Longitudinal automatic control system for a light weight aircraft

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Abstract: *This paper presents the design of an automatic control system for longitudinal axis of a light weight aircraft. To achieve this goal it is important to start from the mathematical model in longitudinal plane and then to determine the steady-state parameters for a given velocity and altitude. Using MATLAB Software the mathematical model in longitudinal plane was linearized and the system transfer functions were obtained. To determine the automatic control design we analyzed the stability of the linearized model for each input. After the stability problem was solved, using MATLAB-Simulink Software we designed the control system architecture and we considered that the objective for a stable flight was to continuously adjust the pitch angle θ through control of elevator and velocity through control of the throttle. Finally, we analyzed the performance of the designed longitudinal control system and the results highlighted in graphs outline that the purpose for which it was designed was fulfilled.*

Key Words: *control, system, MATLAB, Simulink, longitudinal, aircraft*

1. INTRODUCTION

Light weight aircrafts flying around the world and the number or licensed pilots on this category of aircrafts is increasing from year to year. Flight safety of these aircrafts depends on skills and experience of the pilots but it is more important to have a flight stabilization system (autopilot) on board.

Autopilots do not replace a human operators, but assist them in controlling the vehicle, allowing them to focus on broader aspects of operation, such as monitoring the trajectory, weather and systems [1].

This work is focused on the modelling of longitudinal automatic control system which is able to ensure rapid damping of oscillations due to the effect of the disturbance or the elevator command and to maintaining constant speed and steering angle.

To create a link with the real world of lightweight aircrafts we chose an aircraft with the following characteristics: weight is equal to 1000 kg, moment of inertia about a lateral axis is equal to 10 000 kg.m^2, the wing reference surface is equal to 20 m^2 and the wing reference length is equal to 2 m.

Without wind tunnel testing of the scaled model or flight tests we use analytical prediction to determine aerodynamic coefficients and that was quite complicated.

2. MATHEMATICAL MODEL OF THE AIRCRAFT IN LONGITUDINAL PLANE

The evolution of a lightweight aircraft in longitudinal plane, without roll and yaw movements can be described as a system of differential equations of the following form:

$$\begin{cases} \dot{v} = \dfrac{T\cos\alpha}{m} - \dfrac{\rho v^2 S C_x}{2m} - g\sin\theta \\ \dot{\theta} = \dfrac{T\sin\alpha}{m} + \dfrac{\rho v S C_z}{2m} - \dfrac{g\cos\theta}{v} \\ \dot{q} = -\dfrac{\rho v^2 b S C_{m_\alpha}\alpha}{2J_y} - \dfrac{\rho v b^2 S C_{m_q} q}{2J_y} + \dfrac{\rho v^2 b S C_{m_\delta}\delta_p}{2J_y} \\ \dot{h} - v\sin\theta \\ \dot{\phi} = q \\ \theta = \phi - \alpha \end{cases} \tag{1}$$

In calculating the aerodynamic forces and moments we used aerodynamic dimensionless coefficients C_x, C_z, C_{m_α}, C_{m_q}, C_{m_δ} which were estimated by analytical prediction.

These aerodynamic coefficients are referred to as stability derivatives because their values determine the static and dynamic stability of the aircraft [2].

The values of aerodynamic coefficients are displayed in the table below:

Table 1. Aerodynamic coefficients

C_x	C_z	C_{m_α}	C_{m_q}	C_{m_δ}
0.05	2.5	5	5	1.5

It is observed that the system of differential equations of motion (1) is strongly nonlinear, being impossibly to be integrated analytically.

The four first order differential equations describe the evolution of four state variables, which are grouped in the state vector, *X*:

$$X = (V\ \ \theta\ \ \phi\ \ q)^T \tag{2}$$

As values of interest that characterize the evolution of the aircraft, we will consider all state parameters, plus angle of incidence α, which are grouped into the system output vector, Y :

$$Y = (V\ \ \theta\ \ \varphi\ \ \alpha\ \ \omega)^T \tag{3}$$

As inputs in the system we have the elevator and throttle commands which add wind as external perturbation.

All these inputs are grouped into the system input vector, *U*:

$$U = \left(\delta_T \;\; \delta_c \;\; w\right)^T \tag{4}$$

With previous specifications, the aircraft can be represented as shown in Figure 1.

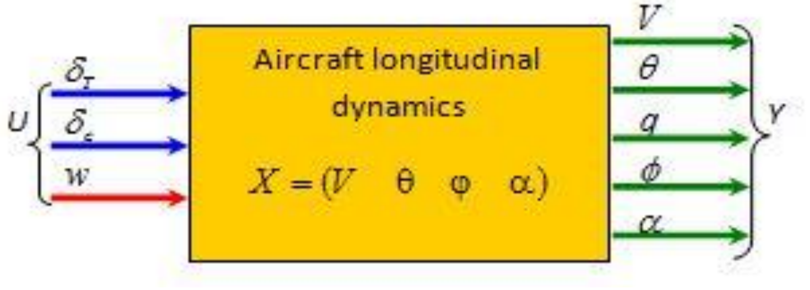

Fig. 1 System State-Space Representation

3. MATHEMATICAL MODEL LINIARIZATION

Before linearizing the mathematical model it is important to determine the steady-state conditions. Therefore, the steady-state conditions of a dynamic system are characterized by zero values of derivatives of the differential equations of motion as follows:

$$v_0 = 110 m/s, \theta_0 = 0, h_0 = 1500 m \tag{5}$$

Using differential equations system (1) we achieved to the following flight parameter values in steady-states conditions:

$$T_0 = K_T \delta_{T0} = \rho(1500)\frac{V_0^2 S}{2} C_x = 1.0600 \cdot \frac{110^2 \cdot 20}{2} \cdot 0.05 = 6413[N] \tag{6}$$

$$\alpha_0 = \frac{m g}{T_0 + \rho \frac{V^2}{2} S C_z^\alpha} = \frac{1000 \cdot 9.81}{6416 + 1.0600 \cdot \frac{110^2 \cdot 20}{2} \cdot 2.5} = 0.0299[rad] \tag{7}$$

$$\begin{cases} \phi_0 = \alpha_0 + \theta_0 = \alpha_0 = 0.0299[rad] \\ \omega_0 = 0 \end{cases} \tag{8}$$

After establishing the steady-state conditions we moved to the linearization of the mathematical model.

The parameters that define the evolution of aircraft V, θ, φ, q can be expressed by declinations ΔV, $\Delta\theta$, $\Delta\varphi$, q, compared to the values corresponding to the steady state conditions V_0, θ_0, φ_0, q_0=0:

$$\begin{pmatrix} V \\ \theta \\ \phi \\ q \end{pmatrix} = X = X_0 + \Delta X = \begin{pmatrix} V_0 \\ \theta_0 \\ \phi_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \Delta V \\ \Delta\theta \\ \Delta\phi \\ q \end{pmatrix} \tag{9}$$

Compared to the values of steady-state conditions, input vector will be expressed similarly:

$$\begin{pmatrix}\delta_T\\ \delta_c\\ w\end{pmatrix}=U=U_0+\Delta U=\begin{pmatrix}\delta_{T0}\\ \delta_{c0}\\ 0\end{pmatrix}+\begin{pmatrix}\Delta\delta_T\\ \Delta\delta_c\\ w\end{pmatrix} \tag{10}$$

The angle of incidence may be defined as declination from the steady-state value:

$$\alpha=\alpha_0+\Delta\alpha \tag{11}$$

Subjecting the differential equation system (1) to steady-state conditions and using the declinations ΔV, $\Delta\theta$, $\Delta\varphi$, q, then proccesing the result and using MATLAB Software for calculating the three matrices of the liniarized system, we get the following:

$$A=\begin{pmatrix}-0.1166[s^{-1}] & -9.6183[s^{-2}] & -0.1917[s^{-2}] & 0\\ 0.0016[m^{-1}] & -2.9733[s^{-1}] & 2.9733[s^{-1}] & 0\\ 0 & 0 & 0 & 1\\ 0 & 160.3250[s^{-2}] & -160.3250[s^{-2}] & -2.9150[s^{-1}]\end{pmatrix} \tag{12}$$

$$B=\begin{pmatrix}19.9911[m\cdot s^{-1}] & 0 & 0\\ 0.0054[s^{-1}] & 0 & 0.0265[s^{-1}]\\ 0 & 0 & 0\\ 0 & 48.0975[s^{-2}] & 1.4575[m^{-1}\cdot s^{-1}]\end{pmatrix} \tag{13}$$

$$C=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \tag{14}$$

Based on state-space model, the input-output model of the aircraft, which is defined by transfer matrix, $H_F(s)$ can be obtained:

$$H_F(s)=C^T\cdot\left(s\,I_4-A\right)^{-1}\cdot B=\begin{bmatrix}H_{TV}(s) & H_{\rho V}(s) & H_{wV}(s)\\ H_{T\theta}(s) & H_{p\theta}(s) & H_{w\theta}(s)\\ H_{T\phi}(s) & H_{p\phi}(s) & H_{w\phi}(s)\\ H_{T\alpha}(s) & H_{p\alpha}(s) & H_{w\alpha}(s)\\ H_{Tq}(s) & H_{pq}(s) & H_{wq}(s)\end{bmatrix} \tag{15}$$

In the above relation, the term $H_{xy}(s)$ represents the transfer function from input *x* to output *y*. The transfer matrix contains 15 similar transfer functions from each of the three inputs to each of the three outputs. The aircraft can be represented input-output on longitudinal plane as shown in figure 2, taking into account the relations for the transfer functions of the angular velocity and the angle of incidence.

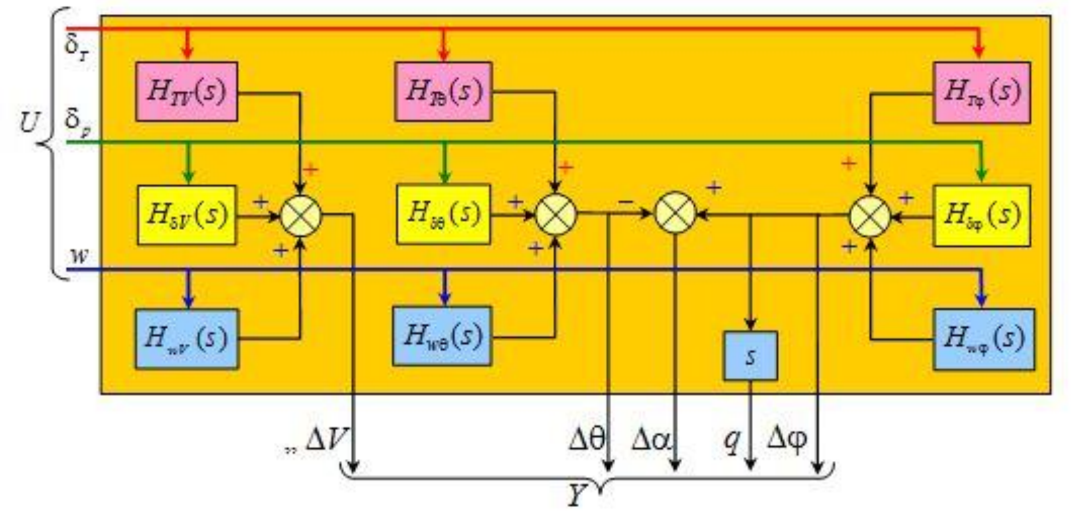

Fig. 2 Simplified Input-Output Representation [3]

4. LONGITUDINAL AUTOMATIC CONTROL SYSTEM DESIGN

Transfer functions previously obtained show that the mathematical model of the aircraft is near the limit of stability or may even be unstable.

Stabilization can be made safe by the negative reaction scheme designed to ensure a stable maintenance of the tangent of trajectory angle, to reduce state oscillations of the system, and to ensure disturbance rejection and maintaining the correct flight path to influence vertical currents [5].

In order to design the control system architecture we considered that the objective for a stable flight is to continuously adjust the pitch angle θ through control of elevator and velocity through control of the throttle.

In the structure below, the controller is represented by the transfer function $H_R(s)$:

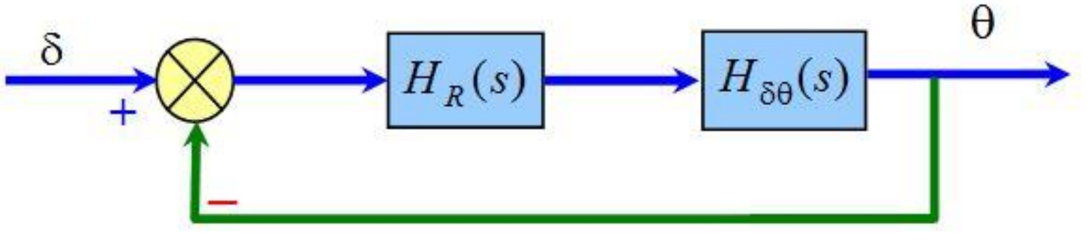

Fig. 3 Simplified Controller Architecture

To ensure the accuracy of the ramp input signal and rejection of disturbances, the controller will have a simple pole in origin.

Consequently, we chose a PI controller, for which the transfer function is of the form [5]:

$$H_R(s) = K_R + \frac{1}{sT_i} = \frac{sK_R + 1/T_i}{s} \tag{16}$$

Controller coefficients, amplification constant K_R and time constant T_i of the integrator were obtained by Ziegler-Nichols method as through relationships:

$$\begin{cases} K_R = 0.45 \cdot K_0 \\ T_i = 0.8 \cdot T_0 \end{cases} \tag{17}$$

$\Rightarrow$

$$\begin{cases} K_R = 0.45 \cdot 6.91 = 3.11 \\ T_i = 0.8 \cdot 0.483 = 0.3864 s \end{cases} \tag{18}$$

because

$$\begin{cases} K_0 = 10^{M_A/20} = 6.91 \\ T_i = 0.483 s \end{cases} \tag{19}$$

The coefficient K_0 and time constant T_0, were obtained considering that the previous controller architecture used a purely proportional regulator.

Actually, K_0 means the maximum constant gain controller to which the system reaches the stability limit and the value of which is equal to the reserve amplitude.

T_0 represents the oscillations time period when the system goes to the stability limit.

The transfer function of the controller was determined using these values:

$$H_{R\theta}(s) = \frac{3.11 \cdot s + 2.63}{s} \tag{20}$$

Below is given the step response of the system to this channel for the controller design, compared to the direct response. [4]

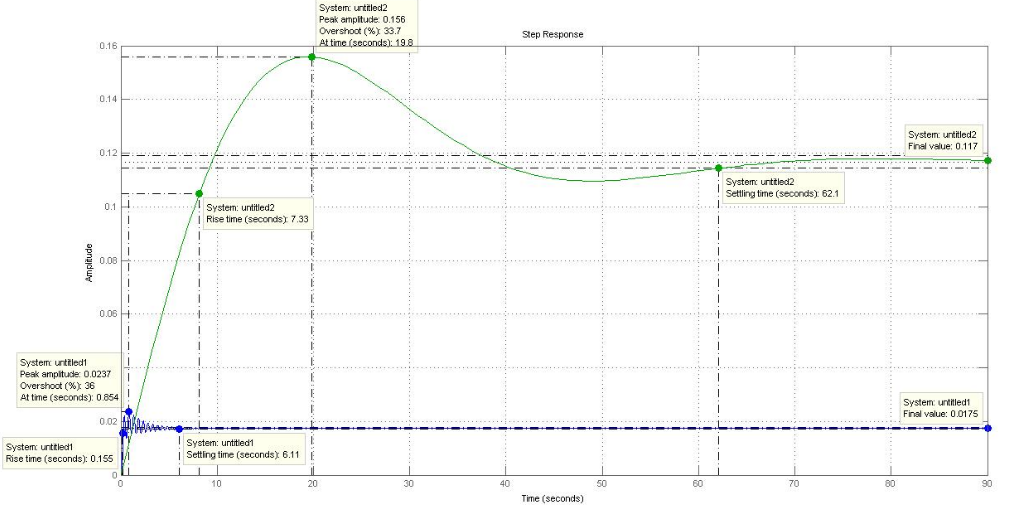

Fig. 4 Step Response of the System to Pitch Channel

The characteristics of the step response can be observed much better. The system responds acurately, in very short period of time, with low overshoot. Fugoid oscillating mode phenomena are eliminated.

5. PERFORMANCE ANALYSIS OF LONGITUDINAL CONTROLLER

The control architecture of the complete system will be:

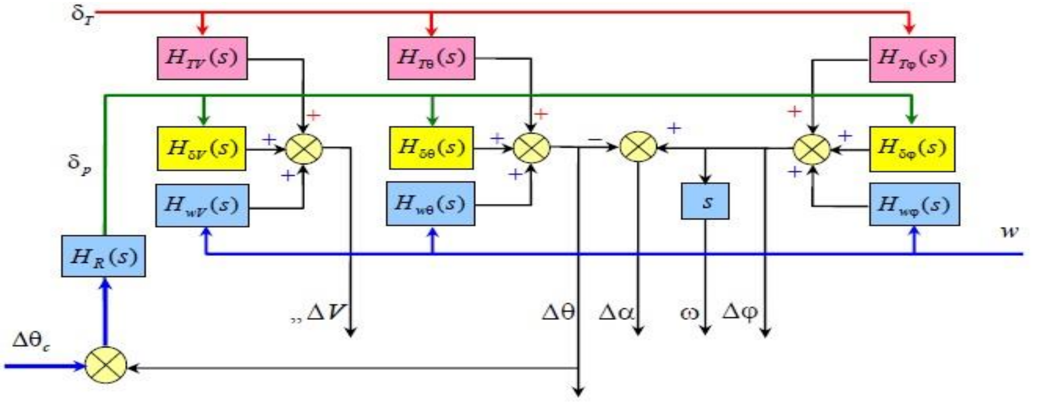

Fig. 5 Control Architecture of the Complete System [3]

To obtain a control architecture model of the complete system in the MATLAB software, in order to implement a feedback loop, then the feedback loop will has to process the state vector using a transfer matrix that ensures the selection of the components.

The formal structure of the complete system configuration for the automatic control will be the following:

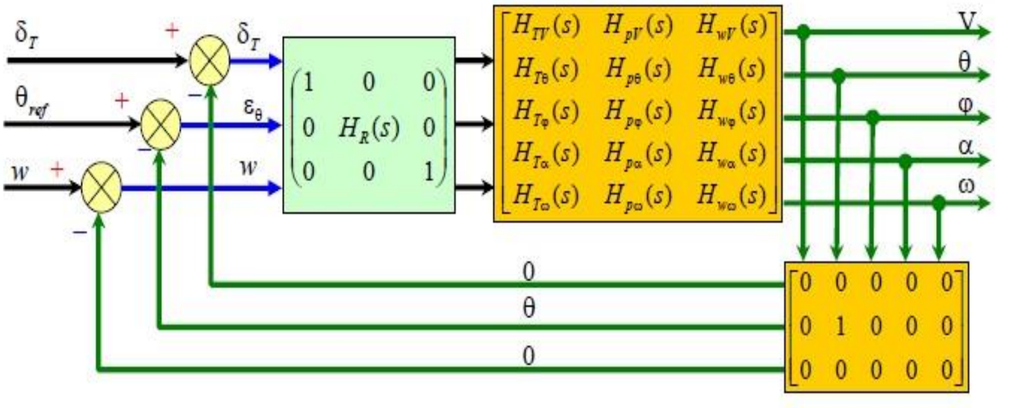

Fig. 6 Formal Structure of Complete System Configuration

Since the MATLAB Simulink module [6] allows the representation systems of nonlinear differential equations and simulates their evolution, the parameters answer obtained after the linearization of the mathematical model is more accurate.

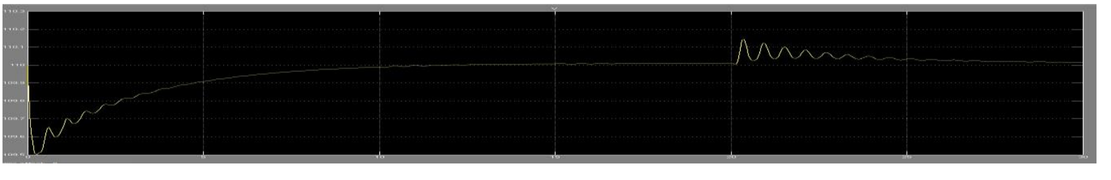

Fig. 7 Velocity Variation

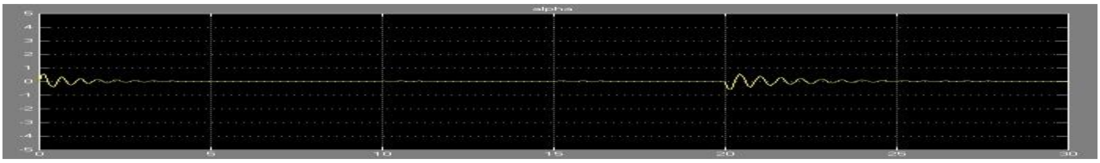

Fig. 8 Incidence Angle Variation

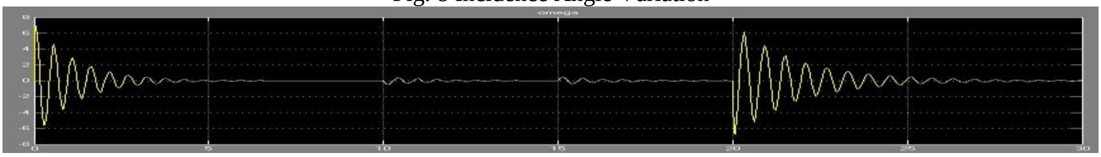

Fig. 9 Angular Velocity Variation

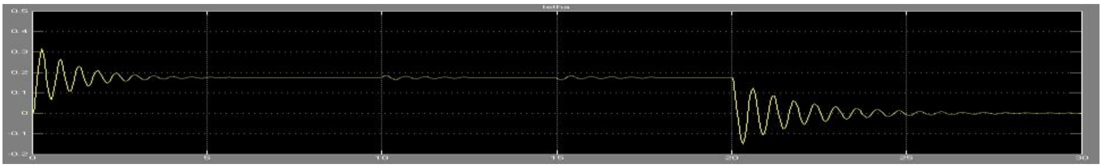

Fig. 10 Pitch Angle Variation

It can be noticed that the control system is able to ensure rapid damping of oscillations due to the effect of the disturbance or the elevator command and to maintain constant speed and steering angle.

6. CONCLUSIONS

We designed a longitudinal automatic control system that has good performance for external disturbances and it is able to maintain a good stability on the pitch axis. Also, we analyzed the performance of the designed longitudinal control system and the results highlighted in graphs outline that the purpose for which it was designed was fulfilled.
As a future work we intend to achieve a similar controller for the lateral channel and to combine it with the already made longitudinal channel automatic system in order to design a controller enabling a full control of any light weight aircraft.

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