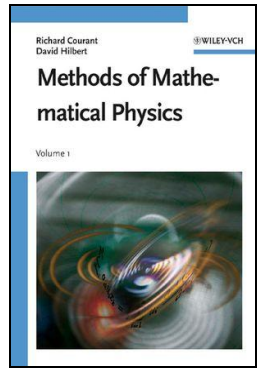
DOI: 10.13111/2066-8201.2013.5.1.15

WHY DAVID HILBERT? (1862-1943)

We must know, we will know (David Hilbert)

***Methods of Mathematical Physics**, **vol. 1** (575 pp, April 1989)*
***Methods of Mathematical Physics, vol. 2**, **Differential Equations** (852 pages, April 1989),* by **Richard Courant** and **David Hilbert − WILEY-VCH**



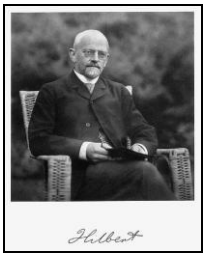
My thought in this issue of INCAS BULLETIN was to draw attention to an exceptional book, available on Internet. I quickly came to feel the need to say a few words about one of the authors, maybe the last from a gallery that should include a list of 10-15 great classics that should begin with Descartes, Newton, Euler, Gauss, Galois and Poincaré. It's about David Hilbert, from whose death at 14 February 2013 were fulfilled 70 years.
Encyclopedias tell us who he was and what he had accomplished in mathematics. But the full significance of his legacy remains a subjective evaluation that depends on each observer. Let we try in a few words such an assessment.

First, Hilbert was a reformer of mathematics, in terms of formalism. *Formalism* considers that mathematical statements may be thought of as statements about the consequences of certain manipulation rules of some strings called “axioms” and some “rules of inference” to obtain new strings from given ones. So, Hilbert elaborates a Program that intended be a complete and consistent axiomatization of all of mathematics. This consistency was based on an arithmetical scaffolding, considered to be philosophically uncontroversial. As it is known, Hilbert’s formalist Program has received a mortal blow by the second of Gödel's incompleteness theorems. It must be said, however, that this outcome has appeared amid a philosophical crisis of mathematics, which neither began nor ended with this unexpected confrontation between Hilbert and Gödel. Anecdotically, we remember the attempted reform of Romanian highschool textbooks of the 1970s, when overnight the Euclid's geometry became unrecognizable for both teachers and schoolboys.

Secondly, Hilbert is the author of the celebrated list of twenty-three problems in mathematics published in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th century mathematics. The list the problems can be found at the address http://en.wikipedia.org/wiki/Hilbert%27s_problems.

Third, the most famous Hilbert's contribution to mathematics and to logic, is ... the Hilbert space. This mathematical concept generalizes the notion of Euclidean space. Unlike the Banach space, a Hilbert space is an abstract (complete) normed vector space possessing the structure of an inner product that allows length and angle to be measured. Here the formalist Hilbert introduced a very intuitive concept. Exact analogs of the Pythagorean theorem and parallelogram law hold in a Hilbert space. Further, the analog of "dropping the altitude" of a triangle is the perpendicular projection onto a subspace, which plays a significant role in optimization problems And so on, an element of a Hilbert space can be uniquely specified by its coordinates with respect to a set of coordinate axes (an orthonormal basis), in analogy with Cartesian coordinates in the plane. When that set of

axes is countably infinite, the Hilbert space can be thought of in terms of infinite sequences that are square-summable. And finally, linear operators on a Hilbert space are simply transformations that stretch the space by different factors in mutually perpendicular directions.



Add herein the special affinity of the Romanian mathematicians in the field: 1) C. T. Ionescu-Tulcea, *Spatii Hilbert*, Ed. Academiei RPR, 1956; 2) V. I. Istratescu, *Inner product structures, Theory and applications*, Mathematics and its Applications, D. Reidel Publishing Co., Dordrecht, 1987; 3) I. Cioranescu, *Geometry of Banach spaces, duality mappings and nonlinear problems*, Mathematics and its Applications, Kluwer Academic Publishers Group, Dordrecht, 1990.

Mathematical personalities such as Richard Courant, Alfréd Haar, Emanuel Lasker (considered by some the greatest chess player of all time), Hugo Steinhaus, Gabriel Sudan (professor at the University Politehnica of Bucharest), Hermann Weyl, Ernst Zermelo were Hilbert's doctoral students.

Last but not least: the books of the great mathematicians of the past are generally inaccessible or hard to read sometimes; instead, the book in two volumes that we present here is a handy treasure. Since these volumes came out in Germany in 1924, and in 1937, respectively, they remained a gold standard in the field. Courant and Hilbert's treatment "restores the historically deep connections between the physical intuition and the mathematical development, providing the reader with a unified approach to mathematical physics". For compliance, we present a summary of the first volume:

I. The algebra of linear transformations and quadratic forms;

II. Series expansions of arbitrary functions;

III. Linear integral equations;

IV. The calculus of variations;

V. Vibration and eigenvalues problems;

VI. Applications of the calculus of variations to eigenvalues problems;

Additional Bibliography.

Dr. math. Ioan URSU
INCAS – National Institute for Aerospace Research "Elie Carafoli"
B-dul Iuliu Maniu 220, Bucharest 061126, Romania
iursu@incas.ro