Modelling and Analysis Library Development for Helicopter Blade and Slender Wings

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Abstract: *In this study, we suggest a framework for optimal design of a composite wing structure with a high aspect ratio at the initial design stage. The optimization framework calculates allowances in the first ply failure, buckling failure and bearing-bypass failure, which are frequent failure modes in the wing structure, using the laminate pattern database built up through past design experiences, and optimizes the weight of the wing structure within the range of the economic minimal safety margin. In order to verify the optimization framework, finite element analysis for the wing structure of an unmanned air vehicle was configured. Subsequently, static analysis and stability analysis were performed to verify robustness and reliability of the optimization framework by entering the composite material laminate data created from the optimization framework into the finite element model of the wing of an unmanned air vehicle. The optimization framework suggested in this study is an automated algorithm that can carry out sizing of various shapes composed of composite material from the concept design stage, and thus reduce the revised and repeated design time.*

Key Words: *Slender wing, Composite, Helicopter blade, Dimensional Reduction, Dimensional Recovery*

1. INTRODUCTION

When it comes to the aircraft structural design process, the finite element analysis is generally facilitated for accurate stress and dynamic analysis. Developing tailored 3-D finite element analysis model similar to the actual structure to have sophisticated analysis takes a lot of time and human resources, and many experiences and know-how required for forming reliable analysis model. In addition, precise three-dimensional model often produces unsatisfying results, in case of identifying qualitative and physical tendency on analysis results or extraction of data for application interpretation such as Aeroelastic analysis. Especially when analyzing the helicopter blade with composite material and aircraft wing structure as a beam structure, it requires to adequately reflect the coupling effect for mutual directions and non-classical effects which hardly seen on general metal structure become very important elements of analysis such as transverse shear deformation, in-plane, out of plane warping and warping constraints even it has high aspect ratio [1, 2]. In addition, same as in preliminary design phase, specific data for structural and aerodynamic configuration are insufficient and if there are frequent changes in configuration, it is merely possible to change three-dimensional structure model each time. Among related studies, Chen [3] pronounced the features of PreComp, VABS (Variational Asymptotical Beam Sectional Analysis), FAROB, CROSTAB and BPED based on comparative study on sectional analysis code for dimension reduction methodology and compared numerical results of sectional analysis. Hu [4] conducted the study on VABS-IDE which optimizes the process of cross-sectional analysis and recovery utilizing GEBT [5] and VABS [6] which calculating one-dimensional beam's movement by non-linear analysis. Hodges et al. [7] introduced a mathematical method and presented a variational asymptotic method based on finite element model. Cesnik, Hodges [8] and Yu [9] further developed variational asymptotic method. It is the core of variational approximation modelling in which cross-sectional analysis on complicated three-dimensional configuration conducted to have two-dimensional elastic coefficient and for applying it to the equation of motion of one-dimensional non-linear beam and copying its movement. From the results of these studies, VABS based on finite element method was developed.

Carrera [10] made a comparison between the movement of thin-wall beam and three-dimensional finite element model with advanced shape function and proved the efficiency of one-dimensional beam. Pollayi and Yu [11] utilized VABS code and looked into the crack problem of composite material rotor blade. Volovoi et al. [12] Volovoi and Hodges [13, 14] presented accurate sectional analysis result of thin-wall beam. Chandra and Chopra [15], Centolanza and Smith [16] produced a successful result of beam equation having displacement with 9 variables. Yu and others [17] utilized variational asymptotic method of beam and tensor to arrange the process of beam modeling and theoretically described recovery relation in detail and made a comparison between the result of sectional analysis of box-type composite beam and results of previous studies.

The analysis library in this paper was conducted based on the discretized element and there is something in common with VABS [18] developed in previous studies. In connection with stiffness matrix derived from dimensional reduction process, one-dimensional model can be made, and recovery analysis process conducted in association with the displacement of beam analysis. By mapping the result of recovery analysis on discretized element, visualized results can be achieved. We would like to present the case studies of analysis library which have efficiency and accuracy by comparing stress calculated by 1-D beam's dimensional recovery analysis with 3-D finite element model.

2. ANALYSIS APPROACH

The basic approach for the problems of high aspect ratio wing and the rotor blade design are described in Fig. 1. Three-dimensional design problems are subdivided into two-dimensional section and one-dimensional beam analysis problems. The basic premise of this approach is that the properties of cross-section showing the structure domain of wing which have high aspect ratio, high ratio of length vs. width and height and rather flat longitudinal direction can be defined. In general, it is applied on wing or rotor blade of high altitude long endurance UAVs which have high aspect ratio. As in Fig. 1, the cross-section of the rotor blade is analyzed based on the structure topology, lay-out, material and ply thickness. After that, it is discretized by elements composed of triangle or quad and material properties are applied and properties of mass and stiffness, shear center and the center of gravity are decided. 4x4 or 6x6 of mass matrix and stiffness matrix are created separately. Then, these mass and stiffness matrix are allocated on beam element of one-dimensional model as a property. As the process described in Fig. 1, the design efficiency of composite material blade can be maximized by dimension reducible modeling process.

Through the stress of dimension reduced one-dimensional beam model, strain and recovery, the analysis results can be visualized by mapping the three-dimensional numerical results with one-dimensional beam model.

By inputting design load of blade, when stress and strain based on the stiffness matrix calculated by dimensional reduction model meets the requirements of strength, vibration and fatigue then, three-dimensional stress or recovery strain maps to section of blade composed with finite element.

The design variables set during these analysis processes can be modified, sectional analysis iterated and can determine design variables which meet the optimized structural requirement and calculate structural margin of safety.

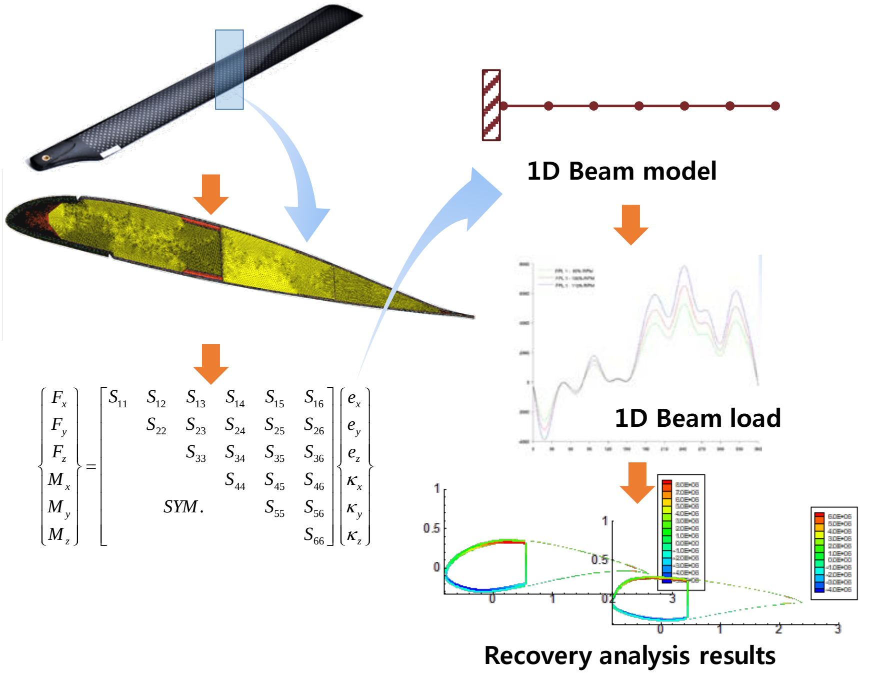

Fig. 1 Blade analysis procedure including dimensional reduction model & recovery analysis procedure

3. THEORY

3.1 Cross-sectional analysis for dimensional reduction

The accuracy of one-dimensional beam analysis can be determined whether the cross-section stiffness matrix is calculated correctly. The power and moment applied to the cross-section of beam have a linear relationship with stiffness matrix Ks. The stress and displacement applied on the one point of cross-section of the beam can be represented as three-dimensional tensor as below and Hooke's law applied. The total displacements are presented as

$$s = Zr + Nu \tag{1}$$

and as described in here, Zr is a displacement from rigid body motion and Nu is a displacement from warping in or out of the face direction. N is a shape function. When it is in micro-displacement, the quantity of strain can be presented as a type of tensor as below.

$$\varepsilon_{\alpha\beta} = 1/2\left(\partial s_\alpha/\partial\beta + \partial s_\beta/\partial\alpha\right), (\alpha, \beta = x, y, z) \tag{2}$$

$\varepsilon_{\alpha\beta}$ is a micro-displacement and shows a type of partial differentiation.

The strain can be arranged in partial diffrentiation of displacement and stress components and described as below.

$$\varepsilon = SZ\psi + Bu + SN\frac{\partial u}{\partial z} \tag{3}$$

In here, u is the nodal warping and displacements, N is the shape functions and $\psi = (T_r + \frac{\partial}{\partial z})r, T_r r = \left[0,0,0,\tau_y,-\tau_x,0\right]$ and the strain-displecement matrix B is as below.

$$B = \begin{bmatrix} \partial/\partial x & 0 & \partial/\partial y & 0 & 0 & 0 \\ 0 & \partial/\partial y & \partial/\partial x & 0 & 0 & 0 \\ 0 & 0 & 0 & \partial/\partial x & \partial/\partial y & 0 \end{bmatrix}^T$$

3.2 A formulation of the principle of virtual work

To calculate F_s which is the inverse matrix of the stiffness matrix, the cross-section equilibrium equation has to be advanced. While doing that, the two-dimenstional paritial differential equation is calculated by facilitating the principle of virtual work [20]. The priciple of virtual work is $\delta W = \delta W_e + \delta W_i = 0$ and is arranged as the matrix below, where W_i is the internal elastic forces work, and W_e is the external forces work acting on the cross section. The virtual work based on internal force is described in formula (4).

$$\delta W_i = \int_\Omega \delta\varepsilon^T \sigma d\Omega = \begin{bmatrix} \delta u_{\partial z} \\ \delta u \\ \delta\psi \end{bmatrix} \begin{bmatrix} M & C & L \\ C^T & E & R \\ L^T & R^T & A \end{bmatrix} \begin{bmatrix} u_{\partial z} \\ u \\ \psi \end{bmatrix} \tag{4}$$

In here, the components of the matrix are arranged as below.

$$A = \sum_{e=1}^{n_e} \int_A Z_e^T S_e^T Q_e Z_e dA, \quad R = \sum_{e=1}^{n_e} \int_A B_e^T S_e^T S_e Z_e dA$$

$$E = \sum_{e=1}^{n_e} \int_A B_e^T Q_e B_e dA, \quad C = \sum_{e=1}^{n_e} \int_A B_e^T Q_e N_e dA$$

$$L = \sum_{e=1}^{n_e} \int_A Z_e^T S_e^T Q_e N_e dA, \quad M = \sum_{e=1}^{n_e} \int_A N_e^T S_e Q_e N_e dA$$

n is the number of discretized element and n_e is the number of nodes.

The virtual work applied by external force of the cross-section can be arranged as below.

$$\delta W_e = \int_A \frac{\partial(\delta s^T p)}{\partial z} dA = \begin{bmatrix} \delta u_{\partial z} \\ \delta \\ \delta\psi \end{bmatrix} \begin{bmatrix} P \\ P_{\partial z} \\ \theta \end{bmatrix} + \delta r^T (\theta_{\partial z} - T_r^T \theta) \tag{5}$$

The virtual work applied by internal and external force can be drawn out as a type of two-dimensional linear partial differential equation and utilized for calculating the stiffness matrix.

$$
\begin{aligned}
&Mu_{\partial^2 z} + (C - C^T)u_{\partial z} + L\psi_{\partial z} - Eu - R\psi = 0 \\
&L^T u_{\partial z} + R^T u + A\psi = 0 \\
&\theta_{\partial z} = T_r^T \theta
\end{aligned}
\tag{6}
$$

3.3 Equilibrium equation of cross-section

The constraints are applied to the formula (6) calculated by virtual work. The relation between the stress and the number of strains is described in formula (7).

$$
Kw = f \rightarrow \begin{bmatrix} K_{11} & K_{12} \\ 0 & K_{11} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{7}
$$

In here, $K_{11} = \begin{bmatrix} E & R & D \\ R^T & A & 0 \\ D^T & 0 & 0 \end{bmatrix}, K_{12} = \begin{bmatrix} (C^T - C) & -L & 0 \\ L^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $w_1 = \begin{bmatrix} u^T & \psi^T & \lambda_u^T \end{bmatrix}^T$,

$w_2 = \begin{bmatrix} \partial u^T / \partial z & \partial \psi^T / \partial z & \lambda_{u\partial z}^T \end{bmatrix}^T$, $f_1 = \begin{bmatrix} 0^T & \theta^T & 0^T \end{bmatrix}^T$, $f_1 = \begin{bmatrix} 0^T & (T_r^T \theta)^T & 0^T \end{bmatrix}^T$. With the formula (7) which derived from the principle of virtual work between the external forces and internal forces, the inverse matrix stated as F_s can be calculated.

$$
\delta\theta^T F_s \theta = \delta\theta^T W^T G W \theta \tag{8}
$$

In here,
$$
G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{bmatrix}, G_{11} = \begin{bmatrix} E & R & D \\ R^T & A & 0 \\ 0 & 0 & 0 \end{bmatrix},
$$
$$
G_{12} = \begin{bmatrix} C & L & 0 \\ L^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, G_{32} = \begin{bmatrix} M & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

The inverse matrix of stiffness matrix can be shown as formula (8) and by utilizing this, the cross-sectional matrix can be calculated as discretized finite element.

$$
F_s = W^T G W, K_s = F_s^{-1} \tag{9}
$$

3.4 Strain and stress recovery

When θ is determined by the equilibrium equation of the beam, the strain of the cross section is arranged as followed. Stress and strain can be calculated based on the Gausian integral or element.

$$
\varepsilon_e = S_e Z_e \psi_e \theta + B_e U_e \theta + S_e N_e U_{\partial z,e} \theta \tag{10}
$$

In here, U_e , $U_{\partial z,e}$, ψ_e is the matrix when calculating the degree of freedom of element and stress can be calculated by the relation of $\sigma_e = Q_e \varepsilon_e$.

4. ANALYSIS LIBRARY DEVELOPMENT OF DIMENSION REDUCIBLE MODELLING AND RECOVERY ANALYSIS

4.1 Analysis library configuration

The procedure of calculating ABD matrix and mass matrix is required to dimensionally reduce three-dimensional composite material blade model into one-dimensional beam. At this time, the process of the two-dimensional discretization is required, and the discretized model has to be constructed with finite elements on the assumed position. The finite element model has to be made by pre-processing tool with the consideration of properties of various composite materials and configuration information. The analysis library of blade and wing structure was developed based on Matlab. Unlike the independent program developed by local researchers, there is no function of blade configuration modelling.

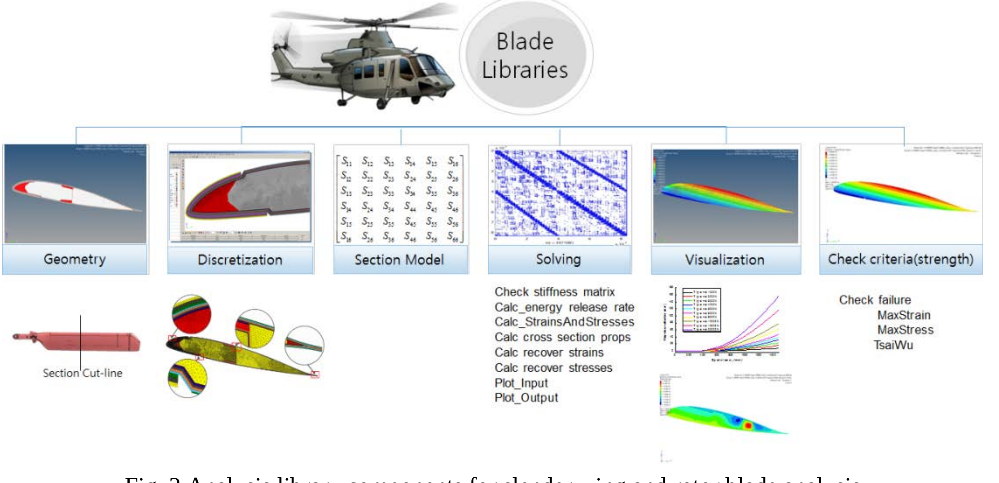

Fig. 2 Analysis library components for slender wing and rotor blade analysis

The actual developed blade modelling is precisely designed by laminate rule internally and externally and it is required to conduct cross-section analysis right after reading the design data in order to correctly describe without distortion of configuration. The blade analysis library includes modules which analyze components and grid generated by the pre-processing analysis program. The hierarchy of the analysis library is described in Fig. 2. The analysis library includes the parts where the nodes and elements are inspected, the stiffness of the configuration is calculated with triangular and square elements, the stress and strain are calculated by inputting load and constraints and the resulted data are visualized. In addition, the max stress, max strain and criteria of Tsi-Wu selected to determine abnormalities of structure.

4.2 Input of Analysis Library

To connect the 1-D beam modelling by dimensional reduction, the number of elements and nodes of the finite element model has to be adequately divided to express the properties of the configuration. It is a pre-requisite to reduce the number of nodes and elements during the process of optimum design, but it is required to have an adequate number of elements to correctly express the coupled properties of the beam. In Fig. 3, the discretization cross-section of the blade is described which is the same as the actual blade. The number of elements, nodes and each element of the triangle and nodes are inputted in the analysis library. The information of nodes (node number, y and z) is entered. As in Fig. 3, the elements of the triangle and square can be applied. To enhance the accuracy of analysis, six (6) elements of triangle and eight (8) elements of square can be applied. This is intended to enhance the accuracy of the stress and strain after the execution of the dimensional recovery by the quadric shape function. This coordinate transformation of the shape function allows the base function within each finite element to be converted to a master element, and vice versa, to be converted to each finite element within the network of any function defined in the master element. By coordinate transformations of defined geometry functions on these standardized master elements, the underlying functions within each finite element can be defined very easily and systematically.

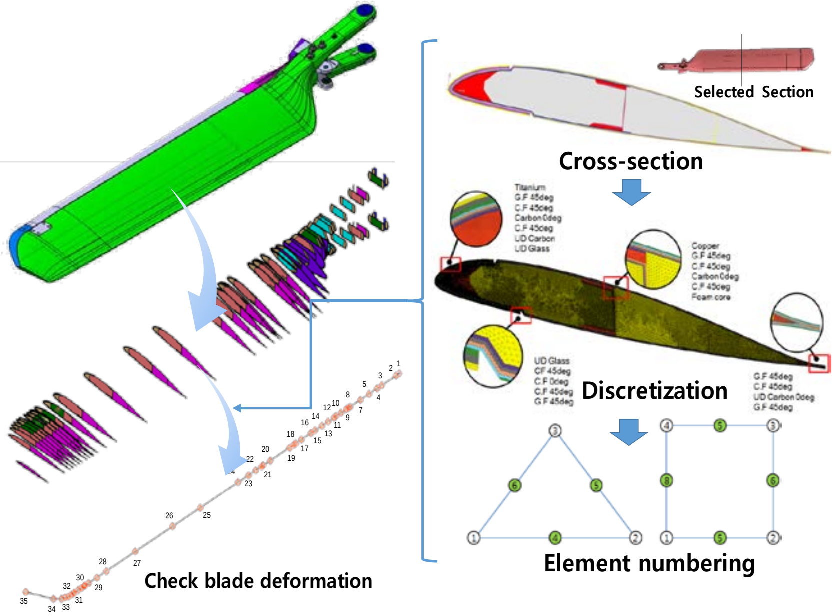

Fig. 3 Example of cross-section discretization of rotor blade and numbering system of triangle and square

The applied quality of material is inputted as isotropic and anisotropic. The material is the same as the value of general composite materials and inputted as in the order below.

$$(E_{11}, E_{22}, E_{33}, G_{12}, G_{13}, G_{23}, \upsilon_{12}, \upsilon_{13}, \upsilon_{23}, \rho)$$

In Fig. 4, the element number, material number, fiber angle and fiber plane angle are inputted to process the applied material information on each element.

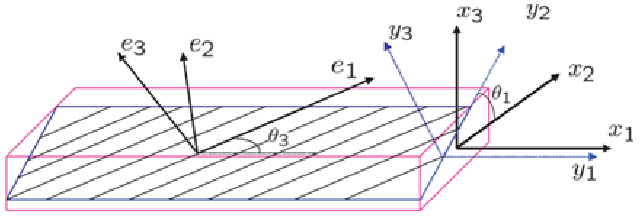

Fig. 4 Direction and angle of composite material layer

4.3 Results of analysis library for slender wing structure

The result of the beam modelling on composite material structure through the dimensional reduction is the geometry information based on the position of the cross-section, stiffness matrix and mass matrix. During the process of the cross-sectional analysis, the extent, inertia value and weighted coefficient are calculated, together with the extension, bending stiffness and tension center and warping vector by facilitating shear stiffness, torsion stiffness and shear center. The result of blade analysis library includes 6 by 6 matrix and stiffness matrix as below and has the same format of stiffness matrix which derived from classic composite mechanics.

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & SYM. & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

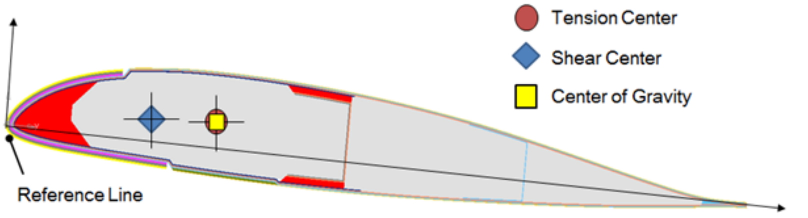

Fig. 5 The configuration and coordinates of cross-section of helicopter rotor blade

Along with the stiffness matrix, the elastic center, center of gravity and mass per unit length are calculated. It is printed as in text format in analysis library. In Fig. 5, the geometry information of the cross-section configuration of the blade and the geometry information based on the axis of the blade lead edge are described. When constructing dimensionally reduced one-dimensional beam model, the model is formed by connecting tension centers of each cross-section based on the cross-section of the blade. Once one-dimensional blade beam model constructed, the distribution of strain and stress can be calculated by inputting load of the blade constructed on each cross-section. The stress and strain calculated in blade analysis library can be mapped on the finite element of cross-section. In addition, it will be printed to match with the format of post-processing program to check the result of the analysis. In Fig. 6 it can be seen the result of visualized strain recovery which was mapped with the result of numerical analysis utilizing analysis library.

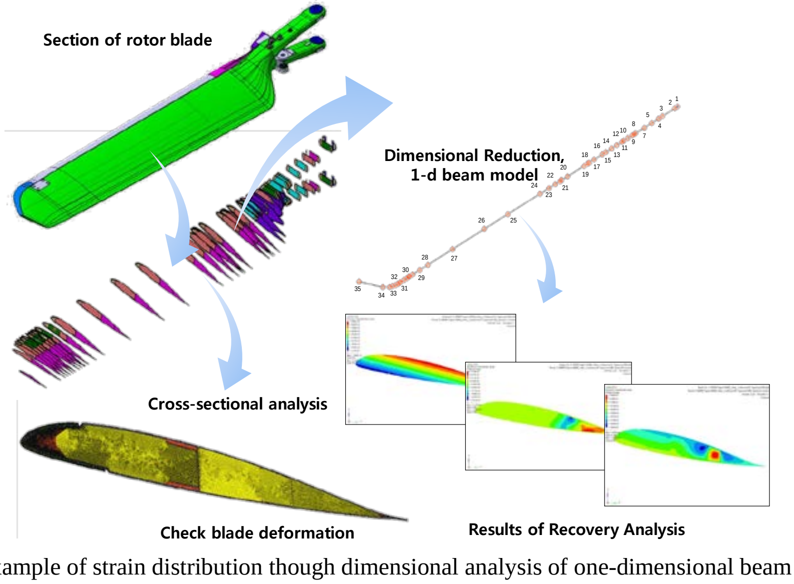

Fig. 6 Example of strain distribution though dimensional analysis of one-dimensional beam model

5. NUMERICAL ANALYSIS FOR VERIFICATION

5.1 The comparison on the result of cross-section analysis on which isotropic and anisotropy materials applied

The section of 1-D model in Fig. 7 with composite materials which stacked at 45 degrees applied on the top and bottom of cross-section.

This is one of the most common configurations which copied the blade configuration since it is the thin structure with three (3) cells. The relative error calculated based on stiffness matrix which is calculated in VABS.

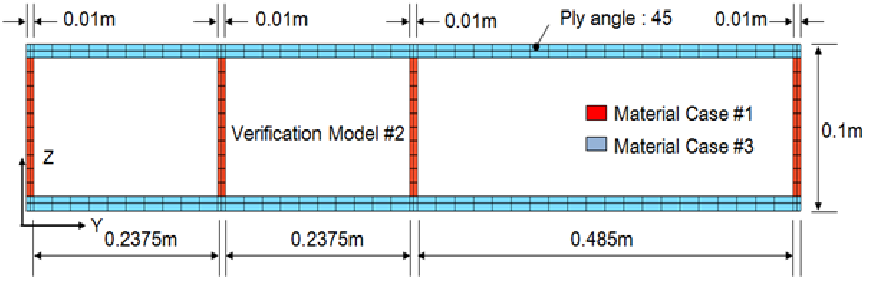

Fig. 7 2nd Cross-sectional analysis verification model of analysis library

The result by cross-section analysis is utilized on one-dimensional beam model. To construct one-dimensional beam model, 6 by 6 of stiffness matrix is connected to the center of elasticity. Once a one-dimensional blade beam model is constructed, strain and stress distribution can be calculated by entering the load of blades produced in each section. Stress results and strain results calculated from the blade analysis library can be mapped to finite elements of the section.

Table 1. Stiffness comparison of cross-section of thin beam calculated by Analysis library and VABS

Stiffness	VABS	Analysis Library	Relative Error(%)
$K_{S,11}$	1.79E+0	1.79E+0	5.58E-10
$K_{S,22}$	3.22E-1	3.22E-1	1.24E-9
$K_{S,33}$	4.37E+0	4.37E+0	2.29E-10
$K_{S,44}$	5.23E-2	5.23E-2	7.65E-10
$K_{S,55}$	3.81E-1	3.81E-1	-7.87E-10
$K_{S,66}$	7.73E-2	7.73E-2	0.00E+0
$K_{S,13}$	-8.48E-1	-8.48E-1	1.18E-10
$K_{S,15}$	2.75E-3	2.75E-3	1.02E-08
$K_{S,24}$	-3.13E-3	-3.13E-3	1.60E-09
$K_{S,26}$	-1.49E-2	-1.49E-2	-2.68E-09
$K_{S,35}$	5.64E-2	5.64E-2	-1.77E-10
$K_{S,46}$	1.83E-2	1.83E-2	5.47E-10

Table 2. Comparison of center of elasticity and shear calculated by analysis library and VABS

Coord.	VABS	Blade Analysis Library	Relative Error(%)
y_t	-1.45E-02	-1.45E-02	0.00
z_t	0.00E+0	0.00E+0	0.00

y_s	-4.30E-2	-4.30E-2	0.00
z_s	0.00E+0	0.00E+0	0.00

As shown in table 4, it can be evaluated that the relative error rate of the analysis library calculated by 2nd verification model and the stiffness matrix calculated by VABS is very small and the result is satisfactory.

As described in table 5, the centers of elasticity and shear which are calculated by analysis library and VABS are the same.

5.2 Comparison of three-dimensional stress and recovery stress of the beam with isotropic material

In Fig. 8, two different models which applied isotropic and anisotropic materials were created to compare the results of three-dimensional finite element analysis, dimensional reduction and recovery analysis.

It is constructed with 10 x 10mm of cross-section and the length is 500 mm of the beam shape.

The material and configuration of the beam described in table 1, case 1 is created with one isotropic material and case 2 is created with two isotropic materials based on neutral cross-section of the beam.

The three-dimensional finite element model consists of 5001 EA of solid elements and the one-dimensional model consists of 40 EA of beam elements. The end is fixed with boundary condition and the other end load of 10N.

In Fig. 8, the configuration of three-dimensional cross-section, one-dimensional variational approximation modelling and applied load are described.

The stress result of three-dimensional finite element model was extracted from a position of 250mm in the centre of the axial direction and the result of one-dimensional variational approximation modelling was also executed recovery analysis on the neutral axis.

Through Fig. 9 and 10, the stress of three-dimensional model and one-dimensional recovery shows the same configuration and it was confirmed that those results had a margin of error less than 1%.

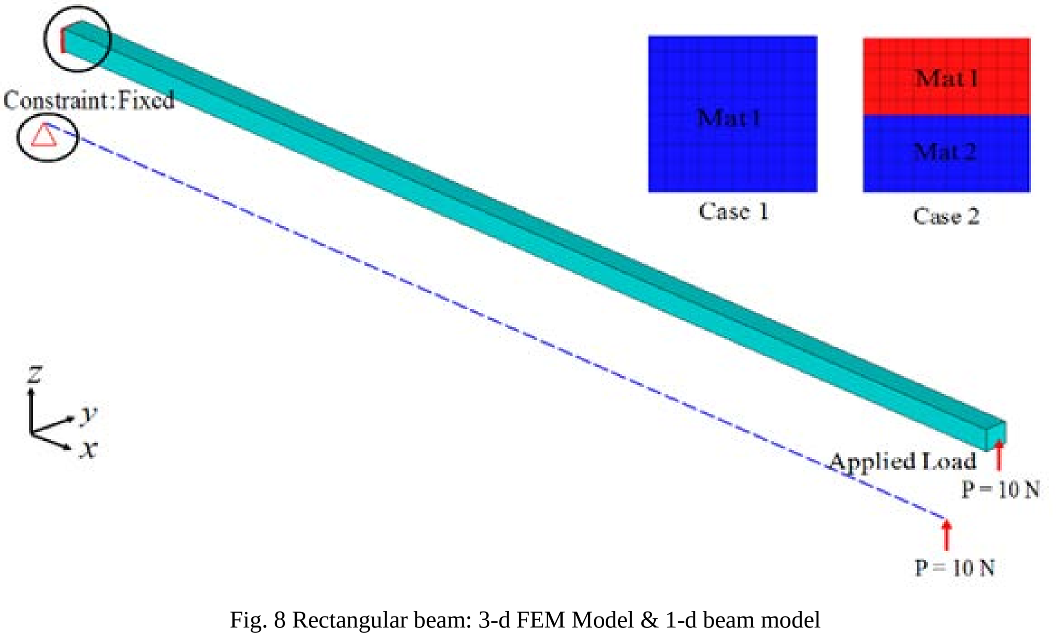

Fig. 8 Rectangular beam: 3-d FEM Model & 1-d beam model

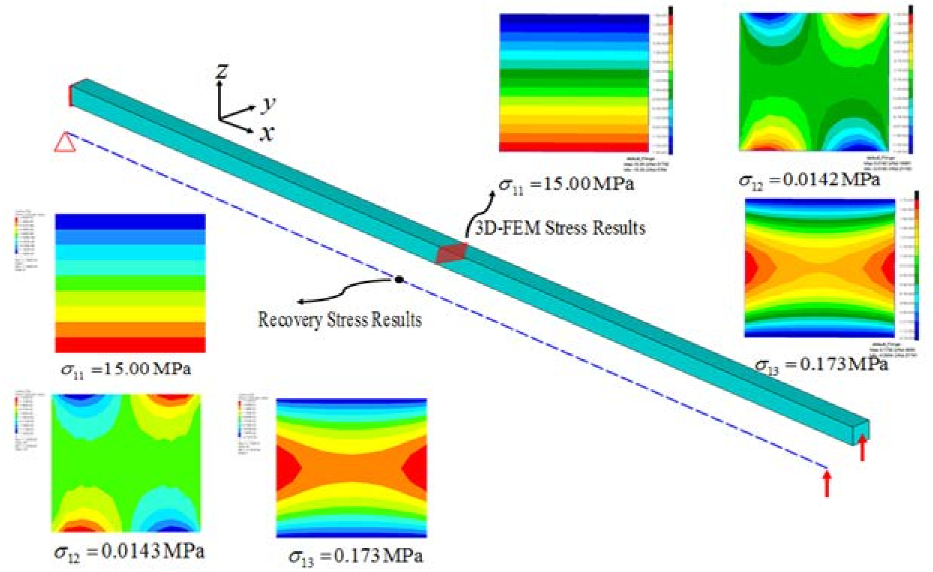

Fig. 9 Stress result at span = 250 mm, Case 1

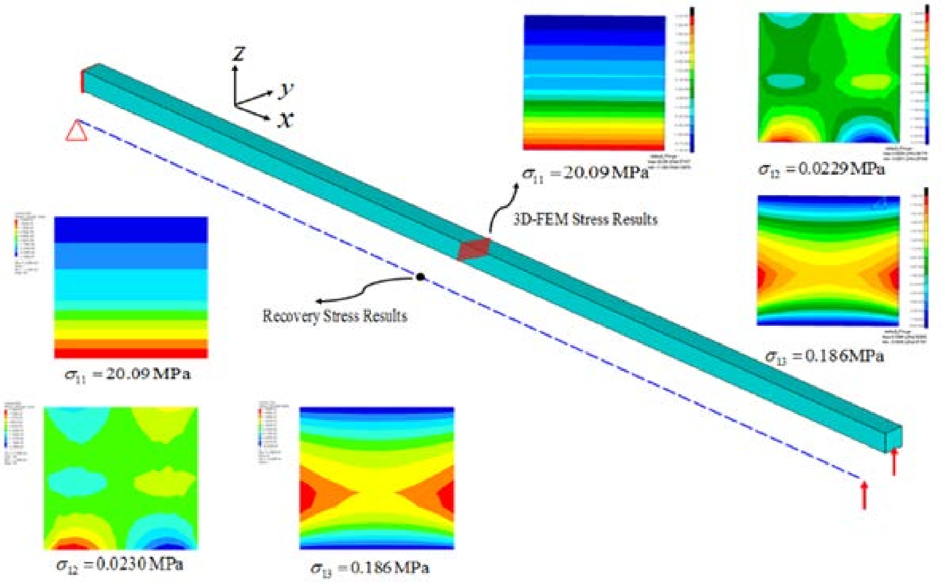

Fig. 10 Stress result at span = 250 mm, Case 1

Table 3. Material Property and Geometry

Properties	Material 1	Material 2
E	71709 MPa	195122 MPa
G	27370 MPa	75842 MPa
ν	0.31	0.28
Length	500 mm	
Width	10 mm	
Height	10 mm	

5.3 The comparison between three-dimensional stress and recovery stress of the beam on which anisotropic material applied

In the past few years, various studies have been conducted for the three-dimensional finite element model and 1-D beam model which consists of composite material. Kovvali [21] constructed a 1-D beam model by using VABS and then recovered the stress on 2-D cross-section and compared to 3-D finite element model in order to validate its efficiency and accuracy. In this paper, the same model was implemented to calculate the three-dimensional stress using the analysis library.

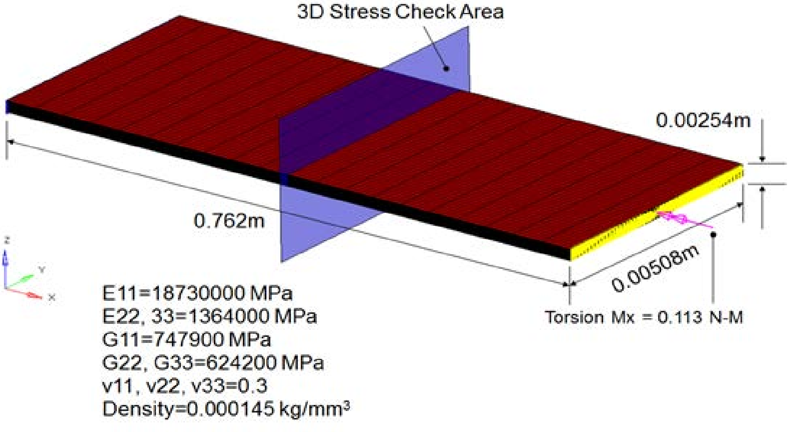

Fig. 11 Stress result at span = 250 mm, Case 1

In Fig. 11, the third verification model, the torsional moment is inputted on the end part of the beam in thin square shape and the other end is covered with 6 degrees of freedom. The 3-D stress extraction point is the center of the middle point of the beam. The stress calculated by VABS and the analysis library from the center towards the width and thickness of cross-section were compared. The stress recovered on the center of one-dimensional beam model with stiffness matrix is the same as VABS and there are discrepancies on some areas which were restored at other point of the end area as in Figs. 12, 13. Apart from this part, the relative error of stress recovered by VABS and analysis library is less than 0.01% and resulted in satisfactory outcomes.

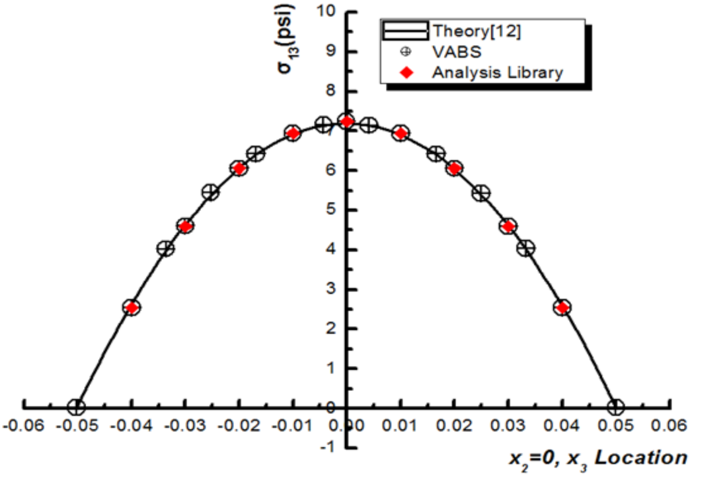

Fig. 12 Three-dimensional stress calculated by VABS and analysis library, σ_{13}, $x_2 = 0$

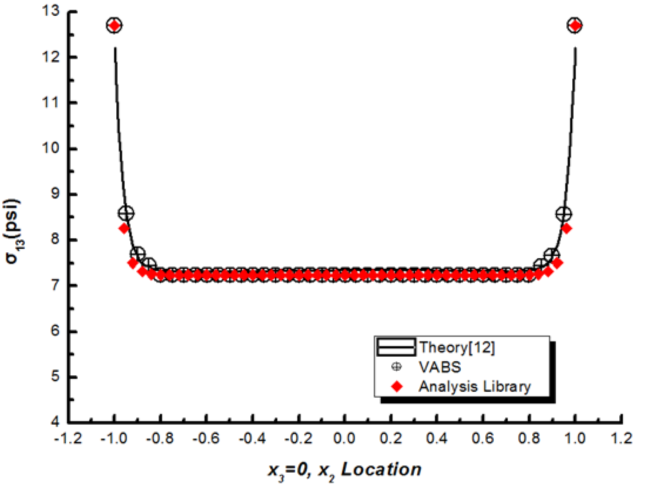

Fig. 13 Three-dimensional stress calculated by VABS and analysis library, σ_{13}, $x_3 = 0$

6. CONCLUSIONS

The library developed in this study creates a one-dimensional model by connecting the stiffness matrix from the process of cross-section dimensional reduction of finely constructed composite material. By inputting the results of the analysis including strain and load, the process of the recovery analysis can be visualized. It can largely save the construction period of the finite element model, the analysis time of Solver and data capacity. The developed library can achieve an accuracy of stress analysis, efficiency and accuracy of analysis on the subject which is hard to construct in 3-D model and frequently revised in design process just like the composite blade. Since it was created in library format, it is easy to apply on the code developed previously and expected to be facilitated by connecting to the precisely designed post-processing program of the composite rotor blade. The developed analysis library can be utilized especially in static and dynamic modelling of wings of high altitude UAVs, helicopter rotor blades, wind blades and tilt-rotors.

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