Mathematical modeling of a V-stack piezoelectric aileron actuation

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DOI: 10.13111/2066-8201.2016.8.4.12

Received: 06 October 2016/ Accepted: 10 November 2016/ Published: December 2016

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Abstract: The article presents a mathematical modeling of aileron actuation that uses piezo V-shaped stacks. The aim of the actuation is the increasing of flutter speed in the context of a control law, in order to widen the flight envelope. In this way the main advantage of such a piezo actuator, the bandwidth is exploited. The mathematical model is obtained based on free body diagrams, and the numerical simulations allow a preliminary sizing of the actuator.

Key Words: piezoelectric stack actuators, flutter counteracting, kinematic scheme, mathematical modeling, numerical simulations

1. INTRODUCTION

Aeroelasticity is the discipline that studies the interaction of inertial, structural and aerodynamic forces on aircraft, buildings, surface vehicles, etc. The interaction between these three forces can cause several undesirable phenomena: divergence (static aeroelastic phenomenon); flutter (dynamic aeroelastic phenomenon); limit cycle oscillations (nonlinear aeroelastic phenomenon); vortex shedding, buffeting, galloping (unsteady aerodynamic phenomena). There is a fairly clear description of “aeroelastic flutter” on Wikipedia: “Flutter is a self-feeding and potentially destructive vibration where aerodynamic forces on an object couple with a structure’s natural mode of vibration to produce rapid periodic motion”. In principle, the variation of eigenvalues of a structural model, driven by aerodynamic flow variation is studied in the linear frame. The flow variation causes the change of the aerodynamic forces that are proportional to the flow velocity of air and degrees of freedom of the system; in turn, this influences the aeroelastic system, thereby modifying their eigenvalues until they become “unstable”. Flutter occurs when the air speed reaches a speed called the flutter speed.

The countering of dangerous phenomena such as flutter and buffeting, for example, was made first by passive techniques: increasing structural rigidity, mass balancing, changing geometry. This led to increase both weight and cost, while reducing overall performance. At some point, the use of primary flight controls and associated actuators was considered efficient to...
combat flutter [1]-[4]. Thus began the active control era [5]. However, as demonstrated in [6], there are many drawbacks when using primary flight controls for auxiliary purposes.

The purpose of this paper is to present a solution to counteract the flutter, based on the use of an actuator consisting in two V-shaped piezo stacks (Fig. 1.). The approach is developing in the project [8] and ends as a demonstrator in wind tunnel.

The advantage of the piezo actuators is their small size, large bandwidth and, not least, high energy density. There is a disadvantage linked to their lower strokes. It is important to note that, for the antiflutter aero-elastic control to be effective, the deflection of the control surface must be at least 5-6 degrees, to the frequency range of at least 25 to 30 Hz, as states the paper [9]. The piezo stacks NAC2022-H98-AO1 were bought from NOLIAC (http://www.noliac.com/) and have the following basic properties: height 98 mm, stroke 148.8 μm, capacitance 19010 nF, maximal force developed 4200 N, maximum operating temperature 150°C, material NCE51F.

![Fig. 1. Left: CATIA 3D view of the V-shaped piezo stacks; right: framing of actuator in the available space in wing](image)

![Fig. 2. CATIA model of the longeron (top left) and wing (top middle); measuring of wing model frequencies (top right) [7]](image)

**Table 1. Natural frequencies, determined by CATIA, and measured, respectively**

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency Hz (longeron)</th>
<th>Frequency Hz (wing)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CATIA computed</td>
<td>CATIA computed</td>
</tr>
<tr>
<td>1</td>
<td>13.59</td>
<td>6.23</td>
</tr>
<tr>
<td>2</td>
<td>43.42</td>
<td>10.21</td>
</tr>
<tr>
<td>3</td>
<td>44.12</td>
<td>20.83</td>
</tr>
<tr>
<td>4</td>
<td>59.14</td>
<td>26.32</td>
</tr>
<tr>
<td>5</td>
<td>78.91</td>
<td>29.83</td>
</tr>
<tr>
<td></td>
<td>5.93 bending</td>
<td>11.70 torsion</td>
</tr>
<tr>
<td></td>
<td>22.73 bending</td>
<td></td>
</tr>
</tbody>
</table>

The actuator is mounted inside the wing, to the far right of the aileron visible in Fig. 2. top middle. The same figure presents a CATIA drawing of the longeron and the experimental facility for measuring natural frequencies of the wing model. Table 1 shows natural frequencies, CATIA computed and measured. In general, the frequency of flutter occurs in...
the domain of the first two frequencies of the wing, thus the bandwidth of the actuator has to well cover this frequency range.

2. KINEMATIC SCHEME OF AILERON ACTUATION WITH TWO V-SHAPED -STACK PIEZOELECTRIC ACTUATORS

The scheme is represented in Fig. 3. The \( xOy \) system of axes, with the \( Ox \) axis along the axis is considered. The coordinates are expressed in [mm]. We mention that the left side of the scheme is actually in a different parallel plane to the right side, the one in which the aileron and the slider crank mechanism, sketched in the point \( P_6 \), are represented.

According to the conception and the rigors of actuation with such actuators, the piezo stacks are arranged along the segments \( PP_1 \) and \( PP_4 \), respectively. When the stack \( PP_3 \) is activated by increasing supply voltage \( V \), \( V_0 + \Delta V(t-\tau) \), which determines its extension to move to the right and slightly below the articulated point \( P_3 \), the stack \( PP_2 \) is supplied with the voltage \( V_0 - \Delta V(t) \)

\[
\Delta V(t-\tau) = \begin{cases} 0, & t \leq \tau \\ \Delta V, & t > \tau \end{cases}
\]

which causes the withdrawal to the left and slightly upward of the articulated point \( P_4 \), therefore to not oppose resistance to movement down of the articulated point \( P_5 \) in the slider crank mechanism. This provides the movement with positive angle (up) \( \delta \) of the aileron. The two stacks are successively active versus passive, in the sense of the presented description. The constant \( \tau \) defines a delay in the application of voltage to “active” stack, to ensure the withdrawal of passive stack.

![Fig. 3. Kinematic scheme of aileron actuation with 2 V-shaped piezo stacks; coordinates in mm](image)

The calculation of the aileron deflection, assuming to neglect the elasticities, the friction and the aerodynamic load on aileron, takes into account the following reasons:
a) simplifying hypothesis: because only one stack is active, consider that the physical model of the actuation is that of Fig. 4, which represents the sequence of the aileron up deflection by activating the piezo stack placed on the segment;
Fig. 4. Above: A simple physical model of aileron actuation; below: the form of voltage signals at the inputs of the two piezo stacks, in a time sequence

b) the lever $PP_3$ is articulated with the lever $PP_3M$, the lever $PP_3M$ is solidary linked with the shaft $PP_0PP_5$, and in the point $P_5$ there is a slider crank type joint for aileron actuation;

c) in the presence of the control voltage $V$, the piezo stack $PP_1PP_3$ develops an expansion movement, and the piezo stack $PP_2PP_4$ develops a movement of withdrawal, if it has previously been extended, and, based on constraints, the point $P_3$ moves on an arc in the point $P_3'$, the lever $PP_3M'$ remaining solidary, without deformations, with $PP_0M'$.

3. THE PIEZO STACK FREE STROKE AND THE AILERON DEFLECTION DETERMINATION WITH RESPECT TO THE APPLIED VOLTAGE

In what follows, we propose a mathematical model of the aileron actuation, with no load at the output of the actuator. Referring to Fig. 4., the angle $\delta$ will be obtained as a function of the control voltage $V$ applied to the piezo stack disposed on the shaft $PP_0PP_3$. As a result of the applied voltage, considering the articulation in the point $P_3$, and the assumption that the length of the segments $PP_0M$ and $PP_3M$ solidary connected at an angle of 90 degrees, remains constant, the point $P_3$ moves in $P_3'$ and the point $M$ moves in $M'$.

The following design data are considered

$x_{P_3} = 103\text{mm}; y_{P_3} = 6\text{mm}; x_{R_1} = 3\text{mm}; y_{R_1} = 11\text{mm}; x_{P_5} = 147\text{mm}; x_M = 103\text{mm}$

$L_1 = PP_1PP_3 = \sqrt{(x_{P_3} - x_{R_1})^2 + (y_{P_3} - y_{R_1})^2}; L_2 = MP_0 = MP_0$

$L_3 = MP_3 = MP_3'; L_4 = PP_0PP_5 = x_{P_5}$
The length of segment $P_0M'$ is

$$x_{M'}^2 + y_{M'}^2 = L_2^2$$

(1)

Now, we transcribe the squareness condition $P_3'M' \perp P_0M'$

$$\frac{y_{M'}}{x_{M'}} \cdot \frac{y_{P_3'} - y_{M'}}{x_{P_3'} - x_{M'}} = -1, \quad y_{M'}y_{P_3'} + x_{M'}x_{P_3'} - (x_{M'}^2 + y_{M'}^2) = 0$$

or, considering the relationship (1)

$$y_{M'}y_{P_3'} + x_{M'}x_{P_3'} = L_2^2$$

(2)

The free stroke of piezostack at the point $P_3$, is approximated with a line passing through the origin (Fig. 5) and having the slope

$$k = \frac{147 \times 10^{-6} m}{200 V} = 0.735 \times 10^{-3} \text{ mm/V}$$

(2')

Therefore, the length of the segment $P_3P_3'$ is

$$\left(x_{P_3'} - x_{P_3}\right)^2 + \left(y_{P_3'} - y_{P_3}\right)^2 = (L_4 + kV)^2$$

(3)

and the length of the segment $P_3'M'$ is

$$\left(x_{M'} - x_{P_3}\right)^2 + \left(y_{M'} - y_{P_3}\right)^2 = L_3^2$$

(4)

We have to solve a system of four equations (1)-(4) with four unknowns $x_{P_3'}, y_{P_3'}, x_{M'}, y_{M'}$.

**Determining the coordinates of $P_3'$**

By developing (4) and using (1) and (2) one obtains

$$x_{P_3'}^2 + y_{P_3'}^2 = L_3^2 + L_2^2$$

(5)

From (3) one obtains

$$x_{P_3'}x_{P_3} + y_{P_3'}y_{P_3} = E(V) := \frac{1}{2}\left[x_{P_3}^2 + y_{P_3}^2 + L_3^2 + L_2^2 - (L_4 + kV)^2\right]$$

(6)

With the notations

$$A = x_{P_3}^2 + y_{P_3}^2, B = -2E(V)x_{P_3}, C = -y_{P_3}\left(L_2^2 + L_3^2\right) + E^2(V)$$

(7)

we determine the coordinates of point $P_3'$

$$x_{P_3'} = -\frac{B + \sqrt{B^2 - 4AC}}{2A}, y_{P_3'} = \sqrt{L_3^2 + L_2^2 - x_{P_3'}^2}$$

(8)

**Determining the coordinates of $M'$**

$$\begin{cases} x_{M'}^2 + y_{M'}^2 = L_2^2 \Rightarrow y_{M'} = \sqrt{L_2^2 - x_{M'}^2} \\ x_{M'}x_{P_3'} + y_{M'}y_{P_3'} = L_2^2 \end{cases}$$

(9)

The expression of $y_{M'}$ from (9_1) is substituted in (9_2)
<table>
<thead>
<tr>
<th>No</th>
<th>Increasing voltage [V]</th>
<th>Displacement [µm]</th>
<th>Decreasing voltage [V]</th>
<th>Displacement [µm]</th>
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<tr>
<td>9</td>
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<td></td>
<td>MUTE(0)</td>
<td>6</td>
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</tbody>
</table>

Piezo stacks measurements: 1 above, 2 below

<table>
<thead>
<tr>
<th>No</th>
<th>Increasing voltage [V]</th>
<th>Displacement [µm]</th>
<th>Decreasing voltage [V]</th>
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<td>153</td>
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<tr>
<td>9</td>
<td></td>
<td></td>
<td>MUTE(0)</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 5. Free stroke measurements for piezo stacks; $k = \tan(\gamma)$

\[
x_{M'}x_{P_5} + y_{P_5}\sqrt{L_2^2 - x_{M'}^2} = L_2^2, \quad y_{P_5}\sqrt{L_2^2 - x_{M'}^2} = L_2^2 - x_{M'}x_{P_5}
\]

\[
y_{P_5}^2\left(L_2^2 - x_{M'}^2\right) = L_2^2 + x_{P_5}^2x_{M'}^2 - 2L_2^2x_{M'}x_{P_5} + x_{M'}^2y_{P_5}^2 + L_2^4 - L_2^2y_{P_5}^2 = 0 \quad (10)
\]

\[
x_{M'}^2\left(x_{P_5}^2 + y_{P_5}^2\right) - 2L_2^2x_{P_5}x_{M'} + L_2^2\left(L_2^2 - y_{P_5}^2\right) = 0
\]

Thus the coordinates of point $M'$ are the following
Mathematical modeling of a V-stack piezoelectric aileron actuation

\[
x_{M'} = -B + \sqrt{B'^2 - 4A'C'} \quad y_{M'} = \sqrt{L_2^2 - x_{M'}^2}
\]

\[A' := x_{P_5'}^2 + y_{P_5'}^2, \quad B' := -2L_5^2x_{P_5'}, \quad C' := L_2^2 \left( L_2^2 - y_{P_5'}^2 \right)\]  

(11)

\[
\begin{align*}
\text{Fig. 6. Evolution of the aileron deflection angle } \delta \text{ (Fig. 4.) with respect to the voltage } V
\end{align*}
\]

**Determining the angle \( \delta \)**

Consider triangles \( P_0M'M', \ P_0P_5P_5', \) Fig. 4 above, as similar, thus Thales theorem gives

\[
\frac{P_0M}{P_0P_5} = \frac{MM'}{P_5P_5'}, \quad \frac{L_4}{P_5P_5'} = \frac{MM'}{L_2}
\]

(12)

\[
P_5P_5' = \frac{L_4 \times MM'}{L_2}
\]

(13)

\[
MM' = \sqrt{\left(x_{M'} - x_M\right)^2 + \left(y_{M'} - y_M\right)^2}
\]

(14)

Finally, let us assimilate the triangle \( P_5P_5'P_6 \) as isosceles, then

\[
\sin \left( \frac{\delta}{2} \right) = \frac{P_5P_5'}{2P_5P_6}, \quad P_5P_6 = 2.5 \text{ mm}
\]

(15)

Therefore

\[
\delta = 2 \arcsin \frac{P_5P_5'}{2P_5P_6}
\]

(16)

Evolution of angle \( \delta \) depending on the voltage \( V \) is given in Fig. 6.

4. COMPUTING THE ACTUATION IN THE PRESENCE OF AERODYNAMIC LOAD

We consider the actuation in the presence of the aerodynamic load on the wing, but with the neglecting of the elasticities and the frictions in the kinematic linkages. The physical model of Fig. 2 is split into 3 “bodies” (Figs. 7, 8). To obtain the actuation mathematical model, the equations of forces and moments will be written using the so-called “free body diagrams”. Most notations are deductible from Figures.
The sense of Ox, Oy axes is given in Fig. 3. When the sense of the forces is evident, it will be used as such in equations. Otherwise, the sense of the reactions in supports will result algebraically:

\[
\begin{align*}
  m_a \ddot{x}_a &= H_1 + F_3 \cos \varphi - F'_3 \sin \varphi \\
  m_a \ddot{y}_a &= V_1 - F_3 \sin \varphi - F'_3 \cos \varphi \\
  J_0 \ddot{\varphi} &= m_a g l_{ag} \cos \varphi + F'_3 l_a \\
  m_s \ddot{x}_{cg} &= H_0 - F_3 \cos \varphi + F'_3 \sin \varphi - F_5 \sin \theta \\
  m_s \ddot{y}_{cg} &= V_0 - m_s g + F_3 \sin \varphi + F'_3 \cos \varphi - F_5 \cos \theta \\
  J_s \ddot{\Theta} &= m_s g l_{scg} \cos \theta - F_3 \sin \varphi \dot{d}_{31} - F'_3 \cos \varphi \dot{d}_{32} + F_5 D \cos \theta \\
  m_e \ddot{x}_{cg} &= -F_5 \sin \theta + H_6 - F_a \sin \delta \\
  m_e \ddot{y}_{cg} &= F_5 \cos \theta + V_6 - F_a \cos \delta - m_e g \\
  J_e \ddot{\Theta} &= l_e F_5 \cos \theta - M_a - m_e g l_g
\end{align*}
\] (17) (18) (19)

Determination of the actuation evolution means the solving of the system of 9 equations (17), (18), (19) with 9 unknowns \(H_0, V_0, F_3, F'_3, F_5, H_1, V_1, H_6, V_6\). Input data are the following: \(l_a = 100.12\,\text{mm} \cong \overline{P_1P_3}\); \(l_e = \overline{P_5P_6} = 2.5\,\text{mm}\); \(l_g = P_6P_7 = 15.77\,\text{mm}\); \(l_{ag} = 100.12/2\,\text{mm}\); \(m_a = 0.079\,\text{kg}\); \(m_s = 0.071\,\text{kg}\); \(d_{32} = 103\,\text{mm}\); \(D = 147\,\text{mm}\); \(l_{scg} = 90\,\text{mm}\); \(m_e = 0.12\,\text{kg}\); \(S_e = 0.11 \times 0.49\,\text{m}^2\); \(c_e = 110\,\text{mm}\); \(\rho = 1.3\,\text{kg/m}^3\); \(U = 0...50\,\text{m/s}\); \(M_a = 0.5pU^2S_eC\) [Nm]; \(C = 0.01...0.1\); \(F_a = 4M_a/c_e\) [N]. The segments \(l_a, l_e, l_g, l_{ag}, d_{32}, D, l_{scg}\) involved in the moments calculation were considered to be constant. Other approximations are inherent.

Fig. 7. Scheme for computing the actuation and introducing the method of “free body diagrams”

Fig. 8a. The body of piezo stack
Measurements of the displacement-force at the level of point $P_3$ were performed (Fig. 10). The displacement of 650 μm corresponds to a half of sinusoid without load. The double, 1300 μm, i.e., 1.3 mm, is an increase of approximate 10 times of the piezo stack stroke given by the provider (148.8 μm; see http://www.mmech.com/noliac-actuators/plate-stacks/nac2022-hxx). The developed maximum force, as measured by loading with resorts becoming increasingly stronger on the other half of the sinusoid, is 230 N. Thus, on the shaft $P_1P_3$, the force of 4600 N (close to 4200 N given by the provider) is checked. In this theoretical-experimental framework, we have to adapt equation (3) to the new situation in which the piezo stack stroke will be affected by aerodynamic load $F_a$. Our reasoning is as follows. At the hinge axis represented by the fulcrum $P_6$, we have the balance of moments.

Add the simplifying assumption of equilibrium of moments at the points $P_3, P_5$

$$4600 \times x \text{[Nm]} = 1.3 \times 10^{-3} \times F_5 \text{[Nm]} \quad (20')$$

Combining the two relationships above, the sought correction of the piezo stack stroke is

$$x \text{[m]} = \frac{1.3 \times 10^{-3} \text{[m]} \times l_e}{4600 \text{[N]} \times l_g} F_a \text{[N]} \quad (20'\ast)$$

$$l_g = 0.25 \times 0.11 - 0.01173 \text{[m]} = 0.01577 \text{[m]}, \quad l_e = 2.5 \times 10^{-3} \text{[m]}$$

Fig. 8c. The body of the actuation, downstream of slider crank

$$F_5 \times P_5P_6 = F_a \times l_g$$

(20)
The evolution of the piezo stack stroke due to application of input voltage $V$ is given by

$$\left( x_R - x_R^2 \right)^2 + \left( y_R - y_R^1 \right)^2 = \left( L_4 + kV - IF_d \right)^2, k = 0.735 \times 10^{-6} \frac{m}{V}$$

$$l := \frac{1.3 \times 10^{-3} \times l_g}{4600 \times l_c} = 1.7827 \times 10^{-6} \frac{m}{N}, M_a = 0.5 \times \rho U^2 S c_e c_e$$

(3')

$$F_a = 4 \times M_a / c_e, c_e = 0.11 \ m, S = 0.11 \times 0.49 \ m^2, U = 0...50 \ m/s, C = 0.01...0.1$$

Therefore, the algorithm for calculating the actuation in the presence of the aerodynamic load $F_a$ requires solving a system of 9 equations with 9 unknowns $H_0, V_0, F_3', F_3, F_5, H_1, V_1, H_6, V_6$, for a set $V_i$ of values in the range $(100; 200)V$ and a set $U_i$ of values in the range $(0 \ldots 50) m/s$.

$$H_1 + F_3 \cos \varphi - F_3' \sin \varphi = 0, V_1 - F_3 \sin \varphi - F_3' \cos \varphi = 0$$

$$m_s g l_{ag} \cos \varphi + F_3' l_a = 0, H_0 - F_3 \cos \varphi + F_3' \sin \varphi - F_5 \sin \theta = 0$$

$$V_0 - m_s g + F_3 \sin \varphi + F_3' \cos \varphi - F_5 \cos \theta = 0$$

$$m_s g l_{scg} \cos \theta - F_3 \sin \varphi d_{31} - F_3' \cos \varphi d_{32} + F_5 D \cos \theta = 0$$

$$-F_3 \sin \theta + H_6 + F_a \sin \delta = 0, F_5 \cos \theta + V_6 - F_a \cos \delta - m_e g = 0$$

$$l_c F_5 \cos \theta - M_a - m_e g l_g = 0$$

(21)

The notations in (3') that do not appear in the Figures refer to aerodynamic moment $M_a$, coefficient of aerodynamic moment $C$, air speed $U$, air density $\rho$, aileron surface $S$, aerodynamic chord $c_e$ of aileron.

The point of application of the aerodynamic force is considered at a quarter of chord versus the leading edge.

Obviously, the solutions will represent stationary regimes that V-piezo stacks actuation can achieve as responses to voltage command signals. The angle values $\theta$ and $\varphi$ are calculated with the same relationships as in the case with no load. The relation (6) is restructured in the form

$$E(V; F_a) := \frac{1}{2} \left[ x_{R1}^2 + y_{R1}^2 + I_2 + I_3 - \left( L_4 + kV - IF_d \right)^2 \right]$$

(6')

Fig. 9. Left: CATIA wing and aileron view; right: typical variation of coefficient $C$
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Fig. 10. Experimental measurements displacement-force, at the level of point \( P_5 \) (see Fig. 3)

Fig. 11. Stationary values of aerodynamic force \( F_a \), depending on air speed \( U \)

Therefore, the algorithm has the following sequences

1. the coordinates of the point \( P'_3 \) (see (8))

\[
x_{P'_3} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \quad y_{P'_3} = \sqrt{L_2^2 + L_3^2 - x_{P'_3}^2}
\]

\( A := x_{P_1}^2 + y_{P_1}^2, \quad B := -2E(V, F_a)x_{P_1}, \quad C := -y_{P_1}^2\left(L_2^2 + L_3^2\right) + E^2(V, F_a) \)

2. the coordinates of the point \( M' \) (see (10))

\[
x_{M'} = \frac{-B + \sqrt{B'^2 - 4A'C'}}{2A'}, \quad y_{M'} = \sqrt{L_2^2 - x_{M'}^2}
\]

\( A' := x_{P_1}^2 + y_{P_1}^2, \quad B' := -2L_2^2 x_{P_1}, \quad C' := L_2^2\left(L_2^2 - y_{P_1}^2\right) \)

3. the trigonometric functions values for angle \( \theta, \varphi \) and the values of angle \( \delta \)
\[
\sin \theta = \frac{|y_{M'}|}{\sqrt{x_{M'}^2 + y_{M'}^2}}, \quad \cos \theta = \sqrt{1 - \sin^2 \theta}
\]
\[
\sin \varphi = \frac{|y_P - y_{P_3}|}{\sqrt{(x_P - x_P)^2 + (y_P - y_{P_3})^2}}, \quad \cos \varphi = \sqrt{1 - \sin^2 \varphi}
\]  
(24)

\[\delta = 2 \arcsin \frac{P_5' P_6'}{2 P_3' P_6}, \quad P_5' = \frac{L_4 \times MM'}{L_2}, \quad L_2 := \sqrt{(y_{M'} y_{P_3} + x_{M'} x_{P_3}), \quad L_4 := P_0 P_5 = x_{P_3}}\]

In all calculation sequences must be fulfilled the condition

\[kV_i - lF_{a,j} > 0\]  
(25)

5. NUMERICAL SOLUTIONS AND CONCLUSIONS

Let's remind that the solutions \(H_0, V_0, F_3, F_3', F_3, H_1, V_1, H_0, V_6\) of the system (21) depend on the command voltage \(V\) and the air speed \(U\). For reasons of printing space, only six of the nine solutions, most representative for the idea of global solving are considered in Figures 5.1-5.6; the solutions are shown in the form of meshes. The maximal moment coefficient \(C = 0.1\) was considered as covering from viewpoint of emphasizing the maximum forces occurring in the kinematic chain. Fig. 11 shows the order of magnitude of aerodynamic force, in other words, of the load to be acted successively by piezo stacks, in a sense or another of the movement. The reference values of the coefficient \(C\) represented in Fig. 9 was provided by Aerodynamics Compartment in the framework of the project internal reports [7]. The notation “control \([V]\)” refers to an open loop command voltage. As shown in Fig. 12, the aerodynamic forces seem to load at a low level the piezo actuator (approximate at one eighth of the maximum force available). Figure 13 shows another important result of modeling, i.e, the angle of deflection values. Compared to values in Fig. 14, they are easily diminished, which is natural given the consideration of aerodynamic force. Figures 14 and 15 show that the reactions in supports are reasonable. Finally, Fig. 16 shows that the calculated force \(F_5\) is quite close to the measured values (Fig. 10).

Fig. 12. Stationary values of the force \(F_3\), developed by piezostack in the presence of the load \(F_a\)
Fig. 13. Stationary values of the angle $\delta$, developed by piezostack in the presence of the load $F_a$

Fig. 14. Stationary values of the force $H_1$

Fig. 15. Stationary values of the force $H_0$
A piezo actuator photo on measurements bench of INCAS Mechatronics Laboratory is shown in Fig. 17. This Laboratory has a tradition of over 45 years in analysis, synthesis, qualification testing and flight clearance for aircraft and helicopters hydraulic servomechanisms [10]-[19]. More recent is research on smart structures [20]-[23].

It is important to note that mathematical modeling presented in this article is original. The results prove that V-piezo stacks design has been well thought out. Further research must develop synthesis of antiflutter control law.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the National Authority for Scientific Research—ANCS, UEFISCSU, through AFDPA research project Contract 289/2014.

The authors also thank their colleague dr. eng. Valentin Butoescu for important contributions in the achievement of the paper results.
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