Transonic Flow Field Analysis of a Minimum Nozzle Length Rocket Engine

Omar ABADA*,[1], Hakim KBAB[1], Sidali HAIF[1]

*Corresponding author
[1]Aeronautical Sciences Laboratory, Institute of Aeronautics and Space Studies, University of Blida 1, BP 270 Blida 09000, Algeria,
abadaomar@ymail.com*

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Abstract: *The aim of this paper is to develop a profile of axisymmetric minimum length nozzle giving a uniform and parallel flow at the exit section. The study is done at high temperature, lower than the dissociation threshold of the molecules. The design is made by the method of characteristics (MOC). The variation of the specific heats with the temperature is considered. The numerical results have been validated with CFD simulation Ansys-Fluent software. The second part of this study is to calculate and analyze the transonic flow field of this supersonic nozzle. The computation of the flow field characteristics at the throat is thus essential to the nozzle developed thrust value and therefore to the aircraft or rocket it propels. An investigation was conducted to analyze the effects of parameters on the position of the sonic line. These parameters include stagnation temperature T0, radius of the nozzle, types of gases, and exit Mach number ME.*

Key Words: *Transonic flow field, Line sonic, Minimum length nozzle, High temperature, Calorically imperfect gas, method of characteristics, numerical simulation*

1. INTRODUCTION

The nozzle is a component utilized to accelerate a supersonic flow up to an exit Mach number (Ms), generating thrust as a result of this acceleration.

For this purpose, a convergent-divergent shape is necessary. Regarding the convergent section, there isn't a universally optimal contour, and guidelines are typically based on experience and subsonic flow theory.

In our case, we simply assume the existence of an appropriate contour for the convergent section, resulting in a sonic flow at the nozzle throat. In this study, our focus is specifically on nozzles that produce a uniform and parallel flow at the exit section, referred to as Minimum Length Nozzles [1, 2].

The objective of this literature review is to explore various studies describing transonic flow phenomena occurring at the throat of supersonic nozzles. Initial attempts to address such flow challenges were undertaken by Mayer T. [3], Taylor G.I. [4] for planar flows, and Hooker S.G for axisymmetric flows [5].

These efforts utilized inverse power series of coordinates (x and y). However, the obtained results were deemed unsatisfactory, as terms beyond the fourth order of y were neglected, despite experimental studies demonstrating the significance of these terms near the wall.

Nonetheless, these methods laid the groundwork for subsequent investigations, particularly Taylor's method, which was employed in nozzles with very small throat radii.

Oswatitsch-Rothstein developed a method focused on studying fluid behavior in the throat region for nozzles with a high R parameter (ratio of upstream curvature radius to throat radius) [6]. Sauer R. proposed another method based on small perturbation theory for transonic flow analysis [7], providing a solution in terms of inverse powers of the R parameter at the first order while neglecting terms beyond the second order.

Further works, such as those by Hall I.M. [8] and Kliegel J.R. & Levine J.N. [9], continued to explore different approaches to solving the equations of transonic flows, with encouraging but often approximation-dependent results.

Experimental studies were also conducted by Back L.H., Massier PF. & Gieren H. I. [10], revealing that the flow at the throat region is transonic. Numerical methods, such as the finite element method, were introduced by Kenneth ET in 1982 to address transonic flow problems around wing profiles [11].

Following this literature review, it is noteworthy that the majority of works in nozzle design rely on using the ideal gas model with constant specific heat Cp.

However, this model does not account for the real behavior of the gas at elevated temperatures exceeding 1000 K [12, 13].

A new model, termed "High-Temperature Perfect Gas (HT)" is proposed to consider the variation of Cp and γ with temperature [14-16].

The study also investigates the influence of triatomic, diatomic, and monoatomic gases on transonic flows at the throat of axisymmetric MLN supersonic nozzles using Computational Fluid Dynamics (CFD) simulations conducted with ANSYS Fluent [16-19]. This research contributes to the ongoing efforts to advance our understanding of transonic flows in supersonic nozzle throats, with potential applications in aeronautics and aerospace engineering.

2. DESIGN METHODOLOGY OF AXISYMMETRIC SUPERSONIC NOZZLE

Fig. 1 shows the various regions of the flow that can have axisymmetric MLN to have uniform and parallel flow to the exit section.

The region ABE is designated as the transition region. It is a complex area, and the region BSE is identified as the uniform region. However, the Mach number is constant at all points in this region. The wall of the nozzle is a priori unknown. It is determined numerically for the desired condition. The search of the wall and the calculation of the internal flow is done by the Method Of Characteristics (MOC) in the case assumptions at HT [14-16]. The High Temperature model HT precisely determines the results in comparison to the Perfect Gas (PG) model, particularly for applications involving high values of M_E and T_0.

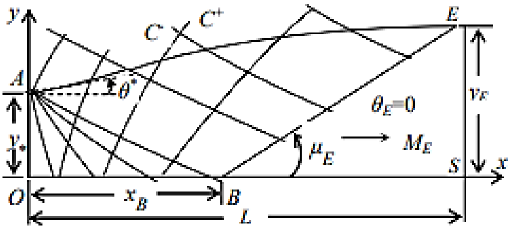


Fig. 1 – Presentation of the axisymmetric Minimum length Nozzle MLN flow field

The calculation is based on the use of the MOC at high temperature model [16]. The compatibilities and characteristics equations, respectively valid on upward and downward characteristic, are represented by:

Following ξ (1–3):

$$-\frac{Cp(T)}{2H(T)}\sqrt{M^2(T)-1}dT+d\theta=\frac{sin\theta sin\mu}{ycos(\theta-\mu)}dx \tag{1}$$

Following η (2–3):

$$-\frac{Cp(T)}{2H(T)}\sqrt{M^2(T)-1}dT+d\theta=\frac{sin\theta sin\mu}{ycos(\theta+\mu)}dx \tag{2}$$

where ν is the Prandtl Meyer function at high temperature (HT) [16].

The use of the method of the characteristics (MOC) let us to introduce a fine mesh in order to approximate each characteristic between two points by straight line segments. The properties (x, y, T, θ, ρ, P) at a flow field point can be determined from two points connected with a point considered by the characteristic lines preceding it.

3. MATHEMATICAL MODEL FOR CALCULATING THE SONIC LINE

The flow through a convergent-divergent nozzle is intricately linked to the geometry near the throat section.

The primary challenge in the numerical or analytical treatment of this transonic region in a fluid field lies in the instability and distinct behavior of the Euler equations governing the flow in this zone.

The evaluation of the effectiveness of information obtained on pressure distribution and Mach number variation critically depends on knowing the efficiency and accuracy of the numerical or analytical method employed.

Over the years, numerous numerical and analytical methods have been developed to study transonic flows in the throat region of a convergent-divergent nozzle. In this study, we will focus on the approach developed by A. Sauer [7], which is based on the theory of small perturbations to analyse the flow in this region.

A. Sauer's approach [7] revolves around the theory of small perturbations applied to compressible flow, both in a two-dimensional context and in axially symmetric flow.

This methodology solves the small perturbation equation by expanding the solution up to the first order in terms of the inverse of R.

Higher-order terms are neglected. While this approach is of substantial interest and has been implemented by designers over several years, its main virtue lies in its applicability to flow around arbitrary profiles.

However, it provides no indication of flow direction or isobar distribution, presenting a major drawback of total divergence for low values of the R parameter.

The schematic figure below illustrates the geometry of a nozzle at the throat level, with a symmetrical contour about the (ox) axis. The sonic line, representing the geometric locus where M=1, takes on a parabolic form.

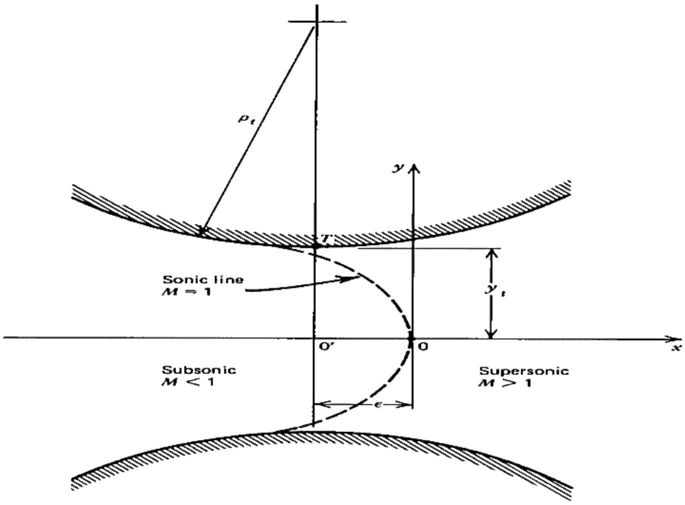


Fig. 2 – Convergent-Divergent Nozzle Configuration

The perturbation equation for a two-dimensional and irrotational planar or axisymmetric flow is expressed by the equation below:

$$(a^2 - \tilde{u}^2)\frac{\partial \tilde{u}}{\partial x} + (a^2 - \tilde{v}^2)\frac{\partial \tilde{v}}{\partial y} - 2\,\tilde{u}\tilde{v}\frac{\partial \tilde{u}}{\partial y} + \delta\, a^2\frac{\tilde{v}}{y} = 0 \tag{3}$$

And the two components of velocity according to Sauer are given by

$$\begin{cases} \underline{u}(x,y) = ax + \dfrac{(\gamma+1)a^2y^2}{2(1+\delta)} + \cdots \\ \underline{v}(x,y) = \dfrac{(\gamma+1)a^2xy}{(1+\delta)} + \dfrac{(\gamma+1)^2a^3y^3}{2(1+\delta)(3+\delta)} + \cdots \end{cases} \tag{4}$$

where

$$a = \sqrt{\frac{1+\delta}{(\gamma+1)\rho_{tu}y_t}} \tag{5}$$

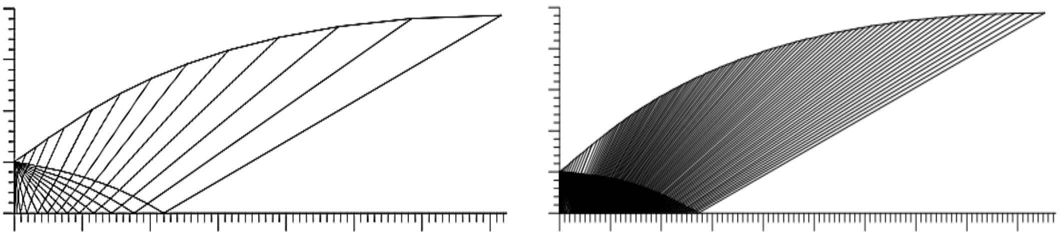
The sonic or critical line has a parabolic shape, and it can be obtained by considering it as a first approximation $\underline{u} = 0$:

$$x_k = -\frac{(\gamma+1)\alpha}{2(1+\delta)}y_k^2 \tag{6}$$

4. RESULTS AND COMMENTS

In Fig. 3, we presented meshes in characteristics in the supersonic nozzle MLN for an exit Mach number $M_E = 2.59$ and $T_0 = 1000$ K with and without condensation effect.

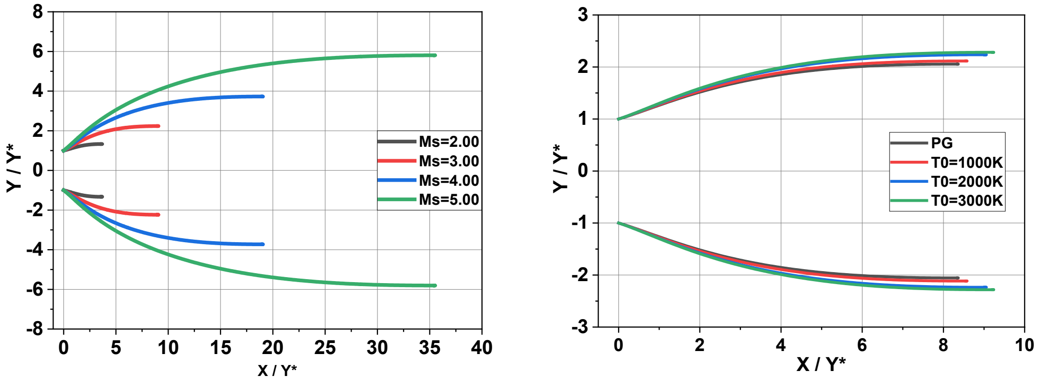
In conclusion, we can say that the good presentation of the contour of the nozzle necessarily depends on the number of points of the mesh, therefore the factor of the condensation of the points on the wall influences the shape obtained and the numerical results of the nozzle.



(a) Coarse mesh with condensation with N=10 (b) Mesh with condensation for N=100

Fig. 3 – Meshes in characteristics of a planar centred expansion nozzle

Fig. 4 shows the shape of the axisymmetric supersonic nozzle rocket engine for the following exit Mach number M_E (1.50; 2.00, 3.00, 4.00 and 5.00) and stagnation temperature T0 (1000K, 2000K and 3000K). The presented figures show that as the exit Mach number M_E and the stagnation temperature T_0 increase, the shape of axisymmetric supersonic nozzle increases.



(a) Exit Mach number M_E (b) stagnation temperature T_0

Fig. 4 – Effect of the shape to the axisymmetric supersonic nozzle MLN

Table 1 – Numerical values for sizing the nozzles giving the exit Mach number M_E

	M_E=2.00	M_E=3.00	M_E=4.00	M_E=5.00
L/y*	3.6497	9.0175	19.0236	35.4889
M *	1.2915	13.0812	1.6632	1.7671
θ*(°)	6.2937	1.5114	17.9718	21.3171
A_E/A^*	1.7711	4.9995	13.8935	33.7028
C_{Mass}	9.0569	34.6950	117.3712	336.1706
C_F	0.1767	0.3478	0.4440	0.4962

Fig. 5 presents the variation of the Mach number through the supersonic nozzle with centered expansion (MLN: Minimum Length Nozzle) where M_E = 3.00 and T_0 = 2000 K for the HT model and which will be compared with the numerical results as follows.

Fig. 6 shows a comparison of the Mach number along the divergent of the nozzle MLN. It shows a rapid increase in the wall Mach number after the throat. After this, a slower progression is observed until the point at which the Mach value has reached the design Mach value at the end of the nozzle (M_E=3.00).

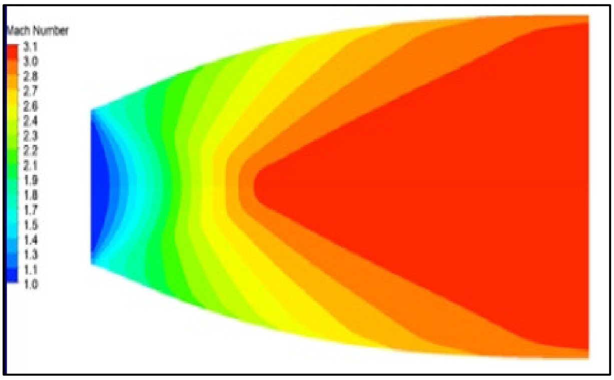


Fig. 5 – Contour of Mach number along the wall

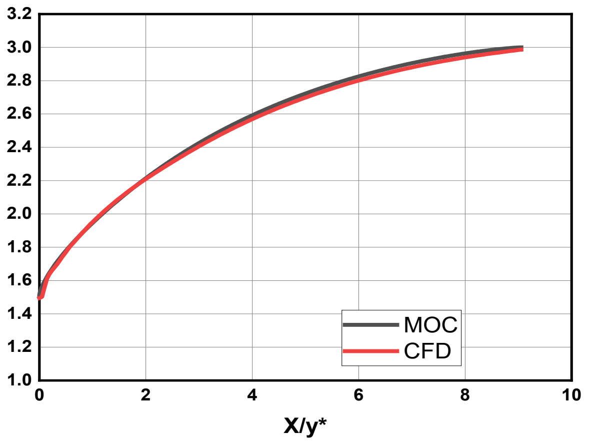


Fig. 6 – Mach number variation on the contour of the nozzle for M_E=3.00 and T_0=2000 K

Fig. 7 presents the pressure ratio variation along the contour of the nozzle. It presents the variation of the pressure ratio P/P_0 which is used to determine the pressure force exerted on the wall of the nozzle.

In addition, Fig. 8 displays a comparison of the pressure ratio of the gas flow at high temperature. The results obtained by simulation (Ansys-Fluent) are similar to the numerical results obtained in Fortran program. A FORTRAN code was developed to design this nozzle using the characteristics method (MOC).

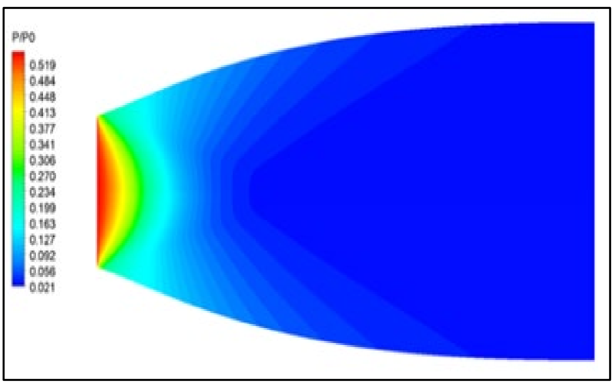


Fig. 7 – Contour of pressure ratio P/P_0 along the wall

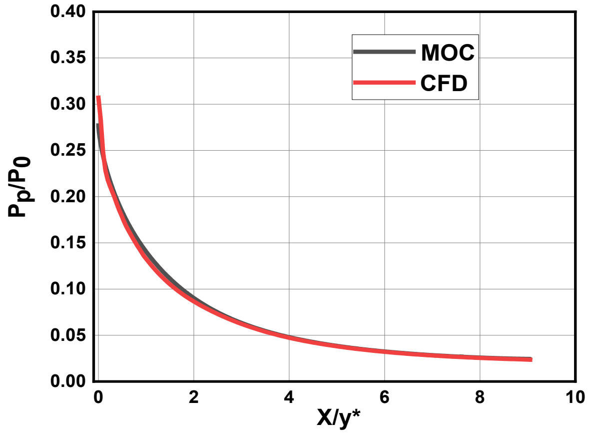


Fig. 8 – Pressure variation on the contour of the nozzle for M_E=3.00 and T_0=2000k

Fig. 9 shows the contours of the density ratio ρ/ρ_0 along the divergent of the nozzle when the exit Mach number is M_E=3.00 and T_0=2000 K. The curves represented by Fig. 10, illustrates a comparison of the density ratio $\boldsymbol{\rho}/\boldsymbol{\rho}_0$ calculated by simulation (Ansys-Fluent) and numerically (Fortran program) according to the axial length of the nozzle.

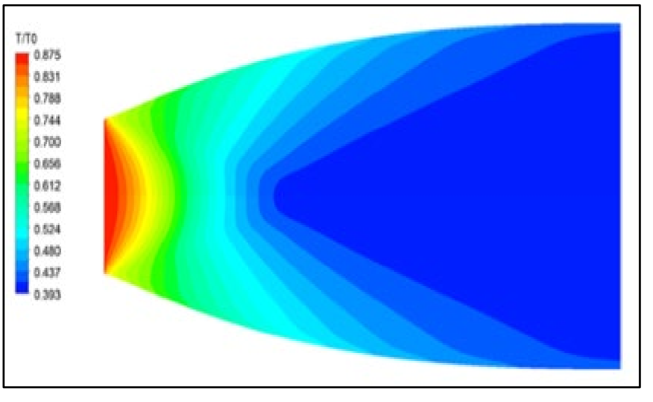


Fig. 9 – Contour of temperature ratio T/T_0 along the wall

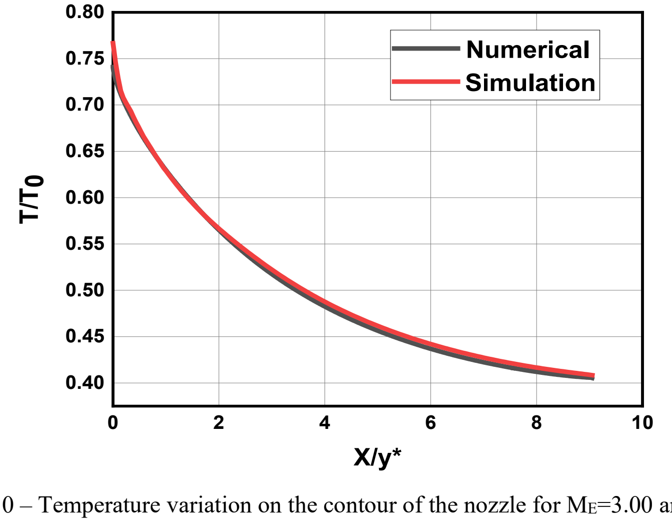


Fig. 10 – Temperature variation on the contour of the nozzle for M_E=3.00 and T_0=2000k

The convergent of a nozzle is the initial section where the flow is compressed before passing into the divergent section. Increasing the radius of the convergent can change the position of the sonic line; therefore, the effect of variation of the radius of the nozzle (convergent) on the sonic line. Fig. 11-13 shows the variation of the sonic line for a PG model and a HT model (1000 K, 2000 K and 3000 K) where M_S =3.00 and R=0.02m, R=0.04m and R=0.06m.

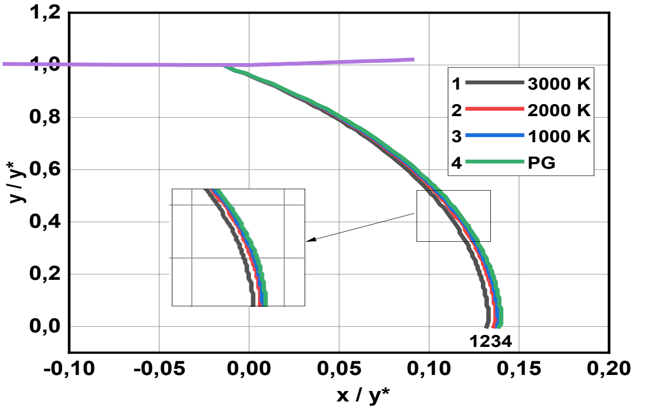


Fig. 11 – Sonic line variation for PG and HT models where M_E=3.00 and R=0.02m

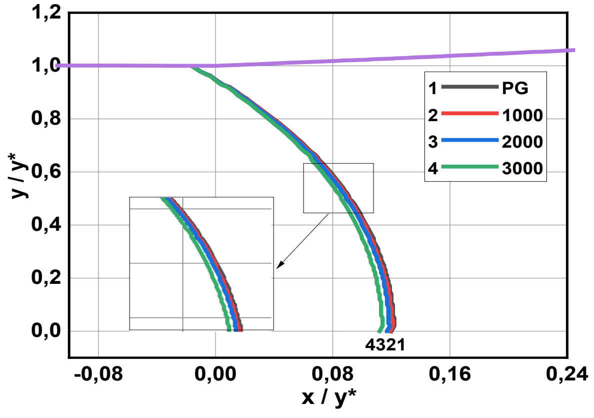


Fig. 12 – Sonic line variation for PG and HT models where M_E=3.00 and R=0.04m

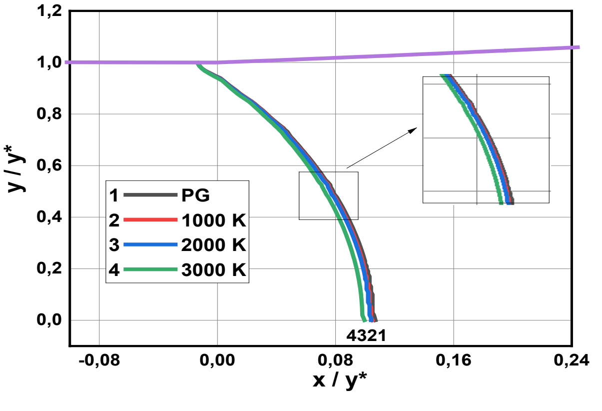


Fig. 13 – Sonic line variation for PG and HT models where M_E=3.00 and R=0.06m

Table 2 shows the variation of the relative error given by the variation of the (convergent) nozzle radius of the PG model compared with the HT model for certain values of T_0.

Table 2 – Effect of variation of the radius of the nozzle (convergent) on the sonic line between the PG model and HT model

Radius of convergent	1000 K	2000 K	3000 K
Error %(R=0.02)	0.7194	2.1580	5.0359
Error %(R=0.04)	0.8330	2.5000	6.6660
Error %(R=0.06)	1.8690	2.8037	6.9008

In a supersonic nozzle, the temperature variation can have a significant effect on the position of the sonic line, which marks the transition between subsonic and supersonic airflow zones.

Fig. 14 illustrates the position of the sonic line along the nozzle at T_0= 3000 K and at an exit Mach number M_E=3.00. This is to say that the sonic line at the HT model is closest to the throat compared to the PG model.

We started with the case where the temperature is T_0=3000 K, then with the case where T=2000 K and finally with T=1000 K.

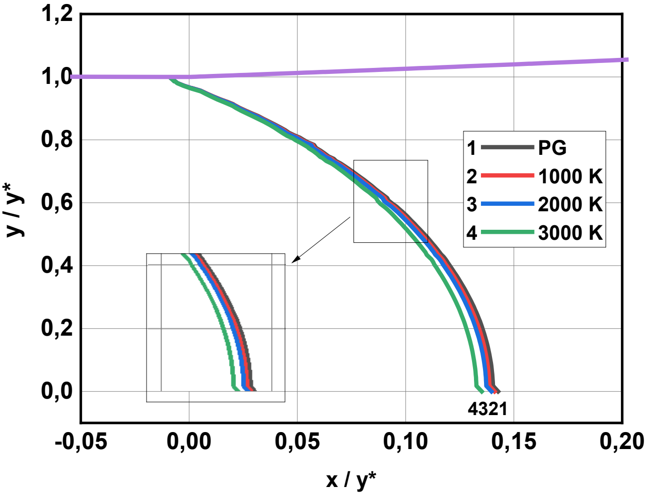


Fig. 14 – Sonic line variation for GP and HT models where M_E=3.00 and T_0=3000 K

The thermochemical and combustion studies of the most used liquid propellants on the satellites and launch vehicles allow to know all gases.

The gas produced from different bipropellant combustions are: triatomic, diatomic and monoatomic types.

In this section, we will study the effects of this gases types on the position of the sonic line. Fig. 15 shows the position of the sonic line along the MLN nozzle for different gases air, H_2O, CO_2.

From the results obtained, the position of the sonic line of CO_2 and H_2O is closer than the air gases. Table 3 shows the variation of the relative error given by the variation of the sonic lines between air and triatomic gases.

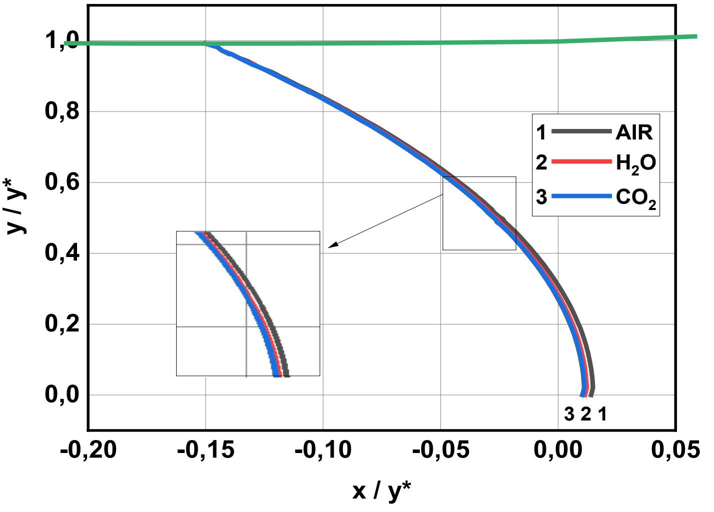


Fig. 15 – Variation of the sonic line for different triatomic gases

Table 3 – Difference on the sonic line between the air and triatomic gases

Type of gas	H_2O	CO_2
Error (%)	19.1323	26.7425

Fig. 16 shows the comparison between different diatomic gases on the position of sonic line. The difference between the values is negligible for the diatomic gases H_2, N_2, OH, CO and N_2 and which are almost identical to those of air, which is also a 99% diatomic gas. Table 4 gives the variation of the sonic lines between air and diatomic gases.

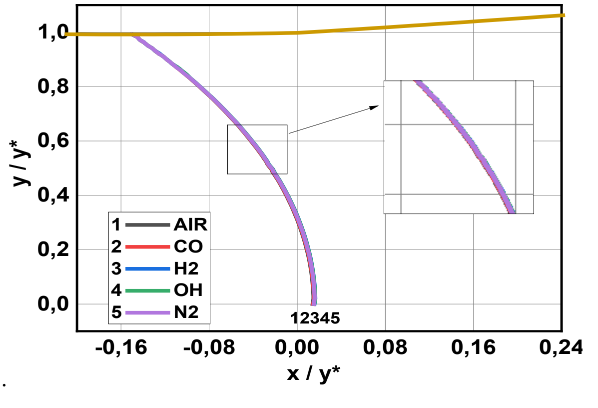


Fig. 16 – Variation of the sonic line for different diatomic gases

Table 4 – Difference on the sonic line between the air and diatomic gases

Type of gas	CO	H_2	OH	N_2
Error (%)	3.168	9.872	9.056	6.516

Fig. 17 illustrates the influence of different monoatomic gases on the position of sonic line for minimum length nozzle. The comparison reveals that the sonic line obtained for a monoatomic gas is positioned downstream from that for air. This indicates that the performance achievable for air is superior to that for a monoatomic gas. Table 5 gives the variation of the sonic lines between air and monoatomic gases.

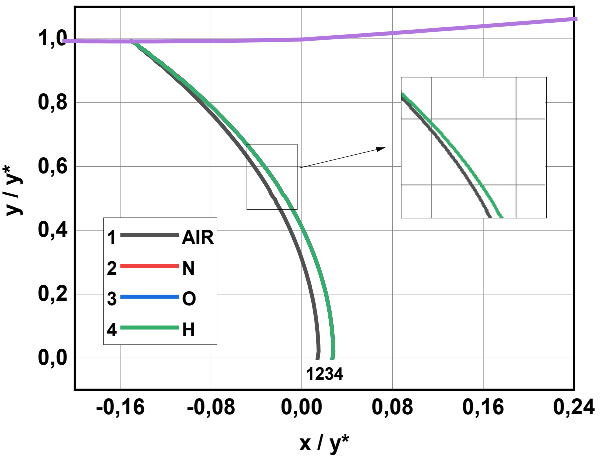


Fig. 17 – Variation of the sonic line for different monoatomic gases

Table 5 – Difference on the sonic line between air and monoatomic gases

Type of gas	N	O	H
Error (%)	48.194	48.156	48.137

Fig. 18 shows the position of the sonic line along the MLN nozzle for different exit Mach numbers M_E=1.5, 2, 3, 4, 5. From the results obtained, the evolution of the sonic line does not show any variation between the different curves of the exit Mach number.

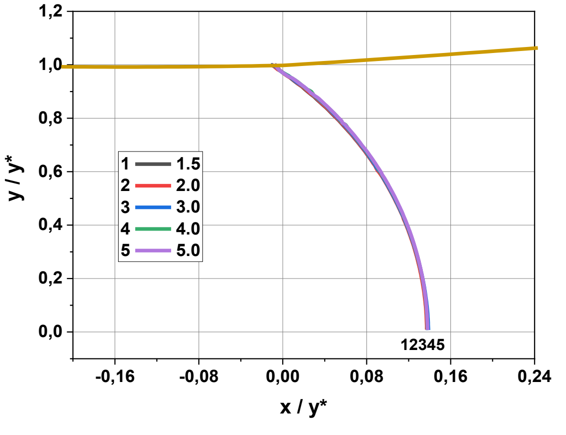


Fig. 18 – Variation of the sonic line for different exit Mach number M_E

5. CONCLUSIONS

In this study, we performed a numerical analysis of the sonic line position of a centered expansion supersonic nozzle (Minimum Length Nozzle). The following findings were obtained.

A FORTRAN code was developed to design this nozzle using the characteristics method (MOC) at high temperature to calculate the internal flow field in the axisymmetric MLN. The CFD study was conducted on this nozzle and compared. The results obtained by simulation (Ansys-Fluent) are similar to the numerical results (M_E, T/T_0, P/P_0, ρ/ρ_0) obtained in Fortran program.

The effect of temperature variation on the shape and appearance of the sonic line has been studied. The result of this study, increasing the radius of the convergent can change the position of the sonic line. An error calculation has been made to clearly illustrate this difference.

A study of the effect of temperature variation has been investigated. From the results obtained, the temperature variation can have a significant effect on the position of the sonic line. The results of this case indicate that the sonic line at the HT model is closest to the throat compared to the PG model. We started with the case where the temperature is T0=3000 K, then the case where T=2000 k and the end where the T=1000 K.

If we consider a gas other than air, for example triatomic gases (CO_2 / H_2O), the difference of the sonic line at the triatomic gases is closest to the throat compared to the air gases.

The diatomic gases (H_2, OH, O_2, NO, N_2 and CO including air) have little effect on variation of the sonic line. On the other hand, if we assume that the gas is monatomic, in this case the sonic lines will be located after that of the air.

The position of the sonic line along the MLN nozzle for different exit Mach numbers M_E=1.5, 2, 3, 4, 5 does not show any variation between the different curves of the exit Mach number.

NOMENCLATURE

The following symbols are used in this paper:

A	=	Section area (m^2)
a	=	Coefficient of the specific heat at constant pressure function
C^-	=	Left-running characteristic
C^+	=	Right-running characteristic
C_F	=	Thrust coefficient
C_{Mass}	=	Coefficient of the mass of the nozzle
C_P	=	Specific heat at constant pressure ($J.Kg^{-1}.K^{-1}$)
H	=	Enthalpy (KJ)
HT	=	Abbreviation for high temperature
L	=	Length of the nozzle (m)
M	=	Mach number
$Mass$	=	Mass through the nozzle normalized by (ρ_M t_M) (m^2)
MLN	=	Minimum Length Nozzle
P	=	Pressure (bar)
PG	=	Abbreviation for Perfect Gas
R	=	Thermodynamic constant of gas
T	=	Temperature (K)
x	=	Abscissa of a section in the nozzle
y	=	Radius of a section in the nozzle
γ	=	Specific heats ratio
ρ	=	Density (Kg/m^3)
θ	=	Flow angle deviation (rad)
μ	=	Mach angle

ν = Prandtl Meyer angle

ε = Tolerance of calculation (desired precision).

ξ = Downward Mach line

η = Right-running Mach line

Indices

0 = Stagnation condition (combustion chamber)

* = Critical condition

S = Supersonic section

E = Exit section

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