Prediction of pilot induced oscillations

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Abstract: *An important problem in the design of flight-control systems for aircraft under piloted control is the determination of handling qualities and pilot-induced oscillations (PIO) tendencies when significant nonlinearities exist in the vehicle description. The paper presents a method to detect possible pilot-induced oscillations of Category II (with rate and position limiting), a phenomenon usually due to a misadaptation between the pilot and the aircraft response during some tasks in which tight closed loop control of the aircraft is required from the pilot. For the analysis of Pilot in the Loop Oscillations an approach, based on robust stability analysis of a system subject to uncertain parameters, is proposed. In this analysis the nonlinear elements are substituted by linear uncertain parameters. This approach assumes that PIO are characterized by a limit cycle behavior.*

Key Words: aircraft pilot coupling, uncertain parameters, Edge Theorem

1. INTRODUCTION

A pilot induced oscillation (PIO) is a complex interaction between the human pilot and the aircraft that leads to sustained and sometimes very large amplitude oscillations of the aircraft. It is characterized by a loss of stability margin in the pilot-aircraft closed loop system. These oscillations can occur about any of the aircraft axes of symmetry.

Many flight test accidents and incidents have been attributed to PIO problems. Most recently, both the F-22 and JAS-39 prototypes have crashed as a result of PIO incidents. Commercial aircraft are also not immune to PIO problems (A-320, Boeing 777). The potential occurrence of PIO is amplified by the use of modern control technology including fly-by-wire systems that determine important modification of the airplane response characteristics.

For example in heavy aircrafts, the problems result in a faster roll rate than normally expected. This combined with delays introduced by the fly-by-wire system cause PIOs.

Detailed analytical studies of PIO incidents are based on pilot behavioral models and closed loop analysis procedures designed to understand and rationalize the phenomena and their associations.

The classification of PIO [15] takes into account some possible different behaviors of the closed loop pilot vehicle system during the PIO. Recently a new category (*IV)* has been added to account for another type of interaction in the pilot vehicle system.

PIO Category I – Essentially Linear Pilot-Vehicle System Oscillations: The effective controlled element characteristics are essentially linear, and the pilot behavior is also quasi-linear and time stationary.

PIO Category II – Quasi-Linear Pilot Vehicle System Oscillations with Rate Limiting or Position Limiting: The closed loop pilot vehicle system has a nonlinear behavior, mainly characterized by the saturation of position or rate limited elements.

PIO Category III – Essentially Nonlinear Pilot Vehicle System Oscillations with Transitions: These PIO depends on nonlinear transitions in either the effective controlled element or in the pilot's behavioral dynamics.

PIO Category IV – Refers to coupling effects between pilot inputs and the aircraft structural modes.

In the present paper an analysis method to predict Category II PIO is considered. These oscillations are induced by nonlinearities determined by rate or position saturations of control surface actuators. This kind of nonlinearity is present in any aircraft, because of the physical constrains of elements such as stick deflections, actuator position and rate limiters, limiter in the controller software. Actuator rate limiting occurs when the input rate to the control surface exceeds the hydraulic and/or mechanical capability of the control surface actuator. Rate limiting has been identified with PIO for two main reasons.

First, it introduces additional phase lag, or delay, between commanded control surface position and actuator control surface. The time delay caused by the additional phase lag can drive the pilot to compensate with faster inputs, worsening the situation. This can ultimately lead to a PIO or unstable situation.

The second reason rate limiting has been identified in PIO is the reduction in gain. The pilot sees this as a reduction in control effectiveness, so he may compensate with larger inputs making the problem worse. These effects often mislead the pilot into thinking the aircraft is not responding to his inputs. These two rate limiting concepts are illustrated in figure 1.

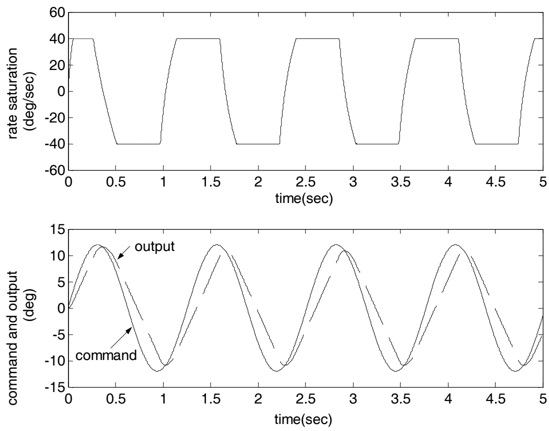

Fig. 1 – Example time history of rate limiting

The resulting PIO has the form of a limit cycle of the nonlinear system, thus limit cycle analysis is a way to analyze the aircraft in order to predict this kind of PIO. Some methods for the analysis of Category II PIO are currently available. *Describing Function* (DF) method, which is the traditional method to analyze the amplitude and frequency of a limit cycle by linearizing the nonlinear elements (see for instance [15]). New methods have been investigated over the last years: *Open Loop Onset Point* (OLOP) method [10] with a modified/enhanced version [9], derived from the describing function method, Robust Stability Analysis Methods [3], [4], [20] also considered in this paper, Time Domain *Neal Smith Criterion* [5], μ -*analysis* based method. Also, solutions to alleviate the effects of PIO have been proposed: phase compensation [17], anti-windup synthesis for PIO avoidance [19], nonlinear pre-filters for PIO prevention [14].

2. ROBUST STABILITY ANALYSIS METHODS

An alternative method to handle saturations is presented in [3], [4] where the nonlinear elements are replaced by linear elements with uncertain gain. Thus an equivalent robustness problem with respect to parametric uncertainty is obtained. This approach has the potential to prevent the computational difficulties present in the describing function analysis and can be used for systems with multi-nonlinear elements. This method is based on Robust Stability Analysis of a linear system, obtained by substituting the nonlinear element with a linear uncertain gain. The uncertain parameter is assumed time-invariant. An approach based on the Edge Theorem is used for the analysis

A classical closed loop scheme for the study of Category II PIO occurrence in the pitch axis is considered.

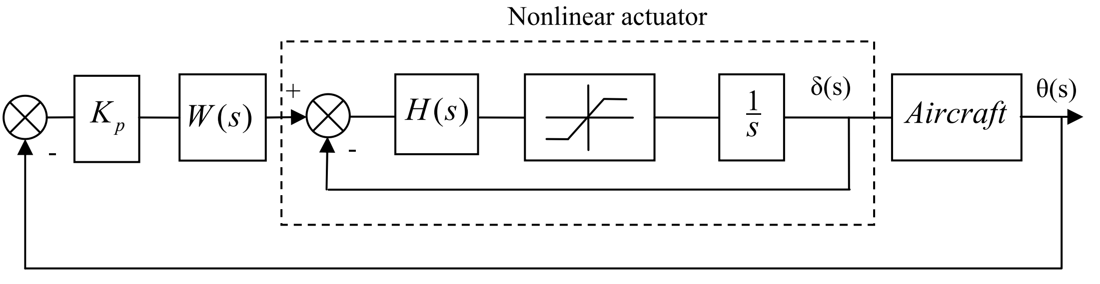

Fig. 2 – Closed loop diagram for Category II PIO analysis

In The blocks in Figure 2 are: K_P the human pilot gain, the normalized filter $W(s)$, the non-linear actuator, whose rate limiting is provided by the saturation (normalized to be symmetric with unitary slope) which precedes the position integrator and the aircraft dynamics transfer function $\theta(s)/\delta(s)$ from the controlled surface position to the variable controlled by the pilot.

As mentioned before, PIO of Category II are mainly determined by saturations. Consider for example a position saturation of an actuator, the behavior of this nonlinearity is equivalent with a linear unknown gain $L \in [L_{\min}, 1]$, as illustrated in Figure 3.

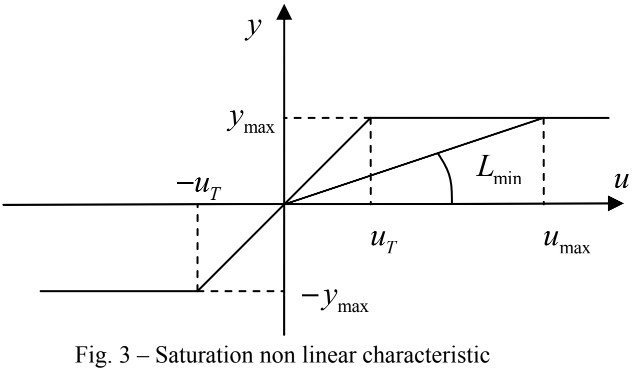

Fig. 3 – Saturation non linear characteristic

Where $y_{\max}$ is the maximum output amplitude; the nonlinearity output is dimensionally an angular rate and will be also denoted by $\dot{\delta}_{\max}$, $u_{\max}$ denotes the maximum input amplitude and $u_T = y_{\max}$ is the linear threshold in input. A large value of the predicted maximal command $u_{\max}$ corresponds to a small value of $L_{\min}$. Therefore for a wide range of the control variable u, a low limit of the gain L is required.

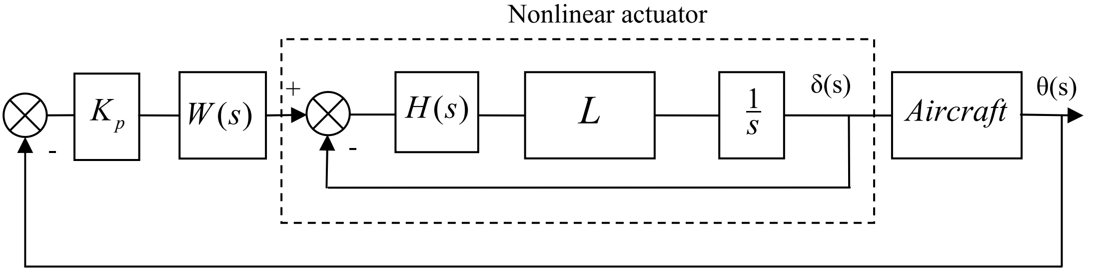

Fig. 4 – Robust stability analysis block diagram

Consider the scheme in Figure 4, where the nonlinear element has been replaced by the linear gain L. When the actuator is not saturated $L=1$ (because the nonlinearity has been normalized to have unitary slope). It is clear that:

$$L_{\min} = \begin{cases} y_{\max} / u_{\max} &, \quad u_{\max} > u_T \\ 1 &, \quad u_{\max} \le u_T \end{cases} \tag{1}$$

therefore $L \in [L_{\min}, 1]$.

Since the pilot gain is variable, two uncertain parameters will be considered in this problem, K_p and L. The PIO detection implies to determine the pairs (L, K_p) separating the stability and instability regions in the parameter plane $L - K_p$.

3. ROBUST STABILITY ANALYSIS METHODS WITH TIME INVARIANT UNCERTAINTIES; AN EDGE THEOREM BASED APPROACH

In this section a method to perform the robust stability analysis of a linear time-invariant system subject to parametric time-invariant uncertainties is presented. This method to determine the maximal domain (L, K_p) for which the resulting system is stable is based on the Edge Theorem.

The notations and definitions used to state this result are presented next. Consider the family of n-degree polynomials:

$$P(s,\delta) = a_0(\delta) + a_1(\delta)s + \cdots + a_n(\delta)s^n \tag{2}$$

where δ is an m-dimensional vector of uncertain parameters.

Assuming that δ_i are independent and $\delta_i \in [\underline{\delta}_i, \overline{\delta}_i]$ where $\underline{\delta}_i, \overline{\delta}_i, i = 1, \ldots, m$ are given, it follows that δ_i lies in an m-dimensional box D.

$$D = \left\{ \delta \in R^m / \delta \in \left[\underline{\delta}_1, \overline{\delta}_1 \right] .... \left[\underline{\delta}_m, \overline{\delta}_m \right] \right\} \tag{3}$$

If the polynomial coefficients $a_k, k = 0, \ldots, n$ are affine functions of $\delta_i, i = 1, \ldots, m$ then:

$$P(D)=\left\{p(s)=\sum_{i=1}^{2^m}\lambda_i P_i(s)\ ,\ \lambda_i\geq 0,\ \sum_{i=1}^{2^m}\lambda_i=1\right\} \tag{4}$$

is the *polytope of polynomials*. The *vertices of the polytope* $p_i(s)\ , i=1...2^m$ are obtained by replacing in (1) the parameters $\delta_i, i=1,\ldots,m$ with their extreme values, in all 2^m possible modes. By $E_{ij}(s,\lambda), i,j=1,\ldots,2^m, i\neq j$ are denoted the *edges of the polytope P(D)*, defined as:

$$E_{ij}(s,\lambda)=\left\{p(s)=\lambda p_i(s)+(1-\lambda)p_j(s)\ ,\ \lambda\in[0,1]\right\} \tag{5}$$

Definition 1. If Δ is a domain in the complex plane, the family of polynomials are called Δ-stable if their roots lie withinΔ.

Theorem 1. (Edge Theorem [7]). The polynomial family (1) with affine coefficient functions $a_i(\delta)$ is Δ-stable if and only if the edges of the polytope $P(D)$ are Δ-stable. Where Δ is a simply connected domain in the complex plane.

4. PILOT INDUCED OSCILLATION ANALYSIS OF X-15 WITH TIME DELAY AND MULTIPLE NONLINEARITIES

This section considers the PIO of X-15 occurred during a landing flare with the pitch SAS off. For this flight the control surface rate was limited to 15 deg/sec. Besides the nonlinear element of the rate limited actuator a time delay block is added. Figure 5 shows the block diagram of this system.

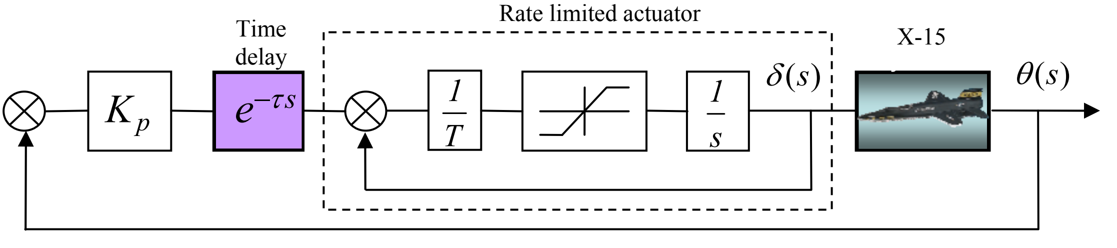

Fig. 5 – Block diagram of the X-15 pilot vehicle system

The numerical values of the elements in the block diagram are:

$$\frac{\theta(s)}{\delta(s)}=\frac{3.476(s+0.0292)(s+0.883)}{(s^2+0.019s+0.01)(s^2+0.841s+5.29)} \tag{6}$$

$$T=0.04\,\text{sec}$$

In Figure 6 is presented the result of Robust Stability Analysis versus a time-invariant uncertain parameter. The stability boundary curve gives the couples (L,K_p) for which the closed loop linear system in Figure 5 is neutrally stable and divides the parameter plane into the stable and unstable regions for different values of the time delay, i.e. the couples of parameters (L,K_p) for which the closed loop system is respectively asymptotically stable (beneath) or unstable (above).

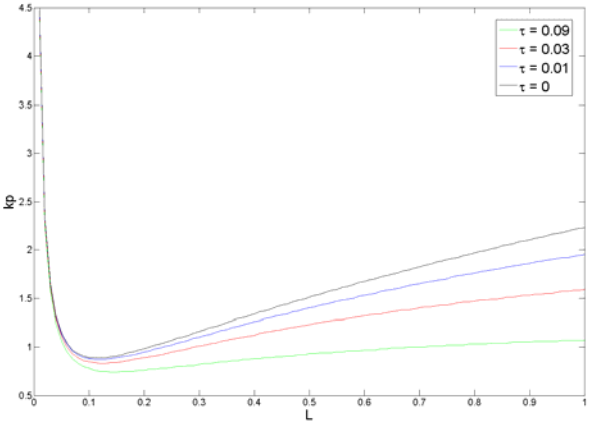

Fig. 6 – Block diagram of the X-15 pilot vehicle system

The results show a reduction of the stability domain due to the presence of the time delay. With an increase of the time delay the stability region becomes smaller.

If we consider besides the nonlinear element of the rate limited actuator a saturation block representing the stick limits the characteristic polynomial (without time delay) of the closed loop system is:

$$\begin{aligned} P(s) = {} & s^5 + (25L_1 + 0.86)s^4 + (21.52L_1 + 5.31)s^3 + \\ & + (132.9L_1 + 86.9K_p L_1 L_2 + 0.1) + \\ & + (2.72L_1 + 79.27K_p L_1 L_2 + 0.05) + \\ & + 1.32L_1 + 2.24K_p L_1 L_2. \end{aligned} \tag{7}$$

In figure 7 the stability boundary is plotted for fixed values of L_2 (corresponding to the stick limits saturation). From figure 7 we can establish that a tight stick limit has a stabilizing effect on the pilot vehicle system.

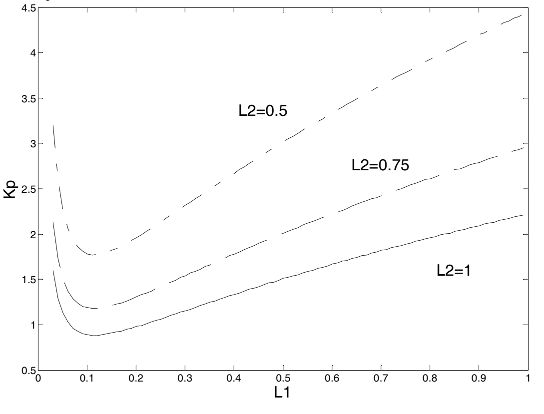

Fig. 7 – Stability boundary with L_2 constant

The time simulation results at four test points are presented in figure 8. In figure 8(d) a PIO occurrence can be observed.

Case (a): $K_p = 2, L_2 = 0.75, L_1 = 0.7;$

Case (b): $K_p = 3, L_2 = 0.75, L_1 = 0.7;$

Case (c): $K_p = 2, L_2 = 0.5, L_1 = 0.4$;

Case (d): $K_p = 1.81, L_2 = 0.75, L_1 = 0.41$.

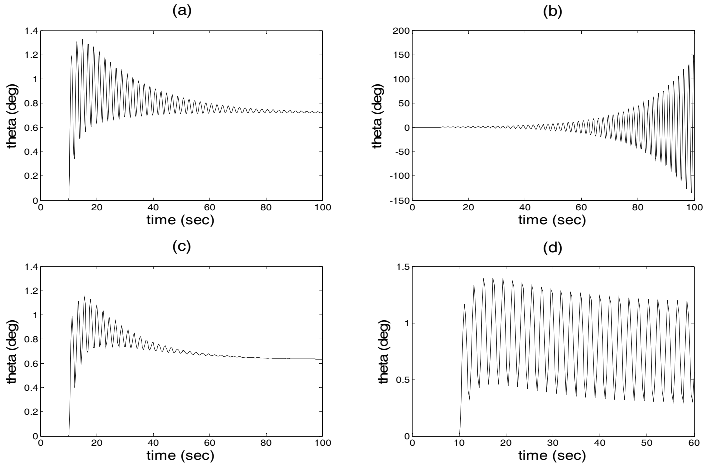

Fig. 8 – Time response of the pitch angle

5. CONCLUSIONS

The Edge Theorem based method is easy to implement in a software procedure. The Robust Stability Analysis is a good tool and can be used in the Category II PIO prediction in alternative to DF analysis.

A further investigation of the following issues is required:

- pilot model to be used for the analysis
- the time varying character of the uncertainties

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