Analysis of structure and algorithm features of new type strapdown gravimetric navigation system

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Abstract: *The paper presents a structure and improved functional algorithms of a strap-down satellite inertial gravimetric navigation system of minimum hardware configuration. Various options of traditional loosely coupled and a new modification of closely coupled architectures were studied, which allowed for authors achieving sufficient precision of vector gravimetry and finding parameters of orientation and navigation. There were also studied potentials of increased accuracy and reliability of SGS as a component of functionally redundant cone-shaped accelerometer modules. The paper described the specifics of use of functional redundancy inertial measuring units. A specific version of a future modification of a closely related architecture is proposed, which opens up additional possibilities for evaluating and correcting errors of a satellite navigation system, which leads to an increase in the overall accuracy and reliability of determining orientation, navigation, and gravimetric parameters. The advantages of use of streamlined redundant raw data sensors were estimated qualitatively and quantitatively.*

Key Words: *functional algorithm, vector gravimetry, inertial measuring unit, optimal estimation, error correction*

1. INTRODUCTION

The effective exploration and development of new mineral deposits both onshore and offshore remains a topical issue. Prospecting and exploration technologies are critical for the fast and efficient development of mineral deposits with the offshore airborne gravimetric survey being one of the key phases. Mobile gravimetry is also employed in geophysical monitoring of the natural and manmade environment, as well for resolving problems of geophysics, geodesy, navigation including the correlation-extreme one (high precision self-contained navigation based on geophysical anomaly data). The problems of gravimetry have critical business, scientific and defense dimensions. As of today such problems are handled mostly with expensive shipborne or airborne scalar gravimetry systems of intermediate class, by surveying the land and water areas of interest in parallel and diagonal sweeps [1]. The airborne gravimetry is developing mainly through improvement of the existing equipment while the general concept of the mobile gravimetry has remained unchanged for decades. Among the advantages of the modern mobile gravimetry is a great deal of expertise accumulated in the areas of hardware, software and technologies, as well as the measurement precision (0.05-0.5 mGal) being satisfactory for the most tasks including search and exploration of mineral deposits.

However, the modern technologies of mobile gravimetry have their limitations such as huge weight and size of the existing gravimetric systems (meters and hundreds kilograms), high energy consumption (hundreds of watts) and high cost (dozens of million Rubles) because gravimetric sensors must be placed on heavy gyroscope-stabilized platforms for proper local vertical orientation.

Consequently, there is a need for research ships, medium-class airplanes and helicopters with certain carrying capacity.

The limited maneuverability of such ships and aircraft reduces the performance, precision and responsiveness of measurements, while operation costs of such platforms with crews drive the total costs of gravimetry dramatically. To solve these problems of the modern mobile gravimetry it is necessary to address its principal drawback i.e. to get rid of an expensive and massive gyrostabilizer used in a gravimetry system for stabilization of a high-precision single-component gravimetric sensor.

This, in its turn, will allow for reducing the mass, size, energy consumption and cost of the system, and will make possible an airborne and shipborne deployment of the system including on automatic small-sized unmanned vehicles.

Therefore, there is a high demand for advanced and cost-efficient airborne gravimetry systems based on modern small-sized unmanned aerial vehicles and employing high-end strap-down satellite navigation, electronics, software and hardware.

The Department of Automatic Orientation and Navigation Systems of Moscow Aviation Institute (MAI) has long been engaged in designing a high-precision small-sized strap-down graviinertial system (SGS) having the best mass and size parameters, reduced energy consumption as compared to the conventional gyrostabilized systems, with commensurable error of measurement of the vertical projection of the acceleration due to gravity.

Moreover, SGS is potentially capable of measuring the horizontal projections of the gravity acceleration (vertical deflections) [2]. The present work describes rational functional algorithms of SGS with a minimum configuration of measuring subsystems, and presents the potentials of increasing its precision through the use of functionally redundant architecture of inertial measuring units.

2. BASIC FUNCTIONAL ALGORITHM OF SGS

A practical configuration of a projected SGS will include a series of raw data sensors and measuring subsystems, computer module with SGS functional algorithms, and memory storage device.

A minimum basic set of SGS measuring units (Figure 1) includes a three-component accelerometer module (AM) and a three-component gyroscope module (GM) as part of inertial measuring unit (IMU) of a strap-down inertial navigation system (SDINS), as well as a differential satellite receiver of GLONASS and/or GPS satellite navigation system (SNS) with Galileo and etc. as an option.

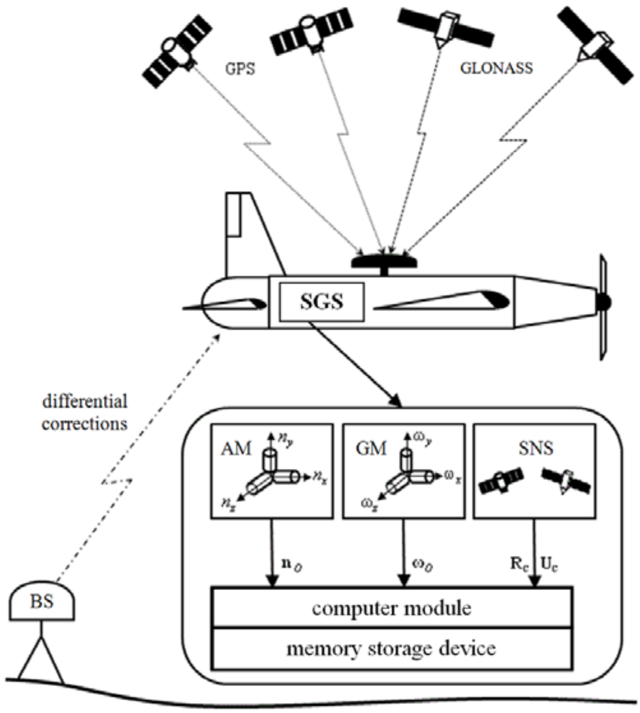

Fig. 1 – Basic configuration of an airborne SGS

In the figure above: BS is a Basic Station; n_x, n_y, n_z are projections of apparent acceleration vector $\mathbf{n}_O$ on an axis of the body axis coordinate system as measured by the Basic Station; ω_x, ω_y, ω_z are projections of vectors of absolute angular rate $\boldsymbol{\omega}_O$ on the axis of the body axis coordinate system as measured by Basic Station; $\mathbf{R}_C$, $\mathbf{U}_C$ are radius-vector of position of an object and vector of its relative velocity as measured by SNS. The SGS computer module implements algorithms of pre-processing of measuring subsystems data, defines parameters of orientation, navigation and vector gravimetry, as well processes and corrects information in the most optimal way through Kalman filtering. The results of and current measuring signals are registered by the memory storage device.

The accelerometer and gyroscope modules of SDINS measure the projections of vectors $\mathbf{n}_O=(n_x, n_y, n_z)^T$ and $\boldsymbol{\omega}_O=(\omega_x, \omega_y, \omega_z)^T$ of the SGS-carrying vehicle in a body axis coordinate system with unit vectors **x**, **y**, **z**. Then, SNS helps find $\mathbf{U}_C$, $\mathbf{R}_C$, and when doing so, both geographic and equatorial coordinate systems can be used. The Equatorial Coordinate System (ECS), as a right-handed system of coordinates with unit vectors $\boldsymbol{\xi}$, $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$ (alternative name is Greenwich coordinate system), originates from the center of the Earth; $\boldsymbol{\xi}$ is situated on the line of intersection of the plane of equator with the Greenwich meridian, $\boldsymbol{\zeta}$ is directed at vector **u** of the Earth angular velocity. The $_C$ index corresponds to the parameters computed from SNS readings, the $_O$ index marks the vectors represented in the projections on the axis of the body axis coordinate system, while no index means vectors on the equatorial coordinate system (ECS). A SNS working in a normal mode will be well enough for the orientation and navigation functions only, while for vector gravimetry it will be practical to apply a more differential mode of measurements with corrections received from a land BS.

For high-precision navigation, the computer module of the SNS receiver calculates and introduces corresponding corrections into the measured pseudo-ranges and pseudo-velocities to offset errors of satellite clocks, as well as ionosphere, troposphere and other errors of SNS

[3]. Then, usually, for finding $\mathbf{R}_C$ and $\mathbf{U}_C$ based on the corrected pseudo-ranges and pseudo-velocities, a well-known least square method is applied. In addition to coordinates and projections of SGS carrier angular velocity, the receiver clock offset τ and its drift are calculated in the dimension of range $\tau_L = c \cdot \tau$ and velocity $\dot{\tau}_L$, were c is the speed of light.

Thus, the pseudo-ranges and pseudo-velocities after an algorithmic offset of basic errors before the start of iteration procedure of the least square method [3] will be written as follows:

$$P_S = d_S + \delta d_S + \tau_L; \tag{1}$$

$$\dot{P}_S = \dot{d}_S + \delta \dot{d}_S + \dot{\tau}_L, \tag{2}$$

where S =0,1,…, N-*1* is the number of a navigation satellite from a N available satellites; $P_S, \dot{P}_S$ are pseudo-range and pseudo-velocity of the satellite; d_S is geometrical range from the receiver antenna to the radiating antenna of the satellite; the δ symbol means the error of the corresponding value, in this particular case this means residual error of pseudo-range and pseudo-velocity after algorithmic compensation of their basic errors caused by all reasons excluding offset and drift of receiver clock. In its turn, the range and its derivative are function of coordinates and relative velocity of the antenna, in this case for true coordinates and velocity (with index $_I$):

$$d_S\left(R_{I_{\xi,\eta,\zeta}}\right) = \sqrt{(R_\xi^S - R_{I_\xi})^2 + (R_\eta^S - R_{I_\eta})^2 + (R_\zeta^S - R_{I_\zeta})^2}; \tag{3}$$

$$\begin{aligned} &\dot{d}_S\left(R_{I_{\xi,\eta,\zeta}}, U_{I_{\xi,\eta,\zeta}}\right) = \\ &= \frac{(R_\xi^S - R_{I_\xi}) \cdot (U_\xi^S - U_{I_\xi}) + (R_\eta^S - R_{I_\eta}) \cdot (U_\eta^S - U_{I_\eta}) + (R_\zeta^S - R_{I_\zeta}) \cdot (U_\zeta^S - U_{I_\zeta})}{\sqrt{(R_\xi^S - R_{I_\xi})^2 + (R_\eta^S - R_{I_\eta})^2 + (R_\zeta^S - R_{I_\zeta})^2}}, \end{aligned} \tag{4}$$

where $R^S{}_{\xi,\eta,\zeta}$, $U^S{}_{\xi,\eta,\zeta}$ are projections of the radius vector of the S satellite's current location and velocity relative to the Earth. The unknown vector to be estimated through iteration procedure of the least square method will take the following form:

$$\mathbf{Y} = \left(\mathbf{R}_C^{\mathrm{T}}, \mathbf{U}_C^{\mathrm{T}}, \tau_L, \dot{\tau}_L\right)^{\mathrm{T}}. \tag{5}$$

In case of absence of a priori information, the zero values are taken as its initial approximation, and when a priori information is available from previous steps of calculations or from other sources, such information will be used to form a vector of initial conditions

$$\mathbf{Y}^0 = \left(R_{C_\xi}^0, R_{C_\eta}^0, R_{C_\zeta}^0, U_{C_\xi}^0, U_{C_\eta}^0, U_{C_\zeta}^0, \tau_L^0, \dot{\tau}_L^0\right)^{\mathrm{T}}, \tag{6}$$

which are corresponded with the initial approximations of pseudo-ranges and pseudo-velocities:

$$P_S^0 = d_S^0 + \tau_L^0; \tag{7}$$

$$\dot{P}_S^0 = \dot{d}_S^0 + \dot{\tau}_L^0, \tag{8}$$

where $d_S^0 = d_S(R_{C\,\xi,\eta,\zeta}^0), \dot{d}_S^0 = \dot{d}_S(R_{C\,\xi,\eta,\zeta}^0, U_{C\,\xi,\eta,\zeta}^0)$ and index ^{0} means initial conditions. The state vector $\mathbf{\Delta}$ estimated through the least squares method includes increments in the vector of initial conditions:

$$\mathbf{\Delta}=\left(\mathit{\Delta}\mathbf{R}_C^{\mathrm{T}},\mathit{\Delta}\mathbf{U}_C^{\mathrm{T}},\mathit{\Delta}\tau_L,\mathit{\Delta}\dot{\tau}_L\right)^{\mathrm{T}}; \tag{9}$$

$$\mathbf{Y}=\mathbf{Y}^0+\mathbf{\Delta}, \tag{10}$$

where Δ means increment in the corresponding values to their initial values.

The assessment $\mathbf{Y}$ can be obtained through one iteration of the least squares method:

$$\widehat{\mathbf{Y}}=\mathbf{Y}^0+\widehat{\mathbf{\Delta}}, \tag{11}$$

by making use of a matrix equation of the least squares method in the form:

$$\widehat{\mathbf{\Delta}}=\mathbf{NB}, \tag{12}$$

where $\mathbf{N}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}$ for a Gaussian estimation or $\mathbf{N}=\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}^{-1}$ for a Gaussian-Markov estimation with a known covariance matrix $\mathbf{P}$ of measurement noise. The matrix and vector of measurement are calculated through the following relations:

$$\mathbf{A}=\begin{pmatrix}
\frac{\partial d_0^0}{\partial R_{C\xi}^0} & \frac{\partial d_0^0}{\partial R_{C\eta}^0} & \frac{\partial d_0^0}{\partial R_{C\zeta}^0} & 0 & 0 & 0 & 1 & 0\\
\frac{\partial d_1^0}{\partial R_{C\xi}^0} & \frac{\partial d_1^0}{\partial R_{C\eta}^0} & \frac{\partial d_1^0}{\partial R_{C\zeta}^0} & 0 & 0 & 0 & 1 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
\frac{\partial d_{N-1}^0}{\partial R_{C\xi}^0} & \frac{\partial d_{N-1}^0}{\partial R_{C\eta}^0} & \frac{\partial d_{N-1}^0}{\partial R_{C\zeta}^0} & 0 & 0 & 0 & 1 & 0\\
\frac{\partial \dot{d}_0^0}{\partial R_{C\xi}^0} & \frac{\partial \dot{d}_0^0}{\partial R_{C\eta}^0} & \frac{\partial \dot{d}_0^0}{\partial R_{C\zeta}^0} & \frac{\partial \dot{d}_0^0}{\partial U_{C\xi}^0} & \frac{\partial \dot{d}_0^0}{\partial U_{C\eta}^0} & \frac{\partial \dot{d}_0^0}{\partial U_{C\zeta}^0} & 0 & 1\\
\frac{\partial \dot{d}_1^0}{\partial R_{C\xi}^0} & \frac{\partial \dot{d}_1^0}{\partial R_{C\eta}^0} & \frac{\partial \dot{d}_1^0}{\partial R_{C\zeta}^0} & \frac{\partial \dot{d}_1^0}{\partial U_{C\xi}^0} & \frac{\partial \dot{d}_1^0}{\partial U_{C\eta}^0} & \frac{\partial \dot{d}_1^0}{\partial U_{C\zeta}^0} & 0 & 1\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
\frac{\partial \dot{d}_{N-1}^0}{\partial R_{C\xi}^0} & \frac{\partial \dot{d}_{N-1}^0}{\partial R_{C\eta}^0} & \frac{\partial \dot{d}_{N-1}^0}{\partial R_{C\zeta}^0} & \frac{\partial \dot{d}_{N-1}^0}{\partial U_{C\xi}^0} & \frac{\partial \dot{d}_{N-1}^0}{\partial U_{C\eta}^0} & \frac{\partial \dot{d}_{N-1}^0}{\partial U_{C\zeta}^0} & 0 & 1
\end{pmatrix}; \tag{13}$$

$$\boldsymbol{B}=\left(P_0-P_0^0,P_1-P_1^0,\ldots,P_{N-1}-P_{N-1}^0,\dot{P}_0-\dot{P}_0^0,\dot{P}_1-\dot{P}_1^0,\ldots,\dot{P}_{N-1}-\dot{P}_{N-1}^0\right)^{\mathrm{T}}. \tag{14}$$

The equation of measurements and vector of measurement noise are written as:

$$\mathbf{B}=\mathbf{A}\cdot\mathbf{\Delta}+\mathbf{V}; \tag{15}$$

$$\mathbf{V}=\left(\delta d_0,\delta d_1,\ldots,\delta d_{N-1},\delta\dot{d}_0,\delta\dot{d}_1,\ldots,\delta\dot{d}_{N-1}\right)^{\mathrm{T}}. \tag{16}$$

Normally, in order to find exact $\mathbf{Y}$, several iterations of the above-mentioned algorithm are repeated until the latest estimation of the state vector reaches the preset minimum value. When starting each next iteration of the least squares method (expressions (11), (12) and (14)), the initial value $\mathbf{Y}$ is assumed to be corresponding to its prior estimation:

$$\mathbf{Y}_0=\widehat{\mathbf{Y}}. \tag{17}$$

A functional algorithm of a SGS of a basic configuration implemented in its computer module represented in ECS build as a close-open scheme and using the IMU and SNS data will take the form as follows [4]:

$$\left.\begin{aligned}
&\dot{\boldsymbol{\Lambda}} = 0{,}5\ \mathbf{T}_{\widehat{\boldsymbol{\omega}}_O}\ \boldsymbol{\Lambda};\\
&\widehat{\boldsymbol{\omega}}_O = \boldsymbol{\omega}_O - \hat{\delta}\boldsymbol{\omega}_{Op} - \mathbf{M}^{\omega}\boldsymbol{\omega}_O + \mathbf{A}_{O/E}^{\mathrm{T}}\mathbf{K}^{\langle 0..2\rangle}\mathbf{Z};\\
&\mathbf{A}_{O/E} = \mathbf{A}_{I/E}[(2\lambda_0^2-1)\mathbf{E} + 2[\boldsymbol{\lambda}\boldsymbol{\lambda}^{\mathrm{T}}] - 2\lambda_0\mathbf{L}_{\boldsymbol{\lambda}}];\\
&\dot{\mathbf{U}} = -2\mathbf{u}\times\mathbf{U} + \mathbf{A}_{O/E}\widehat{\mathbf{n}}_O + \mathbf{g}_{TN} + \widehat{\mathbf{A}} - \mathbf{K}^{\langle 3..5\rangle}\mathbf{Z};\\
&\dot{\mathbf{R}} = \mathbf{U} - \mathbf{K}^{\langle 6..8\rangle}\mathbf{Z};\\
&\widehat{\mathbf{n}}_O = \mathbf{n}_O - \hat{\delta}\mathbf{n}_{Op} - \mathbf{M}^{n}\mathbf{n}_O;\\
&\widehat{\mathbf{A}} = \mathbf{A}_i - \hat{\delta}\mathbf{A}_p - \hat{\delta}\mathbf{A}_{sl} - \mathbf{M}^{A}(\mathbf{R}-\mathbf{R}_0);\\
&\dot{\hat{\delta}}\mathbf{n}_{Op} = \mathbf{K}^{\langle 9..11\rangle}\mathbf{Z};\quad \dot{\hat{\delta}}\boldsymbol{\omega}_{Op} = \mathbf{K}^{\langle 12..14\rangle}\mathbf{Z};\\
&\dot{\hat{\delta}}\mathbf{A}_p = \mathbf{K}^{\langle 15..17\rangle}\mathbf{Z};\quad \dot{\hat{\delta}}\mathbf{A}_{sl} = -\mathbf{D}_{\mu}\hat{\delta}\mathbf{A}_{sl} + \mathbf{K}^{\langle 18..20\rangle}\mathbf{Z};\\
&\dot{\hat{\mathbf{k}}}^{\delta A} = \mathbf{K}^{\langle 21..29\rangle}\mathbf{Z};\ \dot{\hat{\delta}}\mathbf{k}^{\omega} = \mathbf{K}^{\langle 30..32\rangle}\mathbf{Z};\ \dot{\hat{\delta}}\mathbf{k}^{n} = \mathbf{K}^{\langle 33..35\rangle}\mathbf{Z};\\
&\dot{\widehat{\boldsymbol{\Theta}}}^{\omega} = \mathbf{K}^{\langle 36..41\rangle}\mathbf{Z};\ \dot{\widehat{\boldsymbol{\Theta}}}^{n} = \mathbf{K}^{\langle 42..44\rangle}\mathbf{Z};\\
&\dot{\hat{\delta}}\mathbf{R}_{Cp} = \mathbf{K}^{\langle 45..47\rangle}\mathbf{Z};\\
&\dot{\hat{\delta}}\mathbf{U}_{Cp} = \mathbf{K}^{\langle 48..50\rangle}\mathbf{Z};\\
&\mathbf{Z} = \begin{pmatrix}\mathbf{R}-\mathbf{R}_C+\hat{\delta}\mathbf{R}_{Cp}\\ \mathbf{U}-\mathbf{U}_C+\hat{\delta}\mathbf{U}_{Cp}\end{pmatrix}.
\end{aligned}\right\}\qquad(18)$$

where $\boldsymbol{\Lambda} = [\lambda_0, \lambda_1, \lambda_2, \lambda_3]^T$ is the vector of Rodriguez-Hamilton parameters which characterizes the rotation of the body axis coordinate system relating to the inertial coordinate system, $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\lambda_3)^{\mathrm{T}}$; $\mathbf{L}_{\mathbf{a}} = \begin{pmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{pmatrix}$, $\mathbf{T}_{\mathbf{a}} = \begin{pmatrix}0 & -a_1 & -a_2 & -a_3\\ a_1 & 0 & a_3 & -a_2\\ a_2 & -a_3 & 0 & a_1\\ a_3 & a_2 & -a_1 & 0\end{pmatrix}$ – of the matrix made of elements of some vector $\mathbf{a}=(a_1,a_2,a_3)^{\mathrm{T}}$; $\mathbf{A}_{O/E}$ is the matrix of orientation of ECS (E) relating to the body axis coordinate system (O); $\mathbf{M}^{\omega} = \begin{pmatrix}\hat{\delta}k_x^{\omega} & \hat{\Theta}_{xy}^{\omega} & \hat{\Theta}_{zx}^{\omega}\\ \hat{\Theta}_{yx}^{\omega} & \hat{\delta}k_y^{\omega} & \hat{\Theta}_{yx}^{\omega}\\ \hat{\Theta}_{zx}^{\omega} & \hat{\Theta}_{zy}^{\omega} & \hat{\delta}k_z^{\omega}\end{pmatrix}$; $\mathbf{M}^{n} = \begin{pmatrix}\hat{\delta}k_x^{n} & \hat{\Theta}_{xy}^{n} & 0\\ 0 & \hat{\delta}k_y^{n} & 0\\ \hat{\Theta}_{zx}^{n} & \hat{\Theta}_{zy}^{n} & \hat{\delta}k_z^{n}\end{pmatrix}$; $\mathbf{M}^{A} = \begin{pmatrix}\hat{k}_{\xi\xi}^{\delta A} & \hat{k}_{\xi\eta}^{\delta A} & \hat{k}_{\xi\zeta}^{\delta A}\\ \hat{k}_{\eta\xi}^{\delta A} & \hat{k}_{\eta\eta}^{\delta A} & \hat{k}_{\eta\zeta}^{\delta A}\\ \hat{k}_{\zeta\xi}^{\delta A} & \hat{k}_{\zeta\eta}^{\delta A} & \hat{k}_{\zeta\zeta}^{\delta A}\end{pmatrix}$; $k_i^{\omega,n}$ are scale coefficients of gyroscopes or accelerometers with a measuring axis i, $\mathbf{k}^{\omega,n}$ are column vectors made of them; $\Theta_{ij}^{\omega,n}$ are low angles of deviations of measuring axes of gyroscopes or accelerometers from an ideal axis i along axis j of the body axis coordinate system, $\boldsymbol{\Theta}^{\omega,n}$ are column vector made of them; $\mathbf{A}_{I/E} = \begin{pmatrix}\cos(ut) & \sin(ut) & 0\\ -\sin(ut) & \cos(ut) & 0\\ 0 & 0 & 1\end{pmatrix}$ is the matrix of orientation of ECS relative to an inertial coordination system (I); u is module $\mathbf{u}$; t is current time; $\mathbf{E}$ is unit matrix of corresponding dimension; $\mathbf{R}$ is geocentric radius-vector of location of the SGS-carrying vehicle; $\mathbf{U}$ is vector of relative velocity of the SGS-carrying vehicle; $\mathbf{A}_i$ is vector of anomaly of the acceleration due to gravity known with limited accuracy (when initial data is not available, $\mathbf{A}_i$=0); $\mathbf{R}_0$ is initial value of **R**; $\mathbf{g}_{TN}$ – is normal vector of the acceleration due to gravity; $k_{ij}^{\delta A}$ are linear coefficients of error model of anomaly projected on the axis *i* depending on coordinate *j*, $\mathbf{k}^{\delta A}$ is column vector made of them; **A** is vector of anomaly of the acceleration due to gravity; $\mathbf{D}_{\mu}$ is diagonal matrix mad of elements of the vector of coefficients of attenuation **μ** of correlation functions chosen for description of the corresponding projections of random components of the anomaly of the acceleration due to gravity; index *sl* means random components of values while the index *p* corresponds to their systematic constant components; **K** is matrix of coefficients of enhancement of the Kalman optimal filter (KOF); index $^{\langle i,j \rangle}$ indicates that the matrix **K** keeps lines from *i* to *j*; symbol ^ here means estimation of the corresponding value with KOF or a corrected value of some parameter; **Z** is vector of measurement of KOF in true representation [5], [6], [7] for a case of traditional loosely-coupled architecture of the system.

The functioning algorithms results into estimation and correction of the corresponding errors of SDINS and SNS and, eventually, calculate navigation parameters (**R**, **U**), orientation parameters (**Λ**, $\mathbf{A}_{O/E}$), vector of anomaly of the acceleration due to gravity $\hat{\mathbf{A}}$, projections of vectors of the apparent acceleration and absolute angular velocity ($\hat{\mathbf{n}}_O, \hat{\boldsymbol{\omega}}_O$). Notably, for the sake of simplicity, only systematic components of errors of accelerometer modules, gyroscope modules and SNS are to be estimated within this algorithm, represented by a zero order polynomial (constants). When necessary, more complex models can be described by polynomials of the first or second order, as well as by considering a correlating random component of error, for instance, that similar to errors of anomaly of the acceleration due to gravity.

Since the functional algorithm of geophysical SGS is implemented in ECS, $\mathbf{g}_{TN}$ will be described as follows [2]:

$$\mathbf{g}_{TN} = F_{Ngc}\,\mathbf{N}_{gc} + F_R \mathbf{1}_R - \mathbf{u} \times (\mathbf{u} \times \mathbf{R});\ \mathbf{1}_R = \mathbf{R}(\mathbf{R} \cdot \mathbf{R})^{-0,5} = \left[l_\xi, l_\eta, l_\zeta\right]^{\mathrm{T}}; \tag{19}$$

$$\mathbf{E}_{gc} = (\boldsymbol{\zeta} \times \mathbf{1}_R)\left(1 - l_\zeta^2\right)^{-0,5};\ \mathbf{N}_{gc} = \mathbf{1}_R \times \mathbf{E}_{gc}; R = (\mathbf{R} \cdot \mathbf{R})^{0,5}; \tag{20}$$

$$\begin{gathered} F_{Ngc} = g_e(q - e^2)(a/R)^4 l_\zeta (1 - l_\zeta^2)^{0,5}[1 + e^2(0,5e^2 - 30q/14)(q - e^2)^{-1}] \times \\ \times\left\{1 + \left[30q/14 - 1,5e^2 + l_\zeta^2(3,5e^2 - 5q)\right](q - e^2)^{-1}(ea/R)^2\right\}; \end{gathered} \tag{21}$$

$$\begin{gathered} F_R = -g_e\left(\frac{a}{R}\right)^2\{1 - 0,5e^2 - 0,125e^4 + q\left(1,5 - (\frac{15e^2}{28}\right) + [0,5e^2 - 0,25e^4 + \\ +q(-0,5 + 15e^2/14) - l_\zeta^2\left(1,5e^2 - 0,75e^4 + q(-1,5 + 45e^2/14)\right)\big](a/R)^2 + \\ +[0,375e^2 - 15q/28 + l_\zeta^2(0,625e^2 - 25q/28) - l_\zeta^2\left(1 - l_\zeta^2\right)(4,375e^2 - \\ 6,25q)\big]e^2(a/R)^4\}, \end{gathered} \tag{22}$$

where F_{Ngc}, F_R are projections of strength of the normal gravitational field of the Earth on the axis of the following geocentric coordinate system which, in general case, subject to conditions of operation of SGS, can be represented by functions of equatorial, geocentric or geographic coordinates [5, 8]; $\mathbf{E}_{gc}$ (East), $\mathbf{N}_{gc}$ (North), $\mathbf{1}_R$ (geocentric vertical) are unit vectors of a following geocentric coordinate system; g_e is value of normal acceleration due to gravity at equator; q is a ratio of centrifugal force due to Earth' rotation to the gravity at the Equator; $e = (a^2 - b^2)^{0,5}a^{-1}$ and *a*, *b* are the first eccentricity, large and small semi-axes of reference ellipsoid correspondingly.

Parameters of orientation of the SGS-carrying vehicle (heading *ψ*, pitch *ϑ*, roll *γ*) are calculated according to the algorithm [2]:

$$\left.\begin{aligned}&\mathbf{Z}_1=(\mathbf{x}\times\mathbf{r})|\mathbf{x}\times\mathbf{r}|^{-1};\\&\psi=-\mathrm{arctg}(\mathbf{N}_{gg}\mathbf{Z}_1(\mathbf{E}_{gg}\mathbf{Z}_1)^{-1});\\&\vartheta=\mathrm{arctg}(\mathbf{xr}(1-(\mathbf{xr})^2)^{-0,5});\\&\gamma=\mathrm{arctg}(\mathbf{yZ}_1(\mathbf{zZ}_1)^{-1}),\end{aligned}\right\}\tag{23}$$

where unit vectors of the body axis coordinate system **x**, **y**, **z** are the 1st, 2nd and, correspondingly, 3rd columns $\mathbf{A}_{O/E}$; $\mathbf{E}_{gg}$, $\mathbf{N}_{gg}$, **r** are unit vectors of the eastbound, northbound and vertical directions of a geographic coordinate system. These unit vectors can be calculated by simplified relations:

$$\mathbf{r}=-\mathbf{g}_{TN}g_{TN}^{-1};\mathbf{E}_{gg}=\mathbf{E}_{gc};\ \mathbf{N}_{gg}=\mathbf{r}\times\mathbf{E}_{gg},\tag{24}$$

which have a minor methodical error because at non-zero altitudes a normal to a reference ellipsoid and the direction of the normal acceleration due to gravity, in the strict sense, are not in agreement because of presence of a northerly component of normal acceleration due to gravity at non-zero altitudes.

This error is virtually equal to zero near poles, Equator and the surface of the reference ellipsoid, and reaches its maximum at middle latitudes and almost linearly depends on the altitude. For instance, at an altitude of ±10 kilometers and a latitude of 45° the orientation error **r** will be merely ±1.7 arc/sec which satisfies the accuracy requirements of vector gravimetry. However, this error can be completely eliminated by making use of accurate but much more complex relations [4, 5]:

$$\mathbf{r}=\mathbf{1}_R+(1+\Theta^2/4)^{-1}\big(\mathbf{\Theta}\times(\mathbf{1}_R+\mathbf{\Theta}\times\mathbf{1}_R/2)\big),\tag{25}$$

where $\mathbf{\Theta}=2\mathbf{E}_{gg}\mathrm{tg}\left((\varphi_{gc}-\varphi)/2\right)$; $\varphi_{gc}=\arcsin(l_\zeta)=\arccos(1-l_\zeta^2)^{0,5}$ is geocentric latitude; φ is geographic latitude which can be found by SGS though one of the known analytical [5] or numerical [9] methods, by solving algebraic equations relating φ with φ_{gc} and R, or by solving a differential equation in parallel with (18):

$$\begin{aligned}\dot\varphi=&\left((R_\xi\dot R_\xi+R_\eta\dot R_\eta)\mathrm{tg}\varphi-\dot R_\zeta\sqrt{R_\xi^2+R_\eta^2}\right)(1-e^2\sin^2\varphi)\cos^2\varphi\Big/(ae^2\cos^3\varphi\sqrt{(1-e^2\sin^2\varphi)}\times\\&\times\sqrt{R_\xi^2+R_\eta^2}-(R_\xi^2+R_\eta^2)(1-e^2\sin^2\varphi)+\left((R_\xi^2+R_\eta^2)\mathrm{tg}\varphi-R_\zeta\sqrt{R_\xi^2+R_\eta^2}\right)e^2\sin\varphi\cos^3\varphi).\end{aligned}\tag{26}$$

By using functionally redundant accelerometer and gyroscope modules in SGS, the values $\mathbf{n}_O$ and $\boldsymbol{\omega}_O$ are found as follows [2]:

$$\left.\begin{aligned}&\mathbf{n}_O=\mathbf{H}_a\mathbf{a};\ \boldsymbol{\omega}_O=\mathbf{H}_g\mathbf{m};\\&\mathbf{H}_a=(\mathbf{E}_a^{\mathrm{T}}\mathbf{E}_a)^{-1}\mathbf{E}_a^{\mathrm{T}};\ \mathbf{H}_g=(\mathbf{E}_g^{\mathrm{T}}\mathbf{E}_g)^{-1}\mathbf{E}_g^{\mathrm{T}},\end{aligned}\right\}\tag{27}$$

where $\mathbf{H}_a$, $\mathbf{H}_g$ are matrices of (3×k)-dimension of processing of redundant measurements of accelerometer and gyroscope modules from k sensors; **a**, **m** are vectors of redundant measurements of accelerometer and gyroscope modules from k sensors; $\mathbf{E}_{a,g}=[\mathbf{e}_{1O}^{a,g},...,\mathbf{e}_{iO}^{a,g},...,\mathbf{e}_{kO}^{a,g}]^T$ are matrices of (k×3)-dimension of alignment of accelerometer and gyroscope modules from k sensors. And unit vectors of orientation of measuring axes in a body axis coordinate system are lines of matrices of alignment of accelerometer and gyroscope modules:

$$\mathbf{e}_{iO}^{a,g} = [\mathbf{e}_i^{a,g}\mathbf{x}, \mathbf{e}_i^{a,g}\mathbf{y}, \mathbf{e}_i^{a,g}\mathbf{z}]^T, \tag{28}$$

where $\mathbf{e}_{iO}^{a,g}$ – is the unit vector of a measuring axis of accelerometer or gyroscope *i* in projections on the axis of a body axis coordinate system.

3. ADVANCED MODIFICATION OF A TIGHTLY-COUPLED ARCHITECTURE OF THE INTEGRATED SYSTEM

A new modification of a tightly-coupled architecture of SDINS and SNS will open extra potentials of improving the accuracy of the system due to better observability and estimability of systematic errors of SNS resulting from their better corrections and higher accuracy of finding parameters of orientation, navigation and gravimetry. The reliability of SGS will also increase because of their correcting capability remaining even with single available satellite. The special feature of the new modification of a tightly-coupled architecture is a simultaneous use of optimal KOF and the least squares method for processing the readings of measuring subsystems. Thus, a computing module of the satellite receiver, through the least squares method, estimates $\mathbf{R}_C$, $\mathbf{U}_C$, τ_L and $\dot{\tau}_L$ with residual estimation errors which can be found by substituting (15) to (12):

$$\left(\delta\mathbf{R}_C^{\mathrm{T}}, \delta\mathbf{U}_C^{\mathrm{T}}, \delta\tau_L, \delta\dot{\tau}_L\right)^{\mathrm{T}} = \mathbf{NV}. \tag{29}$$

Interestingly, the two bottom lines of the last expression are:

$$\delta\tau_L = \mathbf{N}^{\langle 6\rangle}\mathbf{V} = N_{6,0}\delta d_0 + \cdots + N_{6,N-1}\delta d_{N-1} + N_{6,N}\delta\dot{d}_0 + \cdots + N_{6,2N-1}\delta\dot{d}_{N-1}; \tag{30}$$

$$\delta\dot{\tau}_L = \mathbf{N}^{\langle 7\rangle}\mathbf{V} = N_{7,0}\delta d_0 + \ldots + N_{7,N-1}\delta d_{N-1} + N_{7,N}\delta\dot{d}_0 + \ldots + N_{7,2N-1}\delta\dot{d}_{N-1}, \tag{31}$$

which describe the relation between the residual errors of pseudo-ranges and pseudo-velocities after algorithmic compensation of their main errors, and the estimation errors of the offset of the receiver clock scale due to the least squares method and its drift via elements of **N**-matrix calculated through the least squares method. Thus, considering the estimations of the offset of scale of receiver clock and its drift, the range to a satellite and its velocity relating to the antenna of the moving vehicle, according to SNS:

$$d_{CS} = P_S - \hat{\tau}_L = d_S + \delta d_S - \delta\tau_L; \tag{32}$$

$$\dot{d}_{CS} = \dot{P}_S - \hat{\dot{\tau}}_L = \dot{d}_S + \delta\dot{d}_S - \delta\dot{\tau}_L, \tag{33}$$

have overall errors

$$\delta d_{CS} = \delta d_S - \delta\tau_L = \delta d_S - N_{6,0}\delta d_0 - \ldots - N_{6,N-1}\delta d_{N-1} - N_{6,N}\delta\dot{d}_0 - \ldots - N_{6,2N-1}\delta\dot{d}_{N-1}; \tag{34}$$

$$\delta\dot{d}_{CS} = \delta\dot{d}_S - \delta\dot{\tau}_L = \delta\dot{d}_S - N_{7,0}\delta d_0 - \ldots - N_{7,N-1}\delta d_{N-1} - N_{7,N}\delta\dot{d}_0 - \ldots - N_{7,2N-1}\delta\dot{d}_{N-1}, \tag{35}$$

which depend only on residual errors of the pseudo-ranges and pseudo-velocities. The analysis of observability demonstrates that the errors δd_{CS} and $\delta\dot{d}_{CS}$ are observed values, while their components δd_S and $\delta\tau_L$, $\delta\dot{d}_S$ and $\delta\dot{\tau}_L$ are values observable in combination in pairs with each other. Therefore, within this architecture it is difficult to estimate them individually. However, this fact will not affect the resulting accuracy of function of the integrated system because the latter provides estimation and correction of exactly these combinations which are overall errors of distances to satellites and their velocities relating to the antenna of the moving vehicle calculated from SNS data and through the least squares method. This, in its turn, will allow for improving the accuracy of correction of the integrated system in general. The distances to satellite and its velocity relating to the antenna, as calculated from the data of the integrated inertial satellite system, are:

$$d_{KS}=\sqrt{(R_\xi^S-R_\xi)^2+(R_\eta^S-R_\eta)^2+(R_\zeta^S-R_\zeta)^2}=d_S+\delta d_{KS}; \tag{36}$$

$$\dot{d}_{KS}=\frac{(R_\xi^S-R_\xi)\cdot(U_\xi^S-U_\xi)+(R_\eta^S-R_\eta)\cdot(U_\eta^S-U_\eta)+(R_\zeta^S-R_\zeta)\cdot(U_\zeta^S-U_\zeta)}{\sqrt{(R_\xi^S-R_\xi)^2+(R_\eta^S-R_\eta)^2+(R_\zeta^S-R_\zeta)^2}}= \\ =\dot{d}_S+\delta\dot{d}_{KS} \tag{37}$$

and have overall errors:

$$\delta d_{KS}=\frac{\partial d_{KS}}{\partial R_\xi}\delta R_\xi+\frac{\partial d_{KS}}{\partial R_\eta}\delta R_\eta+\frac{\partial d_{KS}}{\partial R_\zeta}\delta R_\zeta; \tag{38}$$

$$\delta\dot{d}_{KS}=\frac{\partial\dot{d}_{KS}}{\partial R_\xi}\delta R_\xi+\frac{\partial\dot{d}_{KS}}{\partial R_\eta}\delta R_\eta+\frac{\partial\dot{d}_{KS}}{\partial R_\zeta}\delta R_\zeta+\frac{\partial\dot{d}_{KS}}{\partial U_\xi}\delta U_\xi+\frac{\partial\dot{d}_{KS}}{\partial U_\eta}\delta U_\eta+\frac{\partial\dot{d}_{KS}}{\partial U_\zeta}\delta U_\zeta. \tag{39}$$

In the simplest case of an open tightly-coupled integrated system of a new modification, the elements of measuring vector of KOF by range and velocity, can be represented as follows:

$$Z_{dS}=d_{KS}-d_{CS}=\sqrt{(R_\xi^S-R_\xi)^2+(R_\eta^S-R_\eta)^2+(R_\zeta^S-R_\zeta)^2}-P_S+\hat{\tau}_L; \tag{40}$$

$$Z_{\dot{d}S}=\dot{d}_{KS}-\dot{d}_{CS}= \\ =\frac{\left(R_\xi^S-R_\xi\right)\cdot\left(U_\xi^S-U_\xi\right)+\left(R_\eta^S-R_\eta\right)\cdot\left(U_\eta^S-U_\eta\right)+\left(R_\zeta^S-R_\zeta\right)\cdot\left(U_\zeta^S-U_\zeta\right)}{\sqrt{(R_\xi^S-R_\xi)^2+(R_\eta^S-R_\eta)^2+(R_\zeta^S-R_\zeta)^2}}- \\ -\dot{P}_S+\hat{\dot{\tau}}_L. \tag{41}$$

Similarly, for the components of the measurement equation:

$$Z_{dS}=d_{KS}-d_{CS}=d_S+\delta d_{KS}-d_S-\delta d_{CS}= \\ =\frac{\partial d_{KS}}{\partial R_\xi}\delta R_\xi+\frac{\partial d_{KS}}{\partial R_\eta}\delta R_\eta+\frac{\partial d_{KS}}{\partial R_\zeta}\delta R_\zeta-\delta d_S+N_{6,0}\delta d_0+\cdots+N_{6,N-1}\delta d_{N-1}+ \\ +N_{6,N}\delta\dot{d}_0+\cdots+N_{6,2N-1}\delta\dot{d}_{N-1}; \tag{42}$$

$$Z_{\dot{d}S}=\dot{d}_{KS}-\dot{d}_{CS}=\dot{d}_S+\delta\dot{d}_{KS}-\dot{d}_S-\delta\dot{d}_{CS}= \\ =\frac{\partial\dot{d}_{KS}}{\partial R_\xi}\delta R_\xi+\frac{\partial\dot{d}_{KS}}{\partial R_\eta}\delta R_\eta+\frac{\partial\dot{d}_{KS}}{\partial R_\zeta}\delta R_\zeta+\frac{\partial\dot{d}_{KS}}{\partial U_\xi}\delta U_\xi+\frac{\partial\dot{d}_{KS}}{\partial U_\eta}\delta U_\eta+\frac{\partial\dot{d}_{KS}}{\partial U_\zeta}\delta U_\zeta- \\ -\delta\dot{d}_S+N_{7,0}\delta d_0+\ldots+N_{7,N-1}\delta d_{N-1}+N_{7,N}\delta\dot{d}_0+\ldots+N_{7,2N-1}\delta\dot{d}_{N-1}. \tag{43}$$

Thus, switching to a new modification of a closed/open loop tightly-coupled integrated system, as compared to (18), will result into a new form of measuring vector

$$\mathbf{Z}=\begin{pmatrix}\mathbf{d}_K-\mathbf{P}_C+\hat{\tau}_L+\hat{\delta}\mathbf{d}_p-\mathbf{N}^{\langle 6\rangle}\begin{pmatrix}\hat{\delta}\mathbf{d}_p\\ \hat{\delta}\dot{\mathbf{d}}_p\end{pmatrix}\\ \dot{\mathbf{d}}_K-\dot{\mathbf{P}}_C+\hat{\dot{\tau}}_L+\hat{\delta}\dot{\mathbf{d}}_p-\mathbf{N}^{\langle 7\rangle}\begin{pmatrix}\hat{\delta}\mathbf{d}_p\\ \hat{\delta}\dot{\mathbf{d}}_p\end{pmatrix}\end{pmatrix}, \quad (44)$$

where $\mathbf{d}_K=(d_{K0},\dots,d_{KN-1})^{\mathrm{T}}$, $\dot{\mathbf{d}}_K=(\dot{d}_{K0},\dots,\dot{d}_{KN-1})^{\mathrm{T}}$, $\mathbf{P}_C=(P_0,\dots,P_{N-1})^{\mathrm{T}}$, $\dot{\mathbf{P}}_C=(\dot{P}_0,\dots,\dot{P}_{N-1})^{\mathrm{T}}$. Here, in (18), the estimation equations $\delta\mathbf{R}_{Cp}$ and $\delta\mathbf{U}_{Cp}$ should be replaced with the equations of estimation of systematic components of residual errors of pseudo-ranges $\delta\mathbf{d}_p=(\delta d_{p0},\dots,\delta d_{pN-1})^{\mathrm{T}}$ and pseudo-velocities $\delta\dot{\mathbf{d}}_p=(\delta\dot{d}_{p0},\dots,\delta\dot{d}_{pN-1})^{\mathrm{T}}$ after algorithmic compensation of main errors in the SNS computer module, caused by all reasons except the influence of the offset and drift of the receiver clock:

$$\dot{\hat{\delta}}\mathbf{d}_p=\mathbf{K}^{\langle 45\dots(45+N-1)\rangle}\mathbf{Z}; \quad (45)$$

$$\dot{\hat{\delta}}\dot{\mathbf{d}}_p=\mathbf{K}^{\langle (45+\mathrm{N})\dots(45+2N-1)\rangle}\mathbf{Z}, \quad (46)$$

where, similar to (18), the errors are represented with constant values, but, when necessary, more complex models can be assigned to them including with correlated random component. Here we should note that the observability and estimability of the integrated system, though not being part of this paper, have shown that in case of loosely-coupled architecture, a good estimability of SNS errors by coordinates is possible only with highly accurate initial alignment of SDINS or with a high velocity and extremely intense maneuvering of the moving vehicle. At the same time, within the considered case of a tightly coupled architecture for good estimability of SNS errors by ranges to satellites, neither preparatory highly accurate alignment of SDINS nor movement of a SGS-carrying vehicle will be required (including a parked vehicle). Eventually, in most cases the presented modification of a tightly coupled architecture of a system provides for a better resulting accuracy of finding coordinates of the moving vehicle and vertical anomaly of the acceleration due to gravity. The estimability and quality of error corrections of the relative velocity, orientation and horizontal projections of anomaly of the acceleration due to gravity will remain at close levels for both cases.

4. ERROR MODEL OF FUNCTIONALLY REDUNDANT ACCELEROMETER MODULES

Vector gravimetry is defined as finding the vector of the acceleration due to gravity from the solution (18) based on the subsystems data. For this purpose, first of all, it is necessary to have a highly accurate dimension of an apparent acceleration of the vehicle. One of the ways of improvement of the accuracy and reliability of measurement of the apparent acceleration is to use accelerometer modules with redundant functionality. Thus, to study potentials of the improvement of the apparent acceleration accuracy, let us focus on the specifics of the design and operation of functionally redundant accelerometer modules. The mathematic model of an instrumental error of the measurement of the apparent acceleration vector (below indicated as Δ) with a functionally redundant accelerometer module has the form [10]:

$$\Delta\mathbf{n}_O=\mathbf{H}_a(\Delta\mathbf{a}-\delta\mathbf{E}_a\mathbf{n}_O); \quad (47)$$

$$\delta\mathbf{E}_a^{\mathrm{T}}=[\delta\mathbf{e}_1^a,\dots,\delta\mathbf{e}_{\mathrm{i}}^a,\dots,\delta\mathbf{e}_k^a]; \quad (48)$$

$$\delta\mathbf{e}_{\mathrm{i}}^{a} = \left[\alpha_{iy}\mathbf{e}_{iz}^{a} - \alpha_{iz}\mathbf{e}_{iy}^{a}, \alpha_{iz}\mathbf{e}_{ix}^{a} - \alpha_{ix}\mathbf{e}_{iz}^{a}, \alpha_{ix}\mathbf{e}_{iy}^{a} - \alpha_{iy}\mathbf{e}_{ix}^{a}\right]^{\mathrm{T}}, \tag{49}$$

where α_{ix}, α_{iy}, α_{iz} are projections of a small rotation vector $\boldsymbol{\alpha}_i$, characterizing the alignment error of a measuring axis of accelerometer i of an accelerometer module in a body axis coordinate system; $\boldsymbol{e}_{ij}^{a}$ is a unit vector of direction of a measuring axis of the accelerometer i on axis j of a body axis coordinate system.

$$\Delta\mathbf{a} = \Delta\mathbf{a}_0 + \Delta\mathbf{a}_1 + \Delta\mathbf{a}_2 t + \mathrm{Diag}\Delta\mathbf{a}_3\mathbf{E}_a\mathbf{n}_O + \mathrm{Diag}\Delta\mathbf{a}_4(\mathbf{E}_a\mathbf{n}_O)^2 \tag{50}$$

the instrumental errors vector of accelerometers includes: fluctuation of the zero signal $\Delta\mathbf{a}_0$, zero offset $\Delta\mathbf{a}_1$, drift of a zero signal $\Delta\mathbf{a}_2$, error of the scale coefficient $\Delta\mathbf{a}_3$, non-linearity coefficient $\Delta\mathbf{a}_4$ of an output characteristic of the accelerometers; Diag is a diagonal matrix comprised of the vector's elements. The errors of the apparent acceleration vector in a body axis coordinate system, depending on instrumental errors of the sensors of a functionally redundant accelerometer module, correspond to the following relations:

$$\Delta\mathbf{n}_O = \mathbf{H}_a\Delta\mathbf{a}; \tag{51}$$

$$\mathrm{D}_{\Delta\boldsymbol{n}_O} = \mathrm{SpCov}\Delta\mathbf{n}_O; \tag{52}$$

$$\mathrm{Cov}\Delta\mathbf{n}_O = M\left[\Delta\boldsymbol{n}_O\Delta\boldsymbol{n}_O^{\mathrm{T}}\right] = \boldsymbol{H}_a\mathrm{Cov}\Delta\mathbf{a}\mathbf{H}_a^{\mathrm{T}}, \tag{53}$$

where D is dispersion; Sp is matrix spur; Cov is covariance matrix. With measurements of equal accuracy typical for sensors made under the same technology the analytic expression of the dispersion of errors of the apparent acceleration vector will take the form:

$$\mathrm{D}_{\Delta\mathbf{n}_o} = \mathrm{D}_{\Delta\mathbf{a}}\mathrm{Sp}(\mathbf{H}_a\mathbf{H}_a{}^{\mathrm{T}}) = \mathrm{D}_{\Delta\mathbf{a}}\mathrm{Sp}(\mathbf{E}_a\mathbf{E}_a{}^{\mathrm{T}})^{-1}, \tag{54}$$

which, for a cone-shaped accelerometer module from k accelerometers (Figure 2) the measuring axis of which are evenly distributed on the cone generators, will result into:

$$\mathrm{D}_{\Delta\mathbf{n}_O} = \frac{\mathrm{D}_{\Delta\mathbf{a}}}{k}\left(\frac{1}{\cos^2\chi} + \frac{4}{\sin^2\chi}\right), \tag{55}$$

where χ is the cone's semiapex angle. Whence it follows that the accuracy of the accelerometer module increases with an increase in the number of accelerometers in a module. Here it is possible to optimize the design (geometry) of the accelerometer module in terms of specifics of instrumental errors of the module's sensors.

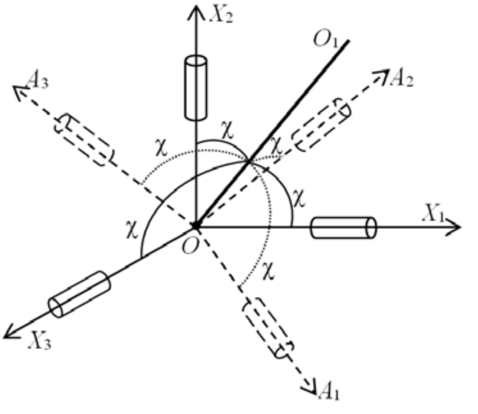

Fig. 2 – Cone-shaped modules comprised of 3 (along axes OX_1, OX_2, OX_3) or 6 (along axes OX_1, OX_2, OX_3, OA_1, OA_2, OA_3) inertial sensors

In this figure the axes OX_1, OX_2, OX_3 are evenly distributed on a generator of the cone surface with semiapex angle χ, the axis OO_1 is the central axis of the cone, the axes OA_1, OA_2, OA_3 also lie on cone's generator and are evenly distributed between axes OX_1, OX_2, OX_3. Let us assume that the model of random errors of accelerometers, besides fluctuation of a zero signal σ_0, also contains a scale coefficient fluctuation σ_M,. In such case the accelerometers error dispersion is

$$\mathrm{D}_{\Delta a_i} = \sigma_0^2+\sigma_M^2 a_i^2 \cos^2\chi=\sigma_0^2(1+K^2\cos^2\chi). \quad (56)$$

Besides, the ratio of the mean square values of the indicated errors is characterized by the coefficient

$$K = \frac{\sigma_M a_i}{\sigma_0}. \quad (57)$$

As shown in [10], the dispersion of a measurement error of vector $\mathbf{n}_O$ takes a minimum value at the semiapex angle χ_{opt} of cone of measuring axes of raw data sensors, which corresponds to the following analytic expression:

$$\chi_{opt} = \arccos\frac{1}{[2(K^2+1)^{1/2}+1]^{1/2}}. \quad (58)$$

In a particular case (K=0) the relation (58) corresponds to the results obtained earlier, for instance, in [11], then, χ_{opt}=54°45′, which corresponds to a traditional orthogonal structure for a three-sensor accelerometer module. If K increases, this value may increase up to 90°. In a general case, the optimum semiapex angle of the cone depends on relation between coefficient of statistic model of measurement errors and the value of measured vector [10].

The estimations of optimum configurations of cone-shaped IMU suggest that as an IMU becomes more redundant, the effectiveness of the considered technical solution increases. For instance, comparing with a three-sensor IMU with K=0, an error influence in a four-sensor IMU will decrease by 14%, in a five-sensor, by 23%, in a six-sensor, by 29%. With a growing K these numbers somewhat decrease: for K=5 and K=10 those are correspondingly 10%, 18%, 23% and 8%, 16%, 21%. However, the undoubted advantage of such technical solution is the potential increased accuracy of measurements of the apparent accelerometer vector in relation to a random error, merely through optimal choice of design of a functionally redundant accelerometer module. Obviously, the module's reliability increases sharply, because, in contrast to a three-sensor module where no single sensor failure is acceptable, a 4-6 sensor module remains operable even if 1-3 sensors fail.

The study of configuration of a cone-shaped accelerometer module in terms of systematic errors of measurements showed that in case of a suboptimum semiapex angle of the cone of an accelerometer module's measuring axes, the requirements to alignment accuracy and offset of sensors' zero for 3-6-sensor modules are somewhat different. Notably, such standards are significantly low for 4-sensor modules (up to – 38%), somewhat low for 5-sensor modules (up to – 8%), significantly higher for 6-sensor modules (within +25%) as compared with a regular three-sensor module. However, as χ goes to optimum value of 54°45′, the requirements for 3-5-sensor modules virtually do not differ, for 6-sensor modules such requirements are higher by mere percents, and become virtual identical for the whole group when exceeded 54°45′.

Therefore, since, depending on K, it is practical to use $\chi \geq$54°45′, it is obvious that the degree of redundancy will not affect systematic errors of the module. Here, the alignment accuracy requirements to functionally redundant IMUs, while being high in general, conflict with feasibility of observing them through high-precision processing alignment bases of measuring elements. Such limitations can be addressed through analytical alignment and calibration of accelerometer modules [12], when an alternative solution is the analytical identification of real parameters of orientation of sensors' measuring axes in a setting coordinate system of IMUs [12], [13], [14], [15], [16].

5. CONCLUSIONS

The paper has studied the structure and principles of design of an airborne strap-down graviinertal navigation system of a basic configuration. There were presented functional algorithms of the system in a case of traditional loosely-coupled architecture of its inertial and satellite components. There was suggested a version of future modification of a tightly-coupled architecture which opens up extra potentials of estimation and correction of errors of a satellite navigation system resulting into increased overall accuracy and reliability of finding parameters of orientation, navigation and gravimetry.

There were also studied potentials of increased accuracy and reliability of SGS as a component of functionally redundant cone-shaped accelerometer modules. The paper shows that, based on requirements to SGS, an optimum cone structure of accelerometer module may be found, having acceptable margins of random and systematic errors. To increase the accuracy and reliability of the system it is practical to combine algorithmic (optimum estimation and correction of errors) and hardware (increase in the number of sensors and finding the optimum cone's semiapex angle) approaches.

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