Dealing with mathematical modeling in applied control

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Control theory – is it a theory of model or control? (Jingqing Han)

Abstract: *The first part of the paper presents some conjectures deduced from o long practice of systems mathematical modeling. So, it is underlined the over-emphasis developed in the sixty years history of classical control on the mathematical model. Two lessons derived from the conservativeness-versatility dualism of the mathematical models versus the mathematical theories in providing the classical control law are advanced. A third reflection concerns the spectacular decreasing of the mathematical model importance in the intelligent control. The second part of the paper shows our arguments in favor of proposed conjectures and intends especially to highlight the new intelligent control paradigm.*

Key Words: *mathematical models, classical versus intelligent control, active and semi active vehicle suspensions, electro hydraulic servos*

1. SOME CONJECTURES ON MATHEMATICAL MODELING

The design of a real world controlled system (CS) is always mediated by the adopted idealization providing an intermediate physical model (PHM). Usually, given a CS, many PHMs can be developed.

Thus, having the first step in the CS’s analysis and design carried out – in other words, making the selection of the associated PHM – , the second step follows: the application of known physical laws which govern the real world of the CSs to the selected PHM, thus leading us to a mathematical model (MM), usually defined as a dynamic system.

Again, for a certain chosen PHM, many MMs can be performed, based on successive operations of fitting, completing or reduction of the initial model. The third step now follows: the treatment of the chosen MM using compatible mathematical instruments (MI) (theories, methodologies, all belonging to the theory of automatic control). This last step results in a solution which, after necessary validation by numerical simulations of MM, ends to be embedded in the designed CS (so called software implementation). And, worthy noticing, the entire described chain is often a recurrent one, until a satisfactory, full achievement (does it indeed exist?)

But let now our reflection focus on MM component of the above inductive-deductive paradigm, more exactly on some empirical findings concerning *the properties of the MMs versus MI*. Therefore, to summarize the ideas: when designing and analyzing a CS, the classical approach involves MMs of the PHM. Thus, generally speaking, the classical problem is for the analyst and designer to find a representative MM. A representative MM must obey to a parsimony principle [1]: the model must be complex enough to capture essential properties and constraints of the CS and, at the same time, simple enough to be compatible with a certain MI.

A handy way to meet these contradictory requirements involves the completion of the following steps: a) the derivation of a MM as complex as possible, in accordance with the physical laws and constraints defining the PHM; b) a certain adjustment – simplification – of the MM in accordance with the rigor of the MI used for the system design and analysis; c) providing CS design and optimized solution; d) iterative validation of the solution by numerical simulation on the *complex* MM and experimental tests on PHM and CS.

The step b) of this strategy requires some elucidation. On one hand: the MM, as a thinking first result, derived from a physical reality, is more flexible than a MI (i. e., a mathematical theory [2]), as a thinking second result, derived from a meta-reality – herein, the mathematical thinking.

But, on the other hand: all MMs, describing the same object – PHM, PHS – will predict approximately the same object performance [3], despite of the used MI.

Our particular interest was focused on the question: why really the products of various MI – from the simplest to most sophisticated – operating on the various MMs associated to PHM and CS, end to be close enough?

Our response is rather philosophical and has been seen as a true conservative property, let us call otherwise the "*firmness*" and "*solidarity*" of the MMs.

In fact, there are always more profound structural-material reasons explaining the limitation of achieved amount of improvement obtained by insertion of a MI in CS [4]. Accordingly, "much more knowledge of CS properties, much more its performance improvement, based on more and more MMs and MIs extensions", cannot be always a realistic endeavor: all these mathematical structures – the MMs – are solidary and firm (in Latin language, *firmitas* = resistance, opposition) not perform miracles!

Possibly, the control MIs could be ranked at least from the viewpoint of two criteria: robustness and cost of implementation!

If the MMs have certain conservative properties able to baffle the researcher's enthusiasm, there is a compensating their property.

This is indeed a complemental property, namely MMs "*flexibility*": to be able to apply a certain MI, a MM can be often "shaped" in view of coping with the constraints of a certain mathematical apparatus which is to be operating on them.

Thus, our empirically lessons [5], [3], [6] concern the behavior of MMs in connection with MI of control synthesis.

Lesson 1. All MMs, describing the same object, will promote in the last analysis a certain "firmness" and "solidarity", in other words, will predict very close CS performance or behavior improvement, in despite of the used mathematical MI.

Lesson 2. The MM, as a thinking first result, derived from a physical reality, is more "flexible" than a MI (i.e., a mathematical theory, as a thinking second result, derived from a meta-reality (herein, the mathematical thinking).

A third lesson concerns classical control versus new paradigm of intelligent control. Classical control systems and intelligent control systems both have certainly their place in control systems theory.

Usually when classical control can be used, it is.

But it is not without significance that the vast majority – over 90% of implemented solutions – yet begins to PID control [7].

This is due to the simple fact that new intelligent control paradigm can be unjustified in some situations. Intelligent control systems excel in areas that are highly non-linear, or when classical control systems fail, or a model of the system is difficult or impossible to obtain.

Lesson 3. The essential part of the intelligent control paradigm is carried out as a free model vision; thus, the control synthesis is free of a few fundamental limitations, such as linearity, time invariance, accurate mathematical representation of plant etc.

While most of reported results in the literature of the field are categorically favorable to the nonconventional, intelligent control, we do not evade that there are many opponents; see, for example, the tempestuous and radical challenge launched in [8], wherein one concludes sententiously: “fuzzy control is a parasitic technology”. On the contrary, our conclusion is that, in various approaches [3], [6], [9]-[13], regarding as applications active and semi-active suspensions of vehicles, ABS system, electro-hydraulic servos actuating primary flight controls, the intelligent control based on neural networks and fuzzy logic worked very well, much better than classical methodologies PID, LQG etc.

2. THE ARGUMENTS

In [3], as foundation of our conjectures – Lessons 1, 2, 3 – has been served the field of active and semi-active road vehicle suspension control synthesis. Several MMs for active and semi-active automotive suspensions were developed and then followed by a large presentation of the derived active and semi-active controllers. So, almost all paradigms, from the old, classical LQG control synthesis, to the post-modern H_∞ robust synthesis, without leaving out the nonconventional paradigm of the neuro-fuzzy control synthesis, were used as control synthesis strategies. Note that in the envisaged problem the performance index includes the ride comfort criterion and the road holding criterion, which appear as two antagonistic performance indices. To compare the final results, the values of the ride comfort and the road holding, corresponding to the passive automotive suspension MMs, were considered as reference points 100%. Concluding histograms of these results are presented in Figs. 1, 2, intending to second once again the previous findings concerning the properties of the MMs and related methodologies operating on them.

Table 1 – Summary of the servo time constants for position references tracking: MHS SMHR case and related presumptive EHS with various control laws [5]

synthesis strategy	servo time constant τ_s [s]	
	EHS	MHS
Back stepping, third order model (2), (4) [5], with load pressure state variable; step reference amplitude: r = 0.255 cm	0.022	0.034 (system (39))
Back stepping, fifth order model (21), (22) [5], two pressures as state variables; step reference amplitude: x_{1s} = 0.255 cm	0.037	0.037 (system (40))
neuro-fuzzy antiwindup synthesis [10], fourth order Wang type model [16], two pressures as state variables; step reference amplitude r = 0.1 cm, and easy modified λ (λ = 0.55) and f (f = 1 daNs/cm)	0.011	0.025
Davison type robust servo synthesis, with antiwindup compensation [15], linear model [17], [3]; step reference amplitude r = 0.3 cm (derived from position transducer gain) and f = 1 daNs/cm, which compensates in some degree the increased r	0.021	0.024 (theoretical servo time constant, derived from linear system)
measured servo time constant [14], step reference amplitude r = 0.12 cm.		0.02

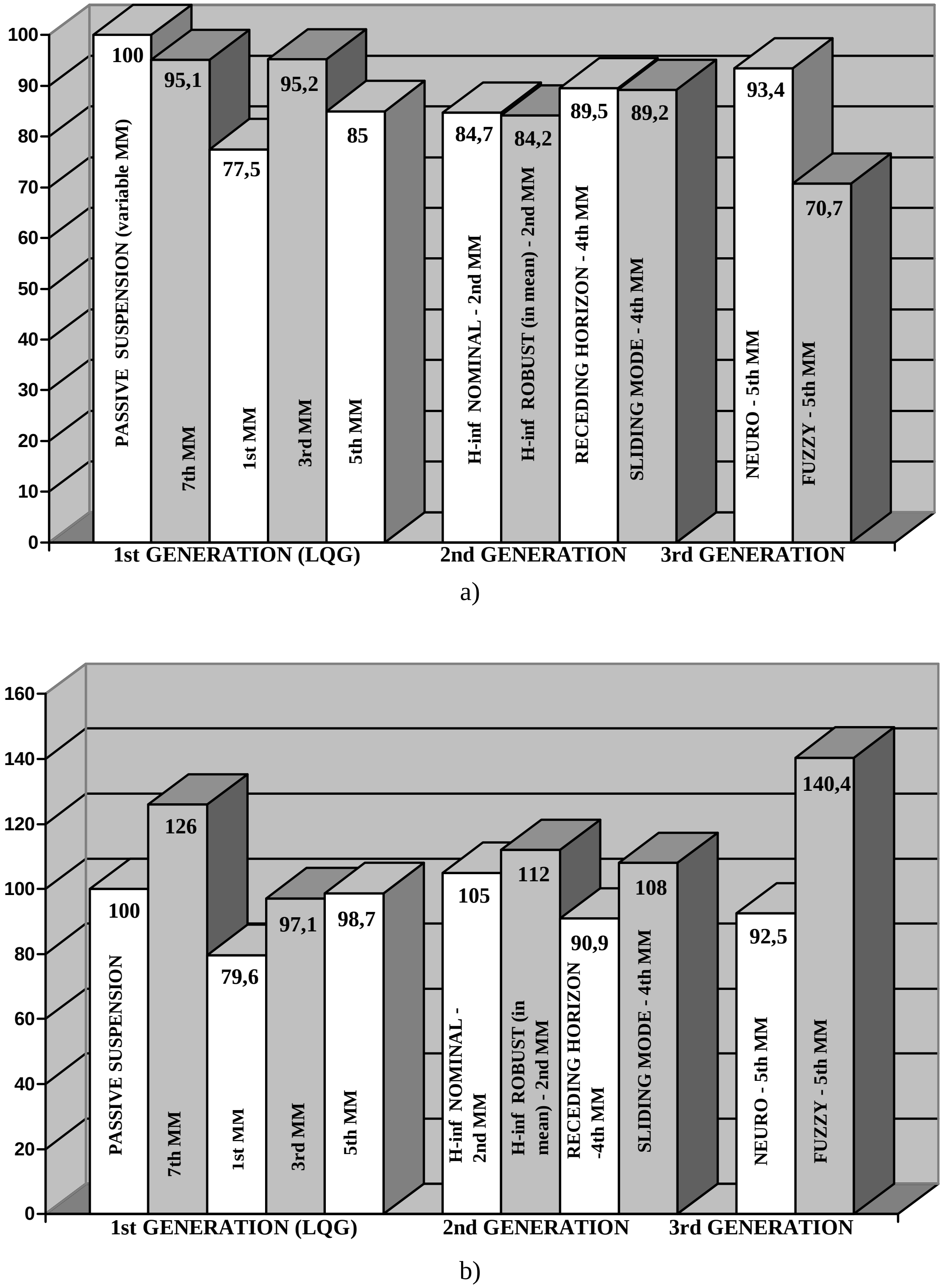
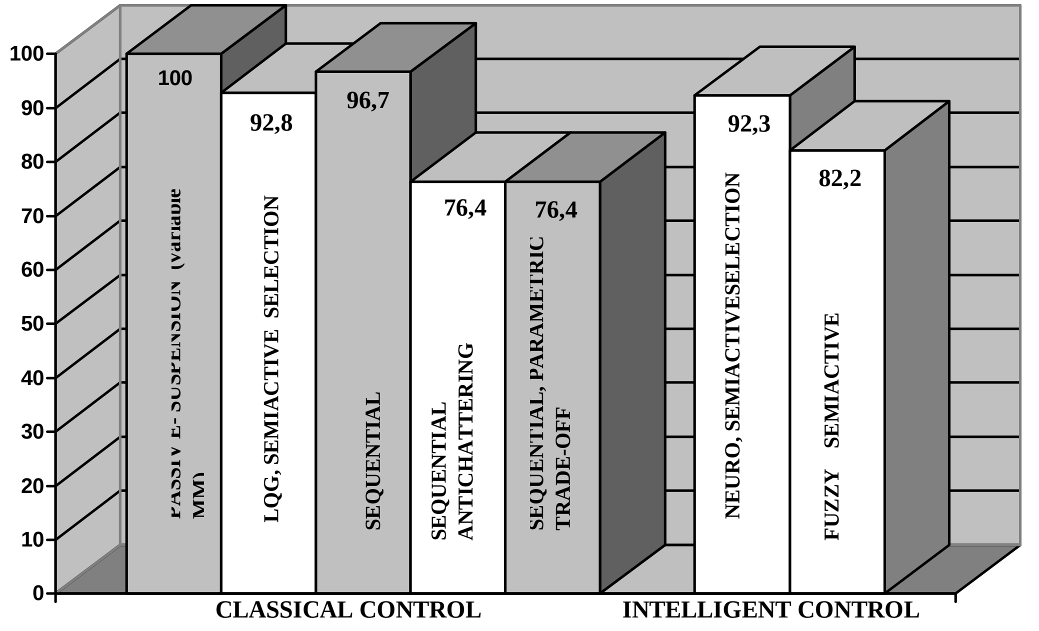
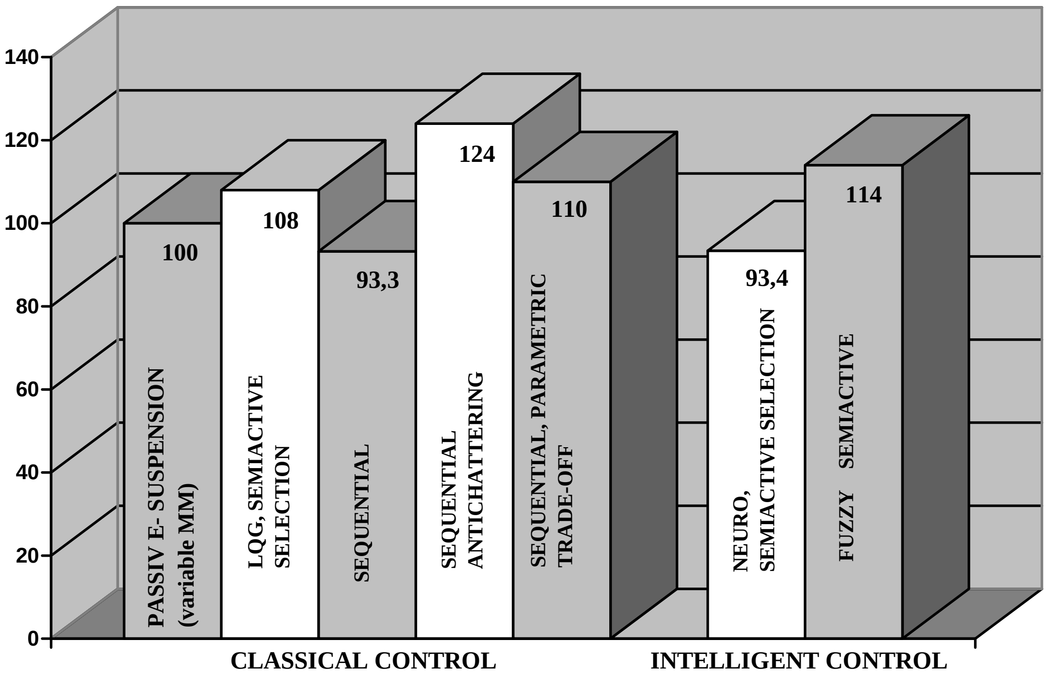


Fig. 1 – Active suspension: ride comfort (a) versus road holding (b), as supplied by synthesis paradigms belonging to three generations of automatic control



a)



b)

Fig. 2 – Semi-active suspension: ride comfort (a) versus road holding (b), as supplied by synthesis paradigms belonging to classical and intelligent control

Similar arguments were provided in [5] (see Table 1) and [6] on electro-hydraulic servos (EHSs) synthesis. Thus, we tried to offer new arguments that lend support to enounced conjectures. This time, several MMs of EHS and associated mechano-hydraulic servo (MHS), obtained from various control strategies, were quantified and compared from the viewpoint of the servo time constant performance and all the values were found to be close to the experimental one [14]. Three case studies on back stepping synthesis for controlling EHS systems are presented. The full state information was considered available. We have illustrated how the main theory can be brought or adapted to design practice as defined by a given MM, and have shown that the back stepping controllers are able to work with a complex plant such as EHS. The performance of the position tracking controllers has been finally compared with that of several other techniques, such as antiwindup neuro-fuzzy synthesis [6], [10] and classical robust synthesis with antiwindup compensation [15]. A certain trend of servo time constants improvement by control synthesis strategies can be attested. Smaller such values can be obtained, but with the risk to compromise a stability-performance trade-off of the servo time response.

3. CONCLUDING REMARK

We apologize for not having cited the paper: Jingqing Han – Robustness of Control System and the Godel's Incomplete Theorem", Control Theory and Its Applications, 16 (suppl.), 149-155, 1999.

Now, in final, we only wish to subscribe to a few words belonging to Karl R. Popper [19]: the objectivity of MMs, as mathematical objects, rests (continues to be) entire by "repeating their construction at will".

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