considering that $h = kT$ and including (22), the control law (7) is rewritten as

$$u(t) = -K\, A_D^k x(t) - K \sum_{j=0}^{k-1} A_D^{k-j-1}\, B_D\, u(t - kT + jT). \tag{23}$$

For $t = nT,\ \ n = 0, 1, 2, \ldots$ the relation (23) is written as

$$u(nT) = -K\, A_D^k x(nT) - K \sum_{j=0}^{k-1} A_D^{k-j-1}\, B_D\, u((n-k)T + jT) \tag{24}$$

or, in a shorten form

$$u(n) = -K\, A_D^k x(n) - K \sum_{j=0}^{k-1} A_D^{k-j-1}\, B_D\, u(n - k + j). \tag{25}$$

Changing the index $i = n - k + j,$ the relation (25) leads to (12)

$$u(n) = -K\, A_D^k x(n) - K \sum_{i=n-k}^{n-1} A_D^{n-1-i}\, B_D\, u(i), \text{ q.e.d. } \blacksquare$$

The disadvantage of the control law (25) is that the gain matrix K was synthesized in the continuous domain.

In this case, the discrete implementation, according to (24), implies a sampling period T sufficiently small, which make things complicated in terms of computational requirements. The alternative is the direct approach of the discrete control synthesis, in which case a reasonable sampling period can be chosen.

2.2 Direct discrete time approach control synthesis

Consider the continuous linear invariant system

$$\dot{x}(t) = Ax(t) + Bu(t),\, x(0) = x_0. \tag{26}$$

The discrete counterpart of the continuous system (26), with the sampling period T and zero-order hold control is represented by the discrete system [8]

$$x(n+1) = A_D x(n) + B_D u(n),\ n = 0, 1, 2, \ldots \tag{27}$$

where

$$A_D := e^{AT}; \quad B_D := \int_0^T e^{A\sigma} d\sigma\, B. \tag{28}$$

If A is invertible, then

$$B_D = A^{-1}(e^{AT} - I)B. \tag{29}$$

Let

$$x(n+1) = Ax(n) + Bu(n-k), \quad x(0) = x_0, \tag{30}$$

be the discrete correspondent of the continuous system (5) with the sampling period *T*, *h=kT*, where for the notation simplicity the matrices corresponding to the discrete system where noted the same as for the continuous system.
It is noteworthy that the solution of the finite difference equation $x(n+1) = Ax(n) + Bu(n),$ $n = 0, 1, 2, \ldots$ is

$$x(n) = A^n x_0 + \sum_{i=0}^{n-1} A^{n-1-i}\ Bu(i). \tag{31}$$

Let also

$$\tilde{u}(n) = -Kx(n) \tag{32}$$

be a control law synthesized in discrete domain ignoring the delay of *k* steps of control, i.e. *A-BK* is stable and the dynamics $x(n+1) = (A - BK)x(n), \quad x(0) = x_0$ satisfies some requirements. The proposed objective is the synthesis of a feedback control law which counteracts the effects of a *k* steps delay of the control, precisely, for *n*>*k* the closed loop system is acting as the following system $x(n+1) = (A - BK)x(n), \quad x(k) = A^k x_0.$
Proposition 3. *The feedback control law for the system* (29) *is given by*

$$u(n) = -K\ x(n+k) = -K\ A^k x(n) - K \sum_{j=n-k}^{n-1} A^{n-1-j}\ B\,u(j). \tag{33}$$

Proof. For $n \le k$ equation (30) becomes $x(n+1) = Ax(n), \ \ x(0) = x_0,$ and for $n \ge k$, the same equation (30), considering the control law $u(n) = -Kx(n+k)$, becomes $x(n+1) = (A - BK)x(n), \ \ x(k) = A^k x_0.$

The state prediction with *k* step of the state, $x(n+k)$, is calculated based on equation (30) and utilizing (31) in which $n \to n+k, \ \ u(i) \to u(i-k).$ As a result, we get

$$\begin{aligned} x(n+k) &= A^{n+k} x_0 + \sum_{i=0}^{n+k-1} A^{n+k-1-i} Bu(i-k) = \\ &= A^{n+k} x_0 + \sum_{i=0}^{n-1} A^{n+k-1-i}\ Bu(i-k) + \sum_{n}^{n+k-1} A^{n+k-1-i}\ Bu(i-k) = \\ &= A^k\ A^n x_0 + A^k \sum_{i=0}^{n-1} A^{n-1-i}\ Bu(i-k) + \sum_{n}^{n+k-1} A^{n+k-1-i}\ Bu(i-k) = \\ &= A^k [A^n x_0 + \sum_{i=0}^{n-1} A^{n-1-i}\ Bu(i-k)] + \sum_{n}^{n+k-1} A^{n+k-1-i}\ Bu(i-k). \end{aligned}$$

Thus,

$$x(n+k) = A^k x(n) + \sum_{n}^{n+k-1} A^{n+k-1-i} B u(i-k). \tag{34}$$

Making the index change $j = i - k$ in (34), it is obtained

$$x(n+k) = A^k x(n) + \sum_{j=n-k}^{n-1} A^{n-1-j} B u(j). \tag{35}$$

Therefore,

$$u(n) = -K\, x(n+k) = -K\, A^k x(n) - K \sum_{j=n-k}^{n-1} A^{n-1-j}\, B\, u(j), \text{ q.e.d. } \blacksquare$$

We emphasize that the relation (35) gives the k-state prediction for the discrete system (30) just as for the continuous system (5) the prediction is given by the relation (8).

As can be seen from (33), the control at step "n" is a function of the current state $x(n)$ as well as the "history" of control over previous k steps with respect to $u(n-1),\ u(n-2), \ldots,\ u(n-k)$. Relation (33) is rewritten as

$$u(n) = -KA^k x(n) - K[B\ AB\ A^2B \ldots\ A^{k-1}B] \begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(n-k) \end{bmatrix}. \tag{36}$$

The following control law is obtained

$$u(n) = -F\, x(n) - \bar{F}\, \bar{x}(n) = -[F\ \bar{F}] \begin{bmatrix} x(n) \\ \bar{x}(n) \end{bmatrix} \tag{37}$$

or

$$u(n) = -\tilde{F}\tilde{x}(n) \tag{38}$$

where

$$F := KA^k, \bar{F} := K[B\ AB\ A^2B \ldots A^{k-1}B], \tilde{F} = [F \quad \bar{F}]; \tilde{x}(n) = \begin{bmatrix} x(n) \\ \bar{x}(n) \end{bmatrix} \tag{39}$$

$$\bar{x}(n) = [u(n-1)\ u(n-2) \ldots u(n-k)]^T.$$

The delay with k step of the control is described by the system:

$$\begin{aligned} \bar{x}(n+1) &= \bar{A}\bar{x}(n) + \bar{B}u(n) \\ \bar{u}(n) &= \bar{C}\bar{x}(n) \end{aligned} \tag{40}$$

where

$$\bar{A} := \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}; \bar{B} := \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; \bar{C} := [0 \quad 0 \quad \ldots \quad 1]. \tag{41}$$

$$\dim(\bar{A}) = k \times k; \quad \dim(\bar{B}) = k \times 1; \quad \dim(\bar{C}) = 1 \times k.$$

From (30) and (40) one obtains the extended system

$$\underbrace{\begin{bmatrix} x(n+1) \\ \bar{x}(n+1) \end{bmatrix}}_{\tilde{x}(n+1)} = \underbrace{\begin{bmatrix} A & B\bar{C} \\ 0_{k\times m} & \bar{A} \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} x(n) \\ \bar{x}(n) \end{bmatrix}}_{\tilde{x}(n)} + \underbrace{\begin{bmatrix} 0_{n+1} \\ \bar{B} \end{bmatrix}}_{\tilde{B}} u(n) \tag{42}$$

or

$$\tilde{x}(n+1) = \tilde{A}\tilde{x}(n) + \tilde{B}u(n), \tilde{x}_0 = \begin{bmatrix} x_0 \\ 0_{k\times 1} \end{bmatrix}. \tag{43}$$

System (42) or (43) represents the open loop system which includes the *k* steps delay of the control.

In contrast to the continuous case, the discrete approach allows to include the delay into an extended linear (finite) system.

The equation of the closed loop system is obtained from (43) and (38)

$$\tilde{x}(n+1) = \left(\tilde{A} - \tilde{B}\tilde{F}\right)\tilde{x}(n), \tilde{x}(0) = \tilde{x}_0. \tag{44}$$

which is rewritten with respect to the previous notations

$$\begin{bmatrix} x(n+1) \\ \bar{x}(n+1) \end{bmatrix} = \begin{bmatrix} A & B\bar{C} \\ -\bar{B}F & \bar{A} - \bar{B}\bar{F} \end{bmatrix} \begin{bmatrix} x(n) \\ \bar{x}(n) \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ \bar{x}(0) \end{bmatrix} = \begin{bmatrix} x_0 \\ 0 \end{bmatrix}. \tag{45}$$

Proposition 4. *Consider the matrix A_0 of the closed loop system,*

$$A_0 := \begin{bmatrix} A & B\bar{C} \\ -\bar{B}F & \bar{A} - \bar{B}\bar{F} \end{bmatrix}. \tag{46}$$

The spectrum of this matrix consists in the eigenvalues of the matrix $A - BK$ plus k eigenvalues placed in the origin of the complex plane (i.e. a null eigenvalue with k order of multiplicity).

Proof. We resume the equation (30)

$$x(n+1) = Ax(n) + Bu(n-k) \tag{47}$$

as well the control law (33) modified by the presence of the reference $x_r(n)$

$$u(n) = -K(x(n+k) - x_r(n)). \tag{48}$$

Applying the *z* transformation [9] from equations (47), (48) we obtain

$$zX(z) = AX(z) + Bz^{-k}U(z) \tag{49}$$

$$U(z) = -K\,(z^k X(z) - X_r(z)). \tag{50}$$

Introducing (50) in (49) we get successively

$$zX(z) = AX(z) - BKX(z) + BKz^{-k} X_r(z)$$
$$\Rightarrow (zI - (A - BK))X(z) = BKz^{-k} X_r(z)$$

$$X(z) = (zI - (A - BK))^{-1} BKz^{-k} X_r(z)$$

$$X(z) = \frac{(zI - (A - BK))^* BK}{z^k \det(zI - (A - BK))} X_r(z) \tag{51}$$

which ends the demonstration. ■

As it could be seen from (51) the predictive control moves the delay term outside the feedback loop and removes it from the design process.

For the particulare case k =1, Proposition 3 can be proved as follows:
from (41) it results that

$$\bar{A} = 0, \quad \bar{B} = 1, \quad \bar{C} = 1 \tag{52}$$

and from (39)

$$F = KA, \quad \bar{F} = KB. \tag{53}$$

Consequently,

$$A_0 = \begin{bmatrix} A & B \\ -KA & -KB \end{bmatrix}. \tag{54}$$

We choose as a transformation of similarity the matrix

$$S = \begin{bmatrix} I_m & 0 \\ -K & 1 \end{bmatrix}; \quad S^{-1} = \begin{bmatrix} I_m & 0 \\ K & 1 \end{bmatrix}$$

$$\Rightarrow S^{-1} A_0 S = \begin{bmatrix} A - BK & 0_{m\times 1} \\ 0_{1\times m} & 0 \end{bmatrix}$$

$$\Rightarrow \det(zI - S^{-1} A_0 S) = \det(zI - (A - BK))z. \quad ■$$

Note. Consider system (5) with (A, B) a controllable, or, at least, stabilizable pair. By considering a state predictor $x_p(t) := x(t+h) = e^{\boldsymbol{A}h} x(t) + \int_{-h}^{0} e^{-As} B(t+s) ds$, the system with control delay (5) can be replaced with the following non-homogeneous system with state delay [1]

$$\dot{x}(t) = Ax(t) + A_d x(t-h) + B_c K \int_{-h}^{0} e^{-As} B_c u(t+s-h) ds, A_d := BKe^{Ah}. \tag{55}$$

3. RESULTS. NUMERICAL SIMULATIONS

The main objective of the numerical simulations is the synthesis of the control law by the predictive feedback method.

The following real design data, characteristic for the EHS integrated in the aileron control chain of the jet fighter IAR99 [2, 3, 10, 11], were used: m = 30kg, f = 3000Ns/m, k = 3x10^{5}N/m, S = 10^{-3}m^2, c_d = 0.63, V_0 = 3×10^{-5} m^3, p_s = 210x10^{5}N/m^2, B = 13x10^{8}N/m^2, ρ = 850kg/m^3, k_{SV} = 2x10^{-4} m/V (maximal opening length of rectangular valve port $x_{5\max} = 2$ mm at a maximal valve input voltage $u_{\max} = 10$ V, with an equivalent valve port width $w = 0.85$ mm),

$k_l = 0.1\times10^{-11}$ m^5/(Ns) and $\tau_{SV} = 7.62\times10^{-3}$. As indicated in the Matlab subroutines, the pairs (A, B), i =1, 2, are not completely controllable, but are stabilizable, inclusively in $x_0 = 0$ the most vulnerable equilibrium point of a EHS [12].

Table 1. *The existence of the solution of the transcendental equation* $e^{(A+P(0))h}P(0) = A_d$

#	*h*	*solution checking*	*conclusion*
1	0.005	1.73×10^{-10}	there is a solution
2	0.01	2.18×10^{-9}	there is a solution
3	0.015	1.99×10^{-10}	there is a solution
4	0.02	7.52	there is no solution
5	0.09	2.74×10^{4}	there is no solution

The introduction of predictive control led directly to the discrete form (35), distinct from the form (55). A point of interest for numerical applications, described below, could be the asymptotic stability assessment of the homogeneous linear state delay system

$$\dot{x}(t) = Ax(t) + A_d x(t-h), A_d := BKe^{Ah} \tag{56}$$

For the case $x_5 > 0$, choosing $\hat{x}_{1,1} = 5\times10^{-3}$ m, the following equilibrium vector point is obtained: $\hat{x}_{1,1} = 5\times10^{-3}$ m, $\hat{x}_{2,1} = 0$ m/s, $\hat{x}_{3,1} = 112.5\times10^{5}$ N/m2, $\hat{x}_{4,1} = 97.5\times10^{5}$ N/m2, $\hat{x}_{5,1} = 0.0018\times10^{-3}$ m, with $u_1 = 0.0925$.

The corresponding eigenvalues of the matrix A (4) are: $\lambda_{1,2} = -97.4 \pm 1726.5i$, $\lambda_3 = -0.3$, $\lambda_4 = -90$, and $\lambda_5 = -131.2$. The control law is obtained as example by a simple LQR synthesis. Taking the weighting matrices Q_J, as zero matrix excepting Q_J (1,1) = 1 and $R_J = 0.0025$, thus K_1 = [6.1005 0.0002 0.0008 −0.0006 3.6293] is the feedback gain. In closed loop, the eigenvalues of matrix A are changing accordingly $\lambda_{1,2} = -97.4 \pm 1726.5i$, $\lambda_3 = -10.2$, $\lambda_4 = -90$, $\lambda_5 = -130.8$.

The stability is ensured if all the zeros of the transcendental characteristic equation $\det(sI - A - A_d e^{-Ish}) = 0$ fulfil the condition $\mathrm{Re}\, s > 0$ (see also [13]). To avoid tedious calculations for analyzing the solutions of this transcendent equation, a result from [14] is used in [1] saying that the asymptotic stability of the equation (54) occurs if there exists at least one solution for the nonlinear algebraic matrix equation $e^{(A+P(0))h}P(0) = A_d$. A simple numerical investigation of this equation is done by using Matlab `fsolve` subroutine and is summarized in Table 2, thus showing the method efficiency as compared to more laborious approaches in [15] in which is used the Lambert function. The result in the Table says something about the risk of loss of the stability for the system $\dot{x}(t) = Ax(t) + A_d x(t-h)$. Unfortunately, it is all about sufficient stability conditions, which can also be very conservative. However, the results from Table 2 can be exploited in the sense that the stability threshold can climb up to h = 0.1s, if we take into account that in the analyzed equation (56) lacks the compensating term $BK\int_{-h}^{0} e^{-As} B_c u(t+s-h)\,ds$.

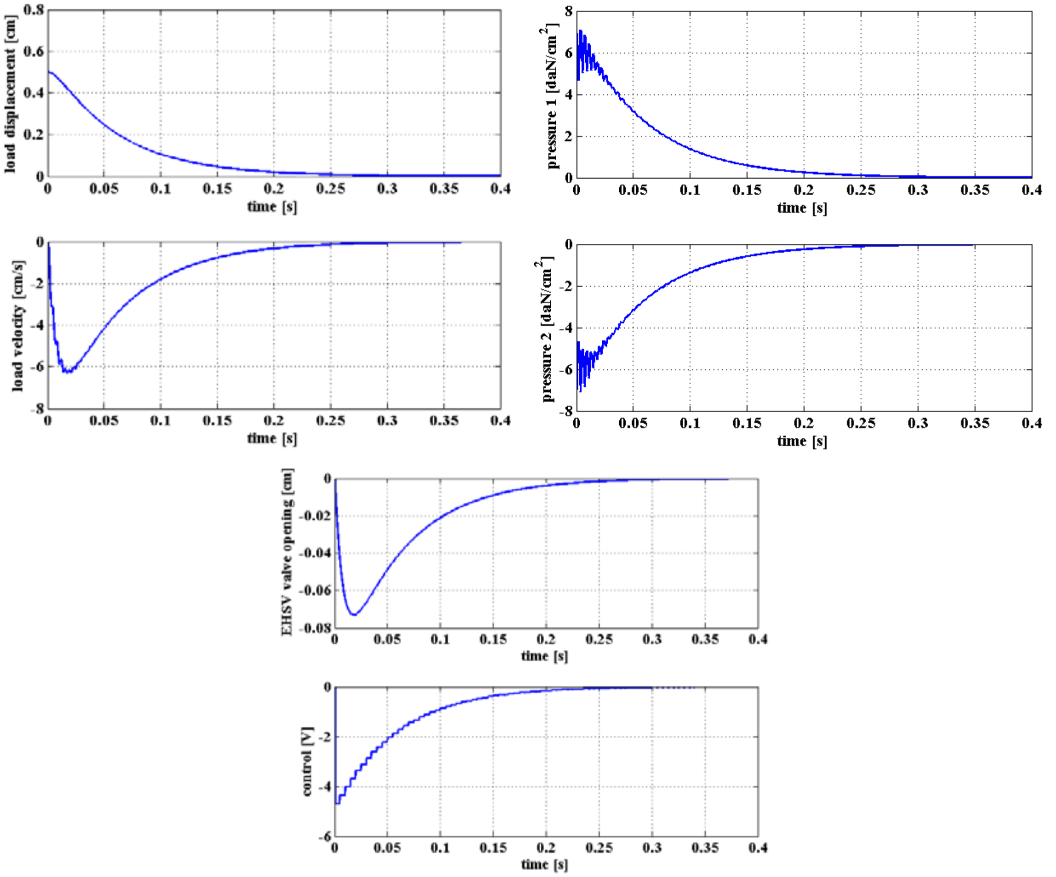
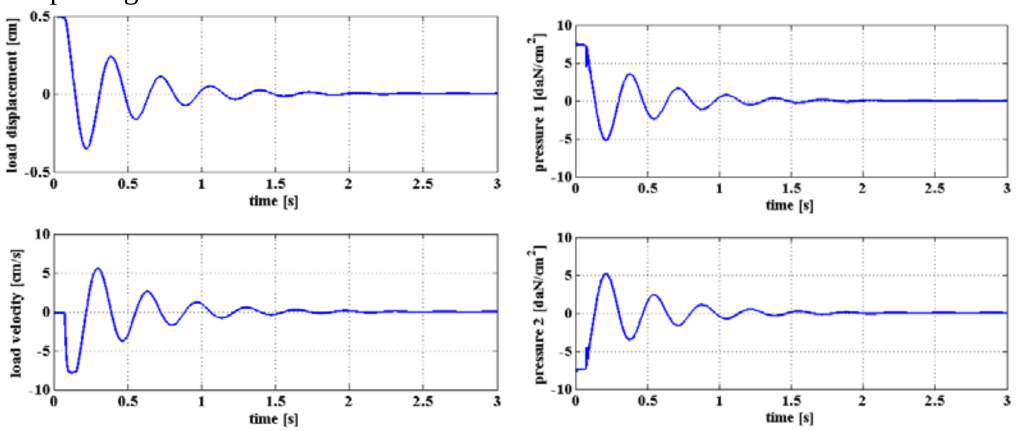


Fig. 2 History of system variables, the case of EHS without delay

Synoptic graphs in Figs. 2-4 give a perspective on the equilibrium stability of the system (5) obtained by numerical simulation. The predictive feedback was synthesized according to Proposition 3 and 4 in which the feedback matrix *K* was obtained using LQR synthesis in discrete time. In Fig. 2 we have the behavior in the presence of equilibrium perturbation with $x_1 = 0.5\text{cm}$ and with the corresponding values for the other variables, values given above. Notice that the system does not switch to $x_5 > 0$, so we do not have to deal with the switching system paradigm.



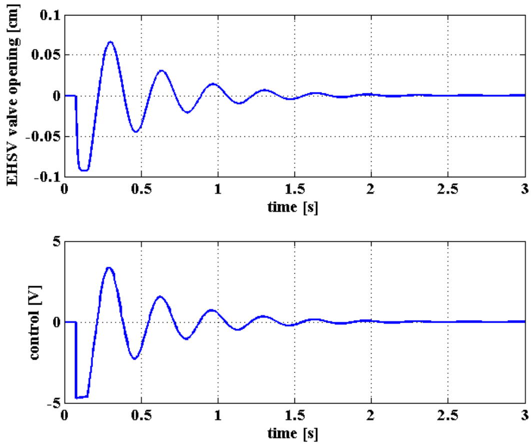


Fig. 3 History of system variables, the case of EHS with 0.075 s delay, without predictive control

In Fig. 3, the performance of the system significantly deteriorates for a delay of 0.075 s. The system is satisfactorily compensated with predictive feedback, as shown in Fig. 4. At a value of approx. 0.1 s a harmful chattering is shown in Fig. 5, which denotes that $h = 0.1\,\mathrm{s}$ is a critical delay for the system, as is expected from the analysis summarized in Table 1.

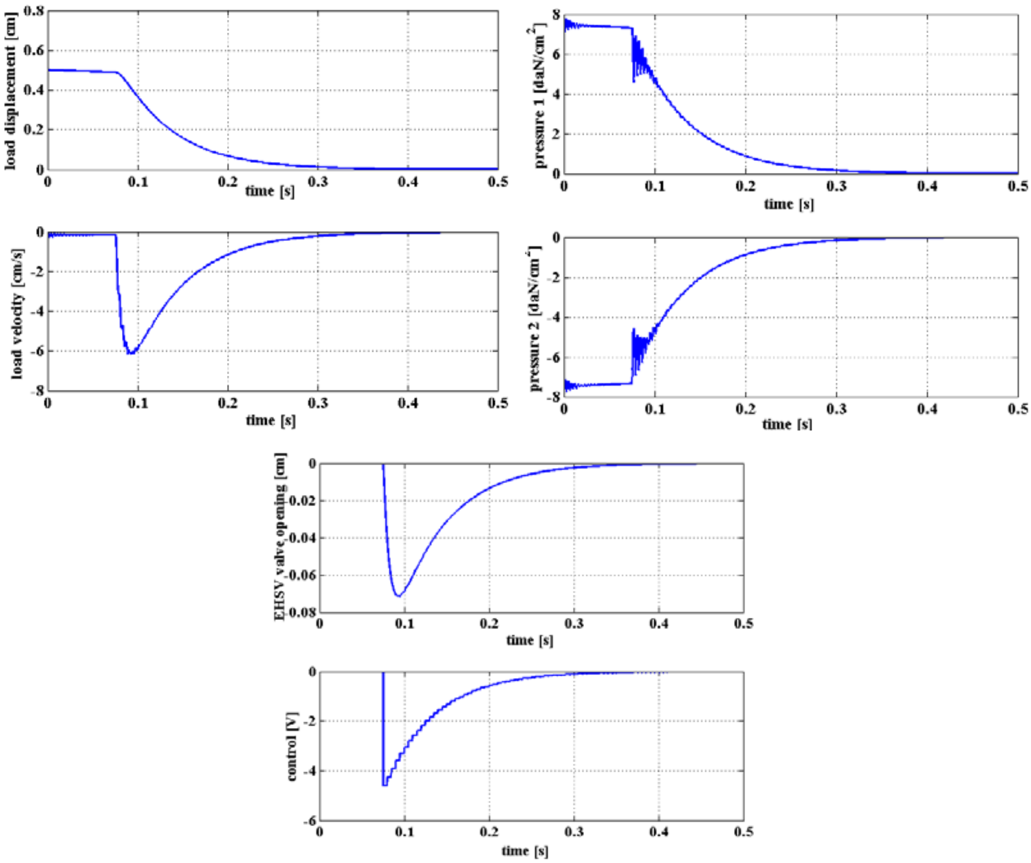


Fig. 4 History of system variables, the case of EHS with 0.075 s delay and predictive feedback

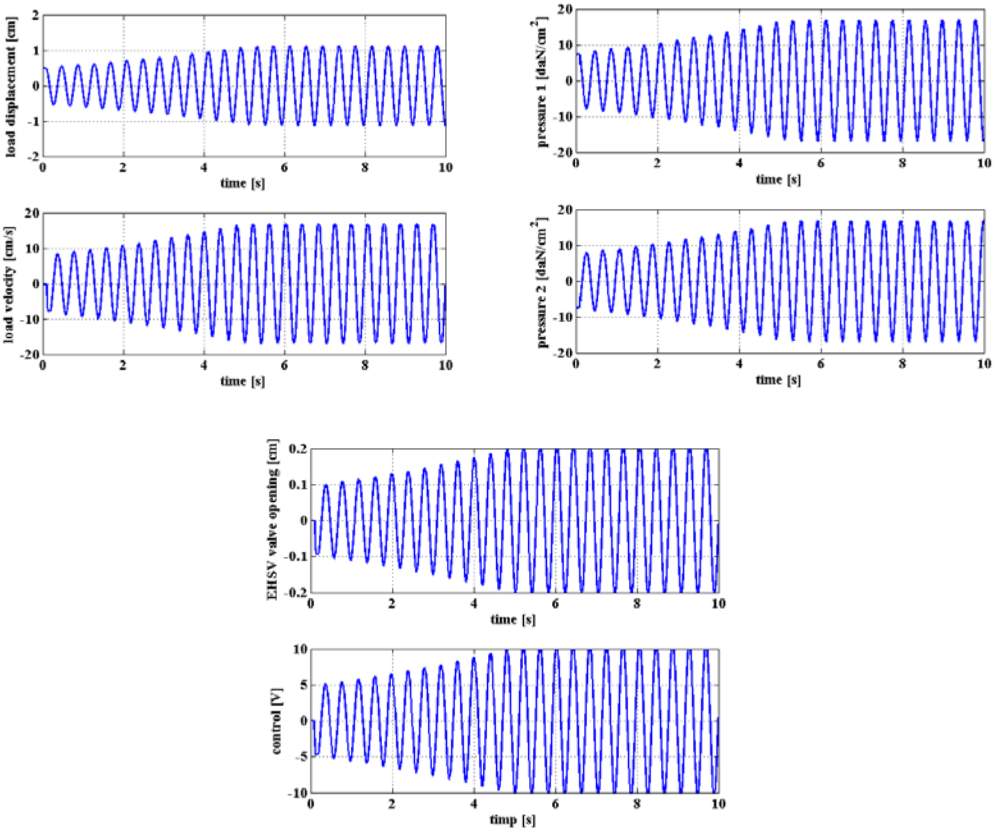


Fig. 5 History of system variables, the case of EHS with 0.1 s delay, without predictive control

4. CONCLUSIONS

The inherent delay in automated systems is a challenge, being a topical field in the literature.In this paper we presented a thorough analysis performed on the mathematical model of electro-hydraulic servomechanism. It was highlighted by analysis and numerical simulations that a delay of 0.1 s on control is a threshold to be avoided, as it can destabilize the system and worsen its stabilization performance even in the presence of a predictive feedback.

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