Tuning of the Wavelet Filters for the IMU Data Based on the PDC Method and the GPS Solution in a Bi-Dimensional Navigation Application

Ioana Raluca ADOCHIEI*,[1], Teodor Lucian GRIGORIE[1],
Felix Constantin ADOCHIEI[2]

*Corresponding author
[1]Military Technical Academy "Ferdinand I",
Blvd George Cosbuc, no. 39-49, Bucharest, Romania,
ioana.adochiei@mta.ro*, ltgrigorie@yahoo.com
[2]"POLITEHNICA" University of Bucharest,
Splaiul Independentei no. 313, 060042, Bucharest, Romania,
felix.adochiei@upb.ro

DOI: 10.13111/2066-8201.2019.11.3.1

Received: 20 July 2019/ Accepted: 08 August 2019/ Published: September 2019

7th International Workshop on Numerical Modelling in Aerospace Sciences "NMAS 2019"
15-16 May 2019, Bucharest, Romania, (held at INCAS, B-dul Iuliu Maniu 220, sector 6)
Section 4 – System design for small satellites

Abstract: *The degradation of navigation accuracy and integrity of GPS in the presence of radio frequency interference, hostile jamming and high dynamical situations, when the satellite signals may get lost due to signal blockage, led to the development of MEMS-INS/GPS integrated navigation systems for various applications of the positioning and navigation technologies. Unfortunately, the short-term advantages brought by the INS systems are overshadowed by their imprecise operation over the long term, mainly due to inertial sensor errors. A critical component of the inertial sensors errors is the noise. To improve the quality of the inertial sensors data, many denoising techniques have been used. Wavelet method has been proven as a useful tool for signal analysis, and it is widely used in signal processing and denoising applications. The here proposed technique is based on a time-frequency approach previously applied in bio-signals processing. In the proposed mechanism, the inertial sensors signals are processed analysed by using an extended version of the Wavelet transform. The optimal levels of decomposition are established for the wavelet filters, based on the evaluation of a parameter called coupling level (CL). It characterizes the coupling dynamics information between the reference signals, provided by a GPS, and the perturbed signal, which are the outputs of the inertial navigation system (INS). The proposed tuning method is experimentally tested in a bi-dimensional navigation application.*

Key Words: *Inertial navigation, INS/GPS, wavelet denoising, tuning, experimental validation*

1. INTRODUCTION

GPS used in standalone or assisted configuration ([1-3]), is still the broadest positioning system, having applications in navigation, geodesy, mapping, timing, and so on. However, usually, for situations when the signals from the *GPS* are lost, due to issues like the presence of radiofrequency interferences, hostile jamming, high dynamical situations or signal blockage ([4-6]), an inertial navigation system (*INS*) is complementary used. Unfortunately, the inertial sensors on the market, used in applications like automated car navigation, assistive navigation, emergency assistance, fleet management, asset tracking, collision avoidance, environment monitoring, and automotive assistance, have a significant error level ([7-9]). An *INS*, based on miniaturized inertial sensors, is accurate for short periods, depending on the error level of the miniaturized inertial sensors. The problem is that miniaturized inertial sensors have lost their accuracy due to their sensing components miniaturization ([5, 6, 10, 11]).

Therefore, there are two directions of study for the researches in the global positioning field, to achieve low-cost, small-size, and high-precision *INS/GPS* navigator, suitable for assistive purposes in *GPS* challenging environments.

The first one is the development of standalone accurate *INS* structures based on optimized inertial sensors or new architectures and algorithms for the error estimation and compensation of sensors. The second one is the development of new *INS/GPS* data-fusion techniques by incorporating artificial intelligence algorithms, to overcome the sensor's limitations by optimizing the model dependencies, prior knowledge dependencies, and linearization dependencies ([5, 11-15]).

This paper proposes a new technique for optimizing an inertial navigator, considered as a single unit, by applying optimized filtering, using the transformed wavelet to process the signals received from inertial sensors; so it is a study for optimizing the entire navigation system at the algorithmic level (component) of the navigation system. The adaptive filters based on the Wavelet transform, positioned between the detection unit (inertial detection unit) and the navigation algorithm, are tuned using the position information received from a *GPS* (considered as the reference navigator) and an extension of the Partial Directed Coherence (*PDC*) method.

The time-variant *PDC* method evaluates the accuracy of the positioning information provided by our navigator (noisy/ perturbed signals) with respect to the positioning information received from the *GPS* (reference signals) using a parameter called the coupling level parameter. When this parameter values have a specific value, the optimal level of decomposition of the wavelet filter is found. The adaptive algorithm for inertial sensors signals de-noising is implemented and experimentally validated for horizontal bidirectional positioning. The results of this optimization revealed a substantial improvement in horizontal positioning accuracy.

2. THE INERTIAL NAVIGATOR STRUCTURE AND MATLAB/ SIMULINK MODEL

The process of measuring the total acceleration of a vehicle and its integration, considering the information received from gyros, allows to determine the speed, position, and attitude of the vehicle (in terms of angle of yaw, roll, and pitch). Inertial navigation allows to determine the position of a noninertial reference system relative to inertial reference systems, through a double integration of its acceleration under the given conditions. However, the errors in inertial navigation systems are mainly caused by sensors' imperfections.

The most significant errors in position, speed, and attitude detection arise from numerical integration of noise and incorrect bias measurements. A constant power characterizes the inertial sensor noise over the entire 0-100 Hz frequency spectrum which includes the dynamics of the monitored mobile systems; therefore, filtering this type of noise is not recommended in the indicated band. To simulate the inertial sensor inputs and outputs as close as possible, we implemented software the mathematical model of the sensors, considering all the characteristic

parameters provided by the manufacturers. Also, the block diagrams of accelerometers and gyro have been designed in Simulink.
To obtain the position and speed information of a monitored vehicle ([6, 8, 9, 16]) it is necessary to perform the numerical integration of the inertial navigation's general equation, relative to the navigation frame which is considered to be the North-East-Down (*NED* - $Ox_ly_lz_l$) local horizontal frame in our investigation.
Therefore, starting from the vehicle attitude information and the relative angular position of the vehicle frame (*SV*) and *NED* frame a transformation of the vehicle acceleration components between the SV, $Ox_vy_vz_v$ frame and *NED* frame is performed (Figure 1) [17]. For the horizontal plane monitoring (the position and speed evaluation) only the *x* and *y*-axes are considered. However, for solving the problem of the horizontal plane navigation, the linear accelerations measurements, along the *x* and *y*-axes and the angular speed measurements, along the *z*-axis, are collected and processed.
In Figure 1 the following notations can be observed: $\vec{r}$ is the vehicle position vector in *NED* frame, $\vec{v}$ is the relative speed of the vehicle reported to the *NED* frame, $\vec{v}_{xv}, \vec{v}_{yv}$ are the vehicle speed components of $\vec{v}$ in *SV* frame, while $\vec{\omega}_{zv}$ is the vehicle's angular speed $\vec{\omega}$ component along the vehicle frame *z*-axis.

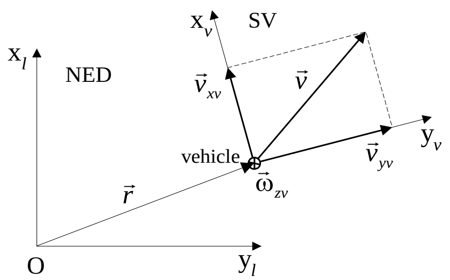

Figure 1. The relative position of the vehicle frame (*SV*) and north-east-down frame (*NED*)

Starting from the specific force vector $\vec{f}$ (the outputs of the accelerometers), Eq. 1 ([6, 8, 9, 16]), which is equal with the kinematic acceleration ($\vec{a}$), when the gravitational field does not influence the accelerometer outputs (the sensitivity axes are located in the horizontal plane), becomes Eq. 2:

$$\vec{f} = \vec{a} = \frac{d\vec{v}}{dt} + \vec{\omega} \times \vec{v}; \tag{1}$$

$$f_{xv} = \frac{dv_{xv}}{dt} + \omega_{zv} v_{yv}, \quad f_{yv} = \frac{dv_{yv}}{dt} - \omega_{zv} v_{xv}, \tag{2}$$

where f_{xv}, f_{yv} are the components of the specific force related to the *x* and *y*-axes of the *SV* frame.
After Eq. 2 integration, the vehicle speed components in *SV* frame are obtained, and further transformed in *NED* frame, by applying the following coordinate changing:

$$\begin{bmatrix} v_{xl} \\ v_{yl} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \cdot \begin{bmatrix} v_{xv} \\ v_{yv} \end{bmatrix}; \tag{3}$$

where the yaw angle ψ value results after the ω_{zv} gyro reading numerical integration as follows:

$$\psi = \psi_0 + \int_{t_{n-1}}^{t_n} \omega_{zv}\, \mathrm{d}t. \tag{4}$$

with ψ_0 denoting the yaw angle initial value. After the vehicle speed components are calculated in the *NED* frame, by using Eq. 5, the vehicle's horizontal positioning is obtained:

$$x_l = x_{l0} + \int_{t_{n-1}}^{t_n} v_{xl}\, \mathrm{d}t, \quad y_l = y_{l0} + \int_{t_{n-1}}^{t_n} v_{yl}\, \mathrm{d}t. \tag{5}$$

The navigator equations are implemented in Matlab/Simulink, and the model from Figure 2 is developed. The model's inputs are the outputs of the inertial sensors, the *SV* accelerations along the *x* and *y*-axes, and the *SV* angular speed along the *z*-axis). The model's outputs are the position and speed of the vehicle relative to *NED* and the yaw angle.
To obtain the driven distances in North and East directions, in terms of latitude and longitude coordinates, the "Flat Earth to LLA" Matlab/Simulink block was used.

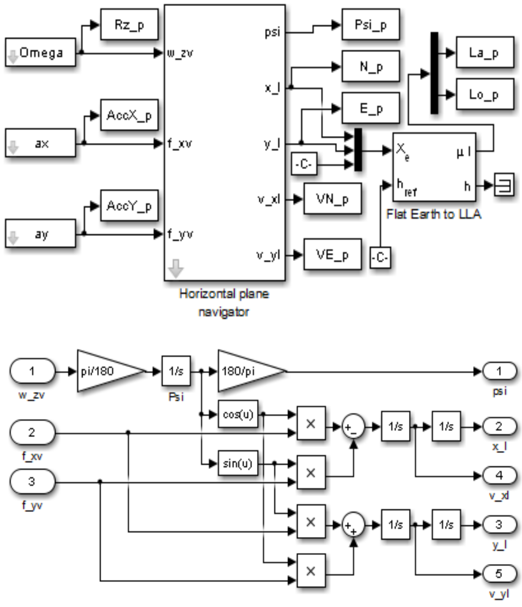

Figure 2. The inertial navigator Matlab/ Simulink model

3. THE PROPOSED WAVELET TUNING METHOD BASIC PRINCIPLE

Denoising is a process to remove noise that is present in the signal of interest. Wavelet method has been proven as a useful tool in signal analysis, and it is widely used in denoising applications ([18-22]). A wavelet filter acts as a filter for mediation or a filter that detects detail when the signal is decomposed by wavelets. Some of the wavelet coefficients, therefore, reflect the details of the data set.
The significance of these details is directly proportional to the amplitude of the waves - if they are low then can be left aside without affecting the basic properties of the dataset. The idea to establish thresholds is to set to zero all coefficients with a value below a certain threshold. For reconstructing the initial data set, these coefficients are used in inverse wavelet transformation (Figure 3 [23]).

Figure 3. Wavelet denoise approach

The wavelet filtering method was already used in several navigation applications. Thus, in [24] was improved the navigation performance of a low-cost *MEMS-INS/GPS* integrated navigation system by applying the thresholding method of the wavelet denoising, Figure 4. Also, the results of comparison for various wavelet thresholding selections, with different level of decomposition for each *GPS* and *INS*, were exposed in [25]. The research of the wavelet method upon the integrated *INS/GPS* navigation systems conducted to different tandems such as wavelet multi-resolution analysis algorithm based ([26]), or wavelet multi-resolution analysis and artificial neural networks ([27]).

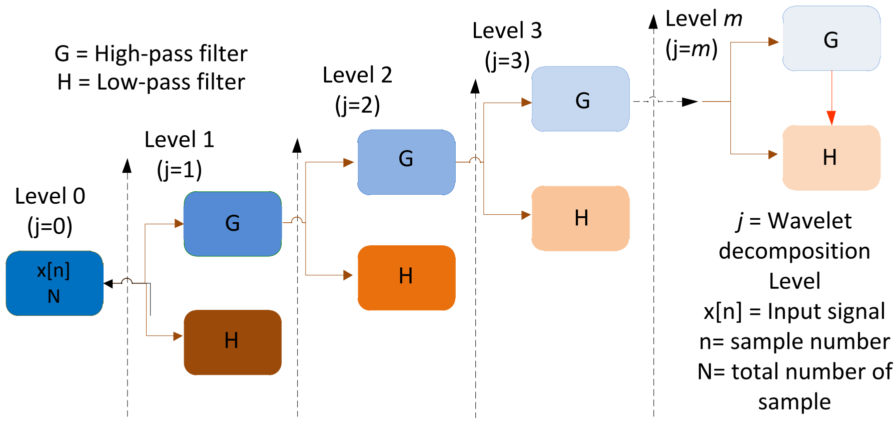

Figure 4. Wavelet's decomposition tree

We introduced a new time-frequency variable approach, an extension of Partial Directed Coherence (*PDC*) method, for assessing the multivariate dynamic systems coupling dynamics information ([28]) by estimating the optimal level of decomposition for the wavelet filter ([29, 30]).

The diagram of the proposed algorithm is presented in Figure 5. Here the disrupted output signals from *INS* toward *PDC* reference signals are illustrated together with the reference signals received from the *GPS*.

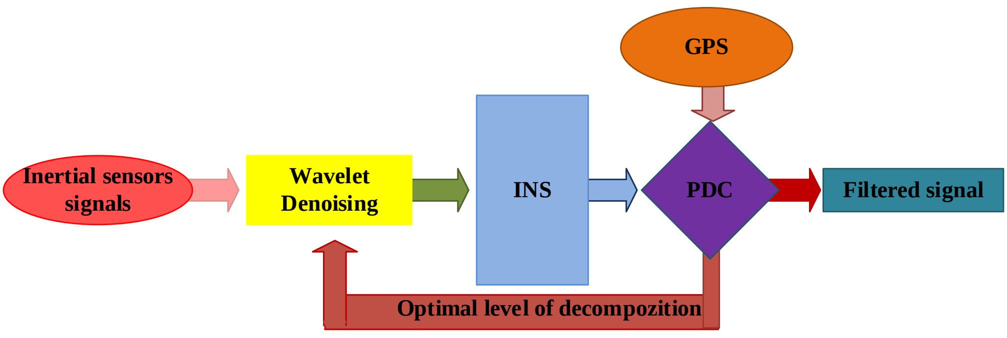

Figure 5. The architecture of the proposed method for processing data from INS

The *PDC* method is based on multichannel autoregressive models ([30]):

$$\begin{bmatrix} x_1(n) \\ \vdots \\ x_N(n) \end{bmatrix} = \sum_{r=1}^{p} \mathbf{A}_r(n) \begin{bmatrix} x_1(n-r) \\ \vdots \\ x_N(n-r) \end{bmatrix} + \begin{bmatrix} w_1(n) \\ \vdots \\ w_N(n) \end{bmatrix}; \tag{6}$$

where N is the time series number, w the white noise, and p the process order. $\mathbf{A}_r$ matrices have the next formulation ([30]):

$$\mathbf{A}_r(n) = \begin{bmatrix} a_{11}(r,n) & \cdots & a_{1N}(r,n) \\ \cdots & a_{ij}(r,n) & \cdots \\ a_{N1}(r,n) & \cdots & a_{NN}(r,n) \end{bmatrix}; \tag{7}$$

a_{ij} parameters denote the linear effect of $x_j(n\text{-}r)$ onto $x_i(n)$. To investigate the time series, ordinary coherence functions are performed ([30]):

$$C_{ij}(f) = \frac{\left|S_{ij}(f)\right|^2}{S_{ii}(f)S_{jj}(f)}, \tag{8}$$

for achieving the relative synchrony degree between the two areas, i and j, under analysis; $\mathbf{S}(f)$ denote the cross-spectral power density matrix of the measured signals $x_i(n)$, i=1÷N ([30]) while and $S_{ij}(f)$ its elements:

$$\mathbf{S}(f) = \begin{bmatrix} S_{11}(f) & \cdots & S_{1N}(f) \\ \cdots & S_{ij}(f) & \cdots \\ S_{N1}(f) & \cdots & S_{NN}(f) \end{bmatrix}; \tag{9}$$

$$\mathbf{S}(f) = \mathbf{H}(f)\mathbf{\Sigma}\mathbf{H}^H(f). \tag{10}$$

where $(.)^H$ is the Hermitian transpose, and $\mathbf{H}(f)$ an adequate filters matrix, with $\mathbf{\Sigma}$ the covariance matrix ([30]):

$$\mathbf{H}(f) = \begin{bmatrix} H_{11}(f) & \cdots & H_{1N}(f) \\ \cdots & H_{ij}(f) & \cdots \\ H_{N1}(f) & \cdots & H_{NN}(f) \end{bmatrix}. \tag{11}$$

For the model described by Eq. (6), it results the joint spectral density estimate:

$$\mathbf{H}(f) = \overline{\mathbf{A}}^{-1}(f) = (\mathbf{I} - \mathbf{A}(f))^{-1}, \tag{12}$$

with the noise signals covariance matrix $w_i(n)$ ([30]);

$$\mathbf{A}(f) = \sum_{r=1}^{p} \mathbf{A}_r z^{-r} \Big|_{z=e^{-i2\pi f}}. \tag{13}$$

The coupling estimation parameter equation, calculated between two time series (X_i and X_j) is given by the formula ([30]):

$$\pi_{ij}(f) \stackrel{\Delta}{=} \frac{\overline{A}_{ij}(f)}{\sqrt{\mathbf{a}_j^H(f)\mathbf{a}_j(f)}}, \tag{14}$$

where $\mathbf{a}_j(f)$ denotes the $\overline{A}(f)$ matrix j^{th} column. The normalization conditions of the π_{ij} parameter, in the frequency domain, is defined as in Eq. (15) ([30]):

$$0 \le \left|\pi_{ij}(f)\right| \le 1, \quad \sum_{i=1}^{N}\left|\pi_{ij}(f)\right| = 1, \tag{15}$$

for all $j=1\div N$ values.

High values, of these measures, denote a direct influence between two signals as follows: values of 1 mean that X_j influences are directional towards X_i, values of 0 are denoting the causal correlation absence between X_j to X_i (X_j does not influence X_i).

One last parameter estimates the two-time series coupling level (*CL*):

$$\begin{aligned} &a = mean\,PDC\,(X_i \to X_j), \\ &b = mean\,PDC\,(X_{i-1} \to X_{j-1}), \\ &CL = \begin{cases} WoptLvl = WactualLvl + 1, \; if \;\; a-b>0, \\ WoptLvl = WactualLvl, \; if \;\; a-b=0, \\ WoptLvl = WactualLvl - 1, \; if \;\; a-b<0. \end{cases} \end{aligned} \tag{16}$$

The wavelet's actual level of decomposition is noted by *WactualLvl*, which is the wavelet's optimal level of decomposition.

The algorithm data flow is presented in Figure 6. The signals received from *INS* and *GPS* as the navigation solutions (for North and East positions) are collected, processed and analyzed by employing the wavelet transform until the optimal levels of decomposition are found and then set up; after this step, more accurate data from future *INS* registered signals can be achieved and interpreted.

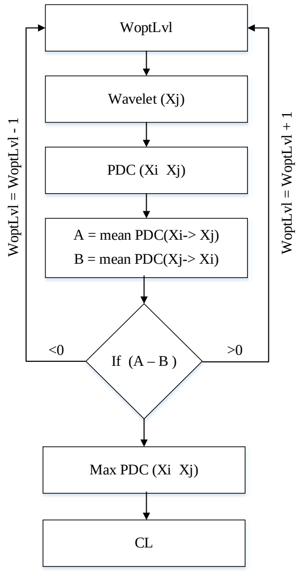

Figure 6. Coupling level estimation

4. EXPERIMENTAL VALIDATION OF THE IMPROVED NAVIGATOR BASED WAVELET DENOISING

For the positioning system's validation step, the inertial navigator model, from Figure 2, and an optimization software routine were implanted in Matlab/ Simulink.

Experimental tests were made, with the proposed *INS* and a *GPS* navigator boarded on a monitored vehicle (Figure 7). Positioning data were simultaneously acquired from these systems. The *GPS* was considered the reference system for evaluating the proposed inertial navigator's errors. The *IMU* had two accelerometers, one along the *x* and *y*-axis, and one gyro along the *z*-axis of the *SV* frame. The initial coordinates, of the starting point, i.e., latitude and longitude, were 44.33 deg and 23.84, respectively. The value of 189 m was considered the reference altitude. The North and East coordinate initial values were considered equal with zero. The inertial sensors data were acquired with a rate of 50 samples/s; therefore, only frequencies until 25 Hz can be analyzed in their spectrums.

Figure 7. Acquiring data during the experimental test

The trajectory of the vehicle during testing is shown in Figure 8.

As previously mentioned, for filtering the signals received from the sensor with the wavelet transform, we set up a tuning methodology of the method and searched for a parameter which denotes the optimal decomposition level. Also, we performed the *INS*'s tuning. The first calculation step was to achieve information on the vehicle attitude estimation to align the *SV* and *NED* frames.

This data was found after establishing the gyro's optimal decomposition level. Next step was to calculate the optimal correlation between the North position given by the *GPS* and the North position estimated with the *INS*. Shortly after was estimated the optimal correlation between the East position indicated by the two measurement units, the *GPS* and *INS*.

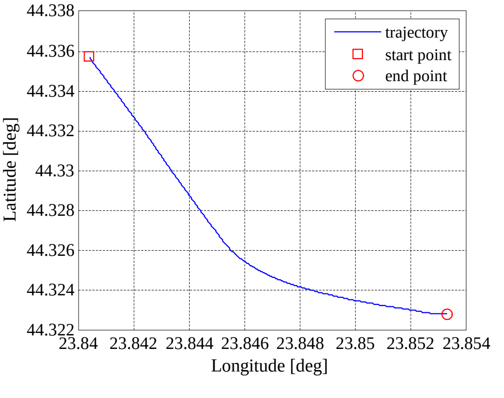

Figure 8. The trajectory of the vehicle during testing

We developed a Matlab software routine which performed tests for 20 decomposition levels for the gyro data, without accelerations data in *x* or *y* axes filtering. The model from Figure 2 was run after each gyro data filtering step. The *CL* coefficient was evaluated at each step for the filtered navigation solution and *GPS* data.

The same procedures were performed for tuning the data received from accelerometers; the targets were to calculate the optimal correlation between the North position offered by *GPS* and by the *INS* system, the optimal correlation between the East position offered by *GPS* and by the *INS* system, respectively. Therefore, the optimization software performs 400 combinations (20×20) between the decomposition levels in the acceleration channels to find the *CL* coefficient.

The values of the *CL* coefficient *CL_N=0.7282*, and *CL_E=0.7209*, while the equivalent decomposition levels for filtering with wavelet transform are *lev_x=13*, and lev_*y*=15.

Therefore, the optimal decomposition levels obtained with the proposed tuning method for the *INS* sensors are lev_*x*=13 for the accelerometer existent on the *x-axis*, lev_*y*=15 for the accelerometer existent on the *y-axis*, and lev_*z*=10 for gyro from the *y-axis*. The filtered data achieved from the inertial sensors versus the unfiltered inertial sensors data resulted as in Figure 9.

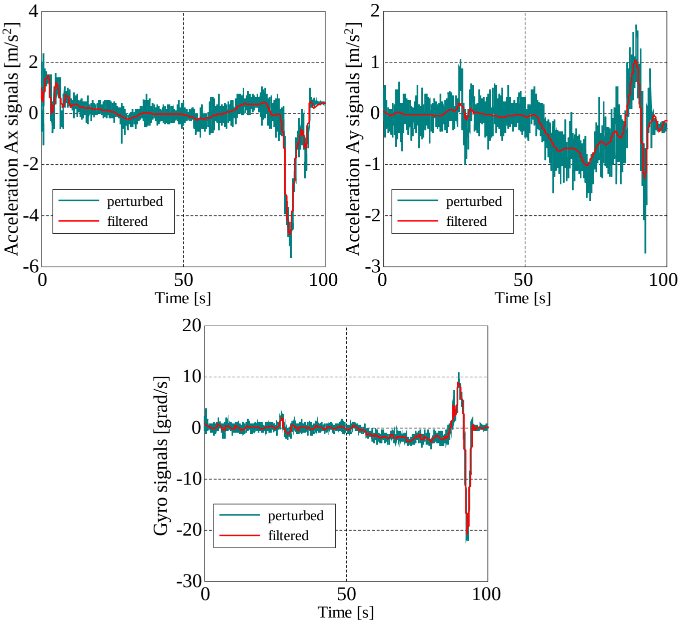

Figure 9. Filtered versus unfiltered inertial sensors data

The graphical results, relating the final coupling level, between the *GPS* and the *INS* solutions, after the sensors' data filtering, are presented in Figure 10 for North and East, and in Figure 11 for Latitude and Longitude, respectively.

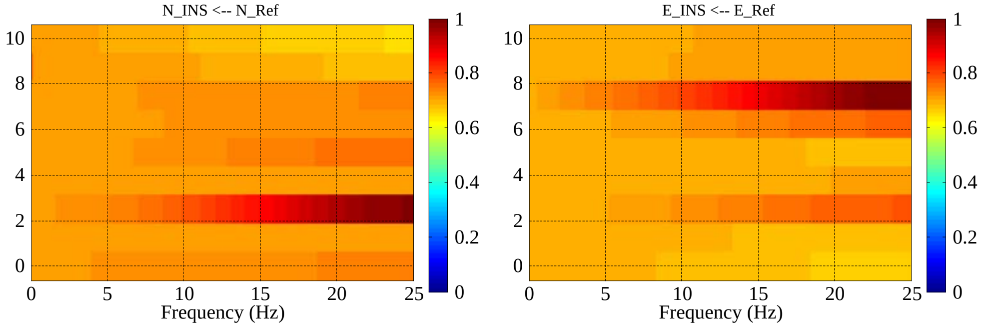

Figure 10. North and East coupling levels: GPS versus filtered INS

By applying the *PDC* method to evaluate *CL* coefficient for the *GPS* Latitude-Longitude solution and filtered *INS* Latitude-Longitude solution, the following values were achieved, *CL_Lat=0.7071* and *CL_Lon=0.7070*, values reflected by the diagrams depicted in Figure 11.

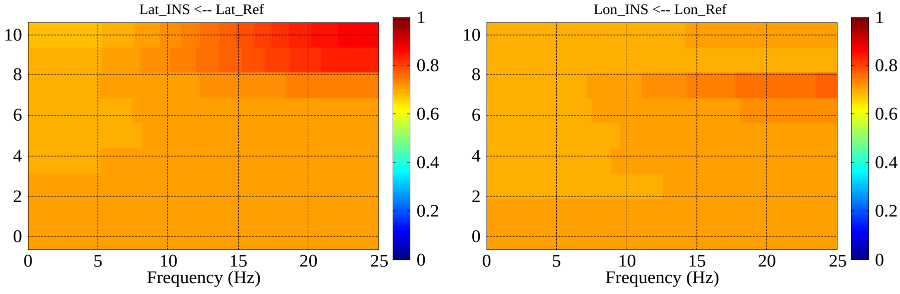

Figure 11. Latitude and longitude coupling levels: GPS → filtered INS

Figure 12 depicts the evolution of the yaw angle obtained from *INS* navigator starting from the unfiltered and filtered sensors data, and the deviation between these two solutions.

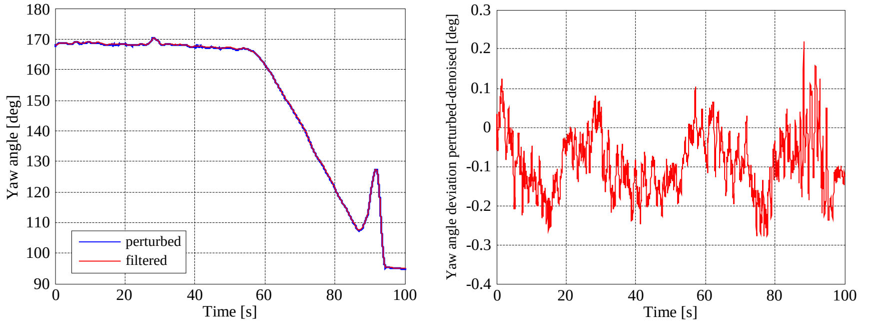

Figure 12. Yaw angle: un-filtered and filtered INSs

A more relevant analysis of the *INS* performance can be made by computing the deviations between *INS* and *GPS* solutions, for nonfiltered and filtered data achieved from sensors. The absolute maximum values of the deviations are presented in Table 1. The same table shows the degree of performance improvement in the form of the ratio of absolute maximum deviations obtained between *INS* unfiltered and *INS* filtered data. From the numerical values, we are concluding that we have achieved an essential improvement in precision positioning and an accurate *INS*.

Table 1. Absolute maximal values of the solutions deviations

Deviations	***INS* un-filtered**	***INS* filtered**	**ratio**
North [m]	89.7261	43.7264	2.052
East [m]	50.6636	35.8058	1.415
North speed [m/s]	2.2992	1.4501	1.585
East speed [m/s]	1.7112	1.7290	0.989
Latitude [deg]	$8.0748*10^{-4}$	$3.9351*10^{-4}$	2.052
Longitude [deg]	$6.3526*10^{-4}$	$4.4896*10^{-4}$	1.415

5. CONCLUSIONS

The development of a new bi-dimensional strap-down inertial navigator was presented in the current research. For testing, this positioning system was boarded on a test vehicle. Experimental tests were made considering *GPS* as the reference navigation system for the new proposed *INS*. This navigator is based on a wavelet transform filtering algorithm which role is to reduce the inertial sensors' perturbations. The filtering processing algorithm was optimized using a short-time variant *PDC* method. In the proposed algorithm, two steps were accomplished for finding the optimal decomposition levels for the three inertial sensors. These steps were related to the navigation problem-solving method of the strap down *IN*Ss. We correlated the data achieved from the *INS* with the data registered by the *GPS*, for each sensor, and found the optimal decomposition level of the wavelet function for each of the three sensors, for the North and East directions. The optimization software performed approximately 400 combinations (20×20) in each case, between the decomposition levels in the acceleration and angular speed channels. A *CL* coefficient was evaluated for each level of decomposition for each type of sensors and their optimum level of decomposition. The filtered and the non-filtered data from the inertial sensor data were studied and graphically illustrated for a better visualization. The *CL* coefficient values were $CL\_N=0.7282$ and $CL\_E=0.7209$, while the equivalent decomposition levels for filtering with the wavelet transform are $lev\_x=13$ for accelerometer x, $lev\_y=15$ for accelerometer y, and $lev\_z=10$ for gyro z.

Moreover, the extended *PDC* method was applied to evaluate the *CL* coefficient existent between the *GPS* Latitude-Longitude solution and filtered *INS* Latitude-Longitude solution; it resulted in $CL\_Lat=0.7071$ and $CL\_Lon=0.7070$. A relevant analysis of the navigator's performances was achieved by computing the deviations between the *INS* solutions and the *GPS* solution for nonfiltered and filtered sensor data. We achieved a visible performance improvement in the ratio of absolute maximum deviations obtained between the noisy sensors signals and filtered sensors signals.

ACKNOWLEDGEMENT

This paper was developed within the program National Project: STAR CDI, "Stratospheric platform for the observation, testing and research of aerospace microsystems" (STRATOTEST), nr 120/ 20.07.2017 from Romanian Space Agency.

REFERENCES

[1] C. Emmerson and e. al., Fork in the road: In-vehicle navigation systems and older drivers, *Transportation Research Part F: Traffic Psychology and Behaviour*, **21**:173–180, Nov. 2013.

[2] B. Krieg-Bruckner and e. al., Navigation Aid for Mobility Assistants, *Proceedings of the Joint CEWIT-TZIacatech Workshop ICT meets Medicine and Health ICTMH 2013*, 2013.

[3] B. Gersdorf and U. Freese, A Kalman Filter for Odometry using a Wheel Mounted Inertial Sensor, *10th International Conference on Informatics in Control, Automation and Robotics ICINCO 2013*, pp. 388-395, Reykjavík, Iceland, 29 - 31 July, 2013.

[4] B. H. Wellenhof, H. Lichtenegger, and J. Collins, *GPS Theory and Practice*, Springer-Verlag/Wien, 2001.

[5] X. He, X. Hu, and M. Wu, Trends in GNSS/INS integrated navigation technology, *Coordinates*, Volume **III**, Issue 3, March 2007.

[6] S. Mohinder, R. Lawrence, and P. Angus, *Global Positioning Systems, Inertial Navigation, and Integration,* John Wiley & Sons, Inc., 2001.

[7] N. Barbour and e. al., Inertial MEMS System Applications, *RTO-EN-SET-116, Low-Cost Navigation Sensors and Integration Technology*, March 2011.

[8] T. L. Grigorie, *Strap-Down Inertial Navigation Systems, Optimization studies*, SITECH, Craiova, Romania, 2007.

[9] T. L. Grigorie and I. R. Edu, *Inertial navigation applications with miniaturized sensors,* SITECH, Craiova, Romania, 2013.

[10] T. L. Grigorie and R. M. Botez, *Modelling and simulation based Matlab/Simulink of a strap-down inertial navigation system' errors due to the inertial sensors*, pp. 305-338, Chapter 11 in the book *Matlab applications for the practical engineer*, InTech, Sept. 2014.

[11] G. Schmidt, INS/GPS Technology Trends, *RTO-EN-SET-116, Low-Cost Navigation Sensors and Integration Technology*, March 2011.

[12] M. Z. Al-Faiz and S. A. Ismaeel, Design of Kalman Filter for Augmenting GPS to INS Systems, *International Conference on Advanced Remote Sensing for Earth Observation; Syst., Tech. Appl.*, pp. 14-20, 2005.

[13] K. W. Chiang, Development of an Optimal GPS/MEMS Integration Architecture for Land Vehicle Navigation Utilizing Neural Network, *Journal of Global Position System and CPGPS student paper competition*, 2004.

[14] S. Lorinda and N. Aboelmagd, Bridging GPS outages using neural network estimates of INS position and velocity errors, *Meas. Sci. Technol.*, **17**: 2783-2798, 2006.

[15] P. Vanicek and M. Omerbasic, Does a navigation algorithm have to use Kalman Filter?, *Canadian Aeronautics and Space J.*, **45**:1-9, 1999.

[16] E. Bekir, *Introduction to Modern Navigation Systems*, World Scientific Publishing Co. Pte. Ltd., 2007.

[17] T. L. Grigorie, I. R. Edu, and F. C. Adochiei, *Inertial positioning precision improvement through sensors denoising by using wavelet functions tuned with DTF method*, 6th International Conference on Information, Intelligence, Systems and Applications (IISA), JUL 06-08, 2015, Corfu, GREECE. doi: 10.1109/IISA.2015.7388108, WOS:000381572400155 http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7388108&isnumber=7387951, 2015.

[18] X.-C. Yin and e. al., Application of Wavelet Transform in Signal Denoising, *IEEE International Conference on Machine Learning and Cybernetic*, 2003.

[19] H. Krim and e. al., On Denoising and Best Signal Representation, *IEEE Transactions on Information Theory*, Vol. **45**, No. 7, pp. 2225-2238, 1999.

[20] H. T. Yang and C. C. Liao, *A Correlation-based Noise Suppression Algorithm for Power Quality Monitoring through Wavelet Transform*, International Conference on Power System Technology, pp 1311-1316, 2000.

[21] H. T. Yang and e. al., *A Wavelet Based Power Quality Monitoring System – Considering Noise Effects*, International Conference on Electric Power Engineering, 1999.

[22] H. T. Yang and C. C. Liao, A De-Noising Scheme for Enhancing Wavelet-Based Power Quality Monitoring System, *IEEE Transactions on Power Delivery*, Vol. **16**, No. 3, pp. 353-360, 2001.

[23] C. Taswell, The What, How, and Why of Wavelet Shrinkage Denoising, in *Computing in Science & Engineering*, Vol. **2**, pp. 12-19, 2000.

[24] C. W. Kang, C. H. Kang, and C. G. Park, *Wavelet Denoising Technique for Improvement of the Low Cost MEMS-GPS Integrated System*, International Symposium on GPS/GNSS, Taipei, Taiwan, October 26-28, 2010.

[25] A. M. Hasan and e. al., Comparative study on wavelet filter and thresholding selection for GPS/INS data fusion, *International Journal of Wavelets, Multiresolution and Information Processing*, Volume **08**, Issue 03, May 2010.

[26] A. M. Hasan, K. Samsudin, and A. R. Ramli, Intelligently Tuned Wavelet Parameters for GPS/INS Error Estimation, *International Journal of Automation and Computing*, **8**(4), 411-420, November 2011.

[27] A. Noureldin, A. Osman, and N. El-Sheimy, A neuro-wavelet method for multi-sensor system integration for vehicular navigation, *Meas. Sci. Technol.*, **15**: 404–412, 2004.

[28] F. C. Adochiei and e. al., *A new normalised short time pdc for dynamic coupling analyses in hypertensive pregnant women*, BMT 2013, GRAZ, 19-21 September 2013.

[29] I. R. Edu, F. C. Adochiei, P. Negrea, C. Rotaru, and T. L. Grigorie, *Tuning Method of the Wavelet Function for Gyro Sensor Signals Denoising with Wavelet Transform*, International Conference of Aerospace Sciences "AEROSPATIAL 2014", Bucharest, Romania, Sept. 2014.

[30] L. A. Baccalá and K. Sameshima, Partial directed coherence: a new concept in neural structure determination, *Biological Cybernetics*, vol. **84**, no. 6, pp. 463-474, 2001.