Intercept Algorithm for Maneuvering Targets Based on Differential Geometry and Lyapunov Theory

Yunes Sh. ALQUDSI*[,1a], Gamal M. EL-BAYOUMI[2b]

*Corresponding author
[1]Cairo University, Nile University, Giza, 12613, Egypt,
yunes.sharaf@pg.cu.edu.eg
[2]Cairo University, Flight Mechanics & Control - Aerospace Engineering Dept.,
Giza, 12613, Egypt,
gelbayoumi@cu.edu.eg

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Abstract: *Nowadays, the homing guidance is utilized in the existed and under development air defense systems (ADS) to effectively intercept the targets. The targets became smarter and capable to fly and maneuver professionally and the tendency to design missile with a small warhead became greater, then there is a pressure to produce a more precise and accurate missile guidance system based on intelligent algorithms to ensure effective interception of highly maneuverable targets. The aim of this paper is to present an intelligent guidance algorithm that effectively and precisely intercept the maneuverable and smart targets by virtue of the differential geometry (DG) concepts. The intercept geometry and engagement kinematics, in addition to the direct intercept condition are developed and expressed in DG terms. The guidance algorithm is then developed by virtue of DG and Lyapunov theory. The study terminates with 2D engagement simulation with illustrative examples, to demonstrate that, the derived DG guidance algorithm is a generalized guidance approach and the well-known proportional navigation (PN) guidance law is a subset of this approach.*

Key Words: *Homing Guidance, Differential Geometry, Proportional Navigation, Intercept, Engagement, Missile, Latax, Air Defense Systems (ADS), Line-Of-Sight (LOS).*

NOMENCLATURE

Basic Latin Letters

a_m , a_t	=	Missile and Target lateral accelerations (m/sec^2).
e_θ	=	Rotational unit vector normal to the LOS.
I	=	Impact point.
n_m , n_t	=	Missile normal and tangential unit vectors.
r'	=	The real closing speed (m/sec).
r_s	=	Sight line range (m).
r_t	=	Target range (m).

[a] Teaching Assistant, Nile University, Egypt.
[b] Professor of Flight Mechanics & Control - Aerospace Engineering Dept.

r_m	=	Missile range (m).
s_m , s_t	=	Length of missile and target trajectories.
t, n, b	=	Tangent, normal and bi-normal unit vectors.
t_s , n_s	=	Tangent and normal unit vectors of the LOS.
t_m , n_m	=	Tangent and normal unit vectors of the missile trajectory.
t_t , n_t	=	Tangent and normal unit vectors of the target trajectory.
t_{md} , n_{md}	=	Missile demand unit tangent and normal vectors.
t_{mc} , t_{tc}	=	Unit vector of the missile and target chords.
v_m , v_t	=	Missile and target velocities (m/sec).

Greek Letters

κ	=	Curvature of the curve (m^{-1}).
κ_m , κ_t	=	Missile and target curvatures (m^{-1}).
τ	=	Torsion of the 3D curve (m^{-1}).
θ_ε	=	Angle between the actual and required missile's heading angle.
θ_m	=	Missile heading angle.
θ_s	=	Sight line angle.
θ_{md}	=	Demand missile-heading angle.
θ_{tcsi}	=	Angle between the LOS and target chord unit tangent vector.
θ_{ms} , θ_{ts}	=	Angles of missile and target relative to sight line angle.
θ_{arc_m} , θ_{arc_t}	=	Missile and the target arc angles.
η , γ	=	Missile to target velocity's ratio.
$\mathbb{Y}$	=	Lyapunov assumed function (rad^2).

List of Abbreviations

ADS	=	Air Defense Systems.
c.g.	=	Center of gravity.
DG.	=	Differential Geometry.
Fn.	=	Function.
LOS.	=	Line of sight.
PN.	=	Proportional Navigation.
SLR.	=	Sight line rate of change.
t_{2go}.	=	Missile time-to-go.

1. INTRODUCTION

Latterly, the missile guidance and control systems became significantly concerned with the effectiveness, accuracy and precision against highly maneuverable targets. Homing guidance is one of the effective approaches for tactical missiles to intercept smart and stealthy targets. The homing guidance configuration is illustrated in figure (1) where the two reference points, the target and the missile, in addition to the impact point, formed the so-called impact triangle, where the triangle's side that connecting the missile c.g. with the target c.g. forms the sight line.

Homing guidance is a common expression referred to missile that steers and directs its motion according to the commands of the missile's onboard seeker, which generates its steering commands based on the reflected or emanating signals from the target. As we know, the equations of motion of the missile are best described using the inertial frame of reference, whereas the representing of the aerodynamics forces and moments is convenient in the body axis frame of reference, which leads to high nonlinearity in the dynamics of both the missile and target [1-4]. By virtue of the differential geometry concepts, we can deal directly with the nonlinearity of the missile systems, which improves the intercept capability of the guidance algorithm.

On the other hand, the well-known PN Guidance algorithm which is considered as a robust approach [5-8], although it is not provide flexibility in choosing the intercept trajectory however this prompts the control designers to produce a more controllable and at the same time robust guidance algorithms [9].

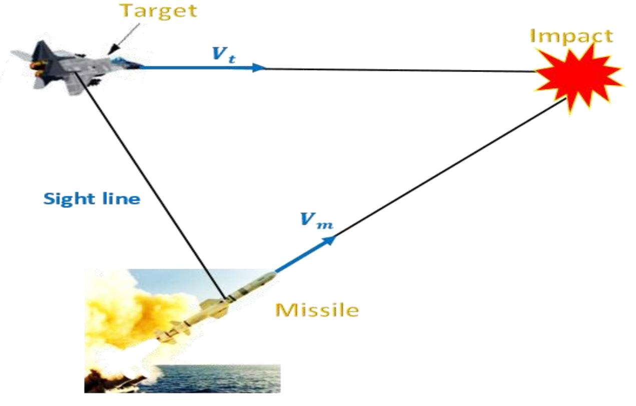

Figure 1: The homing guidance configuration

In order to obtain better control, precise accuracy and flexibility of the intercepting and engagement trajectories for homing missiles guidance and control systems, geometrical approach is used since the concepts of differential geometric control theory provides useful tools for modelling, analysis and design for nonlinear guidance and control systems [10]. The objective is to produce better guidance algorithm to ensure intercepting smaller and highly maneuverable targets with greater flexibility in controlling and choosing the engagement trajectories. Furthermore, this study considers the challenges such as, the complexity and nonlinearity of the missile systems and the restrictions on the missile latax and sensing sensors. Since the interceptor missile's speed plays a key role in determining its aerodynamic maneuverability thus the relative velocity of the missile-to-target also taken into account in a precise and deliberate manner.

2. PRIOR WORK AND LITERATURE REVIEW

There is a great variety of methods used for missile guidance where the accuracy is the most critical factor during its effectiveness. One of the well-known approaches used in the homing missiles is the proportional navigation (PN) guidance law, discussed in many studies such as [11-13]. PN is considered as an experimental law based on the line-of-sight, and there were many trials and modified versions have been developed to improve this approach such as the augmented proportional navigation (APN) [1,4]. In APN a term proportional to the estimated target acceleration was introduced and may give an acceptable performance for a certain maneuvering targets; it has the advantage of decreasing the required induced missile's acceleration due to target maneuvers. Recently, there have been few attempts to improve the missile guidance strategies by using the differential geometry concepts [14-20]. A notable example is the work of [16] White et al. In their paper, they examine the use of DG to the engagement of nonmaneuvering as well as constant curve maneuvering targets. To determine the intercept conditions, they use 2D geometry, where the guidance law of the direct intercepting of nonmaneuvering target is

$$\dot{\theta}_m = -\left(1+\frac{1}{\gamma}\right)\left|\dot{\theta}_s\right|\theta_\varepsilon - \mathrm{K}\,\theta_\varepsilon \qquad \text{and} \qquad \kappa_m = \frac{\dot{\theta}_m}{v_m}$$

where γ is velocity ratio of the missile and the target, θ_ε the error angle between the actual and required missile's heading angle, $\dot{\theta}_s$ the rate of change of the sight line angle, κ_m the intercepting curvature and v_m the missile instantaneous velocity.

They assume that, both the missile and the target have a constant speed and acceleration, and there is an estimator to supply the target motion data such as target range and sight-line-angle. They do not consider the seeker dynamics or uncertainty conditions for instance the noise and time delay. Their work ended by a 2D simulation to illustrate the approach properties. The results show a great accuracy under the ideal condition for both the nonmaneuvering and constant curve maneuvering targets. Another earlier noteworthy work is of [17-20] Chiou et al. In the manuscripts of [17-19] a 3D point mass analysis for both the missile and target kinematics were determined also based on the sight line between the missile and target. The equation of intercepting curvature of the missile is as follows:

$$\kappa_m = b^2\,\kappa_t\,\frac{n_t.e_\theta}{n_m.e_\theta} - A\,\frac{r'\omega}{n_m.e_\theta}$$

Where $=\frac{v_t}{v_m} < 1$, e_θ the rotational unit vector normal to the LOS, n_t ,n_m the missile normal and tangential unit vectors, κ_t the target curvature, ω the rate of the LOS, r' the real closing speed. They introduce in their work a heuristic gain A>2, to enable the approach convergence. The basic idea is based on the condition that the sight line rate "SLR" equals zero, producing an equivalent condition as the PN law. They also develop an extra formula for the missile torsion so as to obtain a well- defined guidance formula. The approach was good only for a certain and specific initial conditions.

3. HOMING GUIDANCE TYPES AND PHASES

The functional missile guidance and control system is required to measure and determine the engagement geometry, well estimating the dynamics of the target using appropriate tracking system, and generating the necessary commands to accurately change the interceptor missile actuators to put it on the collusion course. Additionally, the guidance algorithm needs to actively respond to any mismatch in the intercepting geometry that may emerged due to the target maneuvering or by an initial heading error, thus producing the missile normal acceleration required to correct the missile orientation.

A. Guidance Phases

We may divide the guidance of a missile into three different phases: (1) The booster or launch phase, which begin from the time that the missile leaves its launcher up to burn all the booster fuel. This phase may or may not be controlled, (2) The mid-course phase, which extends until the missile reaches the locked area such that, the last phase can start successfully, and (3) Terminal phase, from the moment the missile seeker locked onto its target until the intercept occurs. This stage required high accuracy and quick reaction to precisely put the missile in the collusion course. Thus, the last phase considered as the critical one, which needs extremely precise equipment to achieve the interception accurately. What is worth mentioning here is that, in the modern ADS and guidance systems there is also additional information collected by third-party sensors. This means you can make use of the missile own sensors or sensors from auxiliary sources connected together in a certain way and work well in an integrated networked system to end up with a qualified and effective system.

B. Homing Guidance Types

In homing guidance, the missile itself acts as receiver and transmitter, in such a type is called active homing. If the missile just receives the target characteristic signals, then it is named passive homing. The third type is the semi-active homing where the target illuminating signal is emanated by launcher or another source as illustrated in figure (2).

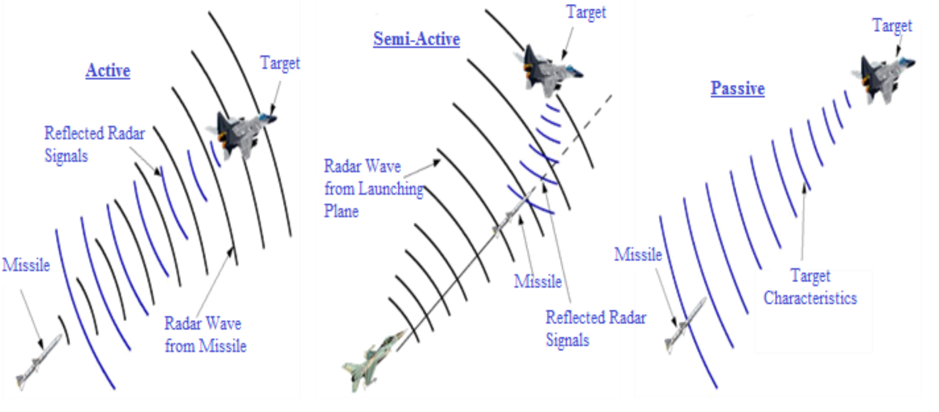

Figure 2: Types of homing guidance systems

If we considered each type individually, then the actively guided system is characterized by the (launch-and-leave) property; however, the disadvantages may be the extra weight, the high cost and the risk that may occur; since its position and motion can be detected because of the radiation that the missile send out. An example of this system type is the European Meteor air-to-air missile (AAM). On the other hand, there is the semi-active homing guidance, which required an external signal source to keep illuminating the target during the entire flight, however this type provides a larger range for the guidance system. The supersonic (Sparrow III) is an example of this system type. The third type "the passive homing arrangement" totally relies on the characteristics of the radiations from the target and should be capable to detect and accurately sense them in order to generate the correct guidance commands, whereas the (Sidewinder missile) is an example of the passive system. At this point, it is appropriate to mention that, the components of the three homing configurations are essentially identical with some differences in terms of their location and the way they are used.

4. DIFFERENTIAL GEOMETRY TOOLS USED IN GUIDANCE LAWS

Using the differential geometry in the control theory offers practical tools for modelling, analysis and design of the linear and nonlinear guidance and control systems. The basic concept of Differential Geometry (DG) in dealing with continuously differentiable curves (smooth curves) in 3D Euclidean space R^3 is to use the calculus to describe the kinematic properties of a particle moving along the mentioned curve or to determine the curve itself geometric properties. On the other side, the differential geometry became an attractive approach to improve and offer useful guidance strategies, since the DG was used in mechanism and machine theory as a functional method of kinematic analysis [21].

One of the essential tools from the differential geometry uses in developing the missile guidance algorithm is the Frenet-Serret formula [6,21], which describes the derivatives of the triple unit vectors t,n and b that representing the curve in the space as a function of the curvature and the torsion.

$$\begin{pmatrix} t'(s) \\ n'(s) \\ b'(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{pmatrix} \cdot \begin{pmatrix} t(s) \\ n(s) \\ b(s) \end{pmatrix}$$

Where ′ means the differentiation with respect to s "the arc length" and κ , τ represent the curvature and the torsion of the 3D curve. The major aspects that have drawn our attention to the use of differential geometric approach in the development of missile guidance systems are (1) DG approach is more generalized guidance algorithm, which deals with curved and straight-line trajectories and the well-known PN guidance law is a subset of DG approach, (2) DG techniques deals with the nonlinear geometry directly, hence the convergence and stability are global, (3) DG approach offers flexibility and a set of possibilities in controlling and choosing the collision courses, and (4) As we will notice, DG guidance law will be stable for all initial conditions as will be shown in the following chapters, which indicates that the capture region involves all the space. On the other side, no restrictions or limits on the capture region, except for the physical limits on the missile such as the lateral acceleration capability as well as the limits on the sensor look angles.

5. ENGAGEMENT KINEMATIC EQUATIONS

According to the postulate stating that, the smallest distance between two points in the space is the direct path i.e. the straight line, we will examine the kinematics of direct intercept of target flying at a constant velocity. Let us consider the sightline joining the missile and the target centers of gravity (c.g.) and the anticipated impact point $\boldsymbol{I}$, where these three points establish what we call the impact triangle (MIT) as illustrated in figure (1).

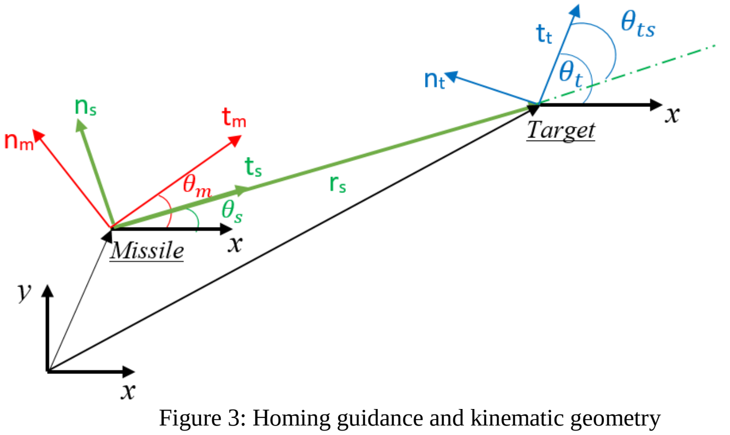

Figure 3: Homing guidance and kinematic geometry

The kinematic geometry is shown in figure (3). Development of the kinematic equations is considered in more details in [22]; here only the final forms of the equations will be mentioned as follows:

$$\dot{r}_s = v_t \cos(\theta_{\mathrm{ts}}) - v_m \cos(\theta_{\mathrm{ms}})\,; \tag{1}$$

$$r_s\dot{\theta}_s = v_t \sin(\theta_{\mathrm{ts}}) - v_m \sin(\theta_{\mathrm{ms}}); \tag{2}$$

$$\ddot{r}_s - r_s\dot{\theta}_s^{\,2} = -a_t \sin(\theta_{\mathrm{ts}}) + \, a_m \sin(\theta_{\mathrm{ms}})\,; \tag{3}$$

$$r_s\ddot{\theta}_s + 2\dot{r}_s\dot{\theta}_s = a_t \cos(\theta_{\mathrm{ts}}) + a_m \cos(\theta_{\mathrm{ms}}); \tag{4}$$

Equations (1, 2) illustrate the relative velocities of the missile and the target along and normal to the LOS. The components of the target-missile relative accelerations, along and normal to the sightline are expressed in equations (3, 4). Easily we can refer equations (1-4) to the inertial coordinate system x and y.

6. GUIDANCE ALGORITHM AND DIFFERENTIAL GEOMETRY

The aim of the guidance algorithm is to define the missile lateral acceleration that is required to produce the missile intercept curvature and to maintain the missile motion along the trajectory of the engagement until the collision occurs. We will derive and examine the guidance algorithm to intercept the maneuvering targets whereas the nonmaneuvering targets are special case. Considering that, the maneuvering targets can intercept either by maneuvering missiles or by direct intercept missile. On the other hand, the nonmaneuvering target may also intercepted by maneuvering or direct intercept missiles.

A unified DG guidance law will be developed by virtue of Lyapunov theory for different engagement scenarios. In the intercept geometry, it's required that, the missile should maneuver until the ratio of the missile-target trajectories length is equal to the ratio of the missile-target velocities i.e.

$$\frac{s_m}{s_t} = \frac{v_m}{v_t} = \eta \tag{5}$$

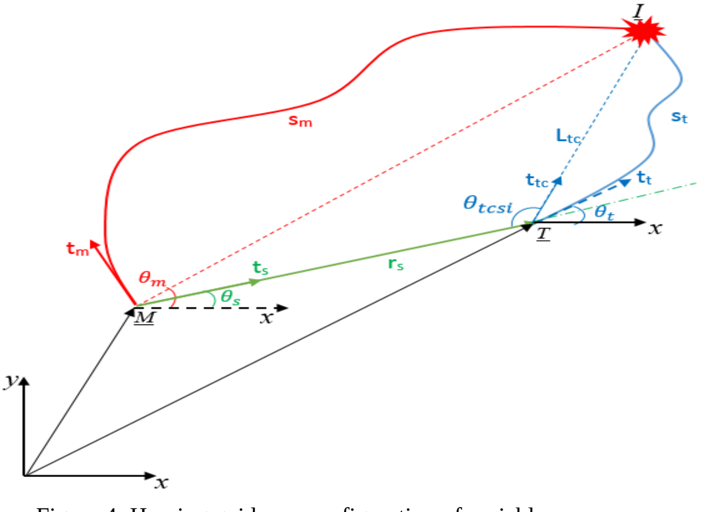

Figure 4: Homing guidance configuration of variable maneuver

In this intercepting configuration, we consider both the missile and the target flying in variable maneuvering curves with constant velocity as illustrated in figure (4) which demonstrates the direct missile-target matching condition for this scenario as follows:

$$s_m t_m = r_s t_s + L_{\mathrm{tc}} t_{\mathrm{tc}}; \tag{6}$$

where; L_{tc} the target chord length

$$L_{\mathrm{tc}} = \alpha_t s_t; \qquad s_m = \eta \ s_t; \qquad \alpha_t = \frac{\sin\left(\frac{\theta_{\mathrm{arc_t}}}{2}\right)}{\left(\frac{\theta_{\mathrm{arc_t}}}{2}\right)} \tag{7}$$

where, θ_{arc_t} the target arc angle.

$$t_m = \frac{1}{\eta}\left(\left(\frac{r_s}{s_t}\right) t_s + \alpha_t \, t_{\mathrm{tc}}\right); \tag{8}$$

Equation (8) represents the vector form of the missile-target matching condition, which indicates that, the missile unit tangent vector is defined base on the LOS unit vector t_s and target-chord unit tangent vector t_{tc}. The equation is also a function of both η and $\frac{r_s}{s_t}$. By using the cosine rule for the impact triangle, we get $\left(\frac{r_s}{s_t}\right)$ as follows:

$$\left(\frac{r_s}{s_t}\right) = \alpha_t \cos(\theta_{\mathrm{tcsi}}) + \sqrt{\eta^2 - \alpha_t^2 \ \sin^2(\theta_{\mathrm{tcsi}})}; \tag{9}$$

where,

$$\theta_{\mathrm{tcsi}} = \pi - \mathrm{abs}(\theta_{\mathrm{tcs}}); \qquad \theta_{\mathrm{tcs}} = \theta_{\mathrm{ts}} + \frac{1}{2}\,\theta_{\mathrm{arc_t}}; \qquad \theta_{\mathrm{ts}} = \theta_t - \theta_s;$$

The mathematical condition to get real value for $\left(\frac{r_s}{s_t}\right)$ is

$$\eta^2 - \alpha_t^2 \sin(\theta_{\mathrm{tcsi}})^2 > 0 \tag{10}$$

Unfortunately, (9) can't be solved explicitly to get $\left(\frac{r_s}{s_t}\right)$ since α_t or $\theta_{\mathrm{arc_t}}$ are function of $\left(\frac{r_s}{s_t}\right)$ as follows:

$$r_{\mathrm{arc_t}} = \frac{1}{\kappa_t}; \qquad s_t = r_{\mathrm{arc_t}} \ \theta_{\mathrm{arc_t}}; \qquad \theta_{\mathrm{arc}_t} = \frac{r_s \kappa_t}{\left(\frac{r_s}{s_t}\right)}; \tag{11}$$

In order to obtain $\left(\frac{r_s}{s_t}\right)$ we need to solve the following equations iteratively.

$$\left.\begin{array}{l} \text{Initial guess of } \left(\frac{r_s}{s_t}\right) \\ \text{From (11) get } \theta_{\mathrm{arc}_t} \\ \text{Now get } \alpha_t \\ \text{From (9) get } \left(\frac{r_s}{s_t}\right)_{\text{new}} \\ \text{Re-iterate until get the solution} \end{array}\right\} \tag{12}$$

From (12) we get $\left(\frac{r_s}{s_t}\right)$,θ_{arc_t}, κ_t , α_t. After obtaining these parameters by substituting in (9) then (8) we will get the demand unit tangent vector t_{m} of the missile collision trajectory.

Now, to generate the guidance algorithm and study its stability we will use the Lyapunov theory. Lyapunov tests whether the system dynamics are stable/asymptotically stable. In other words, in the sense of Lyapunov, if the system starting at state $x_o \in$ domain D and staying within that domain then the system is Lyapunov stable, or in case of asymptotically stable which is more strongly; the system state will return to x_o “equilibrium state” after any disturbance. The Lyapunov stability condition implicitly implies that if the system energy decreases with time, which indicates that the system states would return to the equilibrium state then the system is stable. To study and check the system stability, we assume an energy-like function, positive definite, quadratic, and scalar at the same time its time derivative should be negative to ensure the stability of the system. So, let us assumes Lyapunov $\mathbb{Y}$ function as follows:

$$\mathbb{Y} = \frac{1}{2}\theta_\varepsilon^2; \tag{13}$$

$$\begin{gathered}\theta_\varepsilon = \theta_{\mathrm{md}} - \theta_m; \\ \dot{\mathbb{Y}} = \theta_\varepsilon\dot{\theta}_\varepsilon; \qquad \dot{\mathbb{Y}} = \theta_\varepsilon\left(\dot{\theta}_{\mathrm{md}} - \dot{\theta}_m\right);\end{gathered} \tag{14}$$

Then, we required that

$$\dot{\mathbb{Y}} = -K\mathbb{Y}; \qquad \dot{\mathbb{Y}} = -K\left(\frac{1}{2}\theta_\varepsilon^2\right); \tag{15}$$

From (14 -15) we get:

$$\dot{\theta}_m = \dot{\theta}_{\mathrm{md}} + \frac{K}{2}\theta_\varepsilon; \tag{16}$$

Then, to obtain $\dot{\theta}_{\mathrm{md}}$ from the missile demand unit vector as in (9), and by deriving (8) we get,

$$\dot{t}_{\mathrm{md}} = \frac{1}{\eta}\left(\frac{d}{\mathrm{dt}}\left(\frac{r_s}{s_t}\right) t_s + \left(\frac{r_s}{s_t}\right)\ \dot{\theta}_s\, n_s + \frac{d}{\mathrm{dt}}(\alpha_t t_{\mathrm{tc}})\right) \quad \text{and} \quad \dot{t}_{\mathrm{md}} = \dot{\theta}_{\mathrm{md}}\, n_{md}; \tag{17}$$

where $t_{\mathrm{tc}} = \cos(\theta_{\mathrm{tcsi}})\, t_s + \sin(\theta_{\mathrm{tcsi}})\, n_s;$

$$\dot{\theta}_{\mathrm{md}}\, n_{md} = \frac{1}{\eta}\left(\frac{d}{\mathrm{dt}}\left(\frac{r_s}{s_t}\right) t_s + \left(\frac{r_s}{s_t}\right)\ \dot{\theta}_s\, n_s + \frac{d}{\mathrm{dt}}(\alpha_t t_{\mathrm{tc}})\right) \tag{18}$$

$$\dot{\theta}_{\mathrm{md}} = \left|\frac{1}{\eta}\left(\frac{d}{\mathrm{dt}}\left(\frac{r_s}{s_t}\right) t_s + \left(\frac{r_s}{s_t}\right)\ \dot{\theta}_s\, n_s + \frac{d}{\mathrm{dt}}(\alpha_t t_{\mathrm{tc}})\right)\right|; \tag{19}$$

Then obtaining the guidance law by substituting about $\dot{\theta}_{\mathrm{md}}$ in

$$\dot{\theta}_m = \dot{\theta}_{\mathrm{md}} + \frac{K}{2}\theta_\varepsilon; \tag{20}$$

Therefore, equation (20) demonstrates the guidance law of this scenario, which will guarantee that

$$\dot{\mathbb{Y}} = -K\mathbb{Y}\,, \quad K > 0.$$

Then, the guidance law is globally stable. Applying the Frenet-Serret equation, we find:

$$\kappa_m = \frac{\dot{\theta}_m}{v_m}; \tag{21}$$

After we determine $\dot{\theta}_m$, we get from equation (21) the missile trajectory curvature κ_m, and

$$a_m = v_m^2 \kappa_m = v_m \dot{\theta}_m; \tag{22}$$

Equation (22) calculates the missile lateral acceleration that the missile should produce to guarantee following the determined engagement path. As expressed in the equations, the missile trajectory controlled by defining the missile curvature. The curvature is achieved as a result of the missile normal “Latax” acceleration. To this end, the guidance law shows that there is no restriction on the initial conditions of both missile and target. The limitation will appear only due to physical reasons for instance, limitation on the missile lateral acceleration and on the accuracy and capability of the tracking sensor and the sensor look angle range.

7. NUMERICAL SIMULATION RESULTS

This section will illustrate the simulation results of a missile-target engagement and interception in different cases and conditions to show the significant role and contribution of the differential geometry (DG) approach in the homing missile guidance and control systems. DG offers great approach so as to guarantee missile target precisely interception and gain allowance in choosing the suitable trajectory that the missile can adopt to ensure intercept the target within the missile design limits such as the maximum missile Latax.

The missile guidance and control system and the simplified block diagram are illustrated in figures 5 and 6, respectively, where the missile flight control system and the target dynamics both control the interceptor motion.

The terminal sensor and the state estimator determine the relative geometry and feed the guidance law by the missile range, LOS angle and their rates of change. The guidance algorithm then introduces the steering commands such as the required missile lateral acceleration to the flight control system.

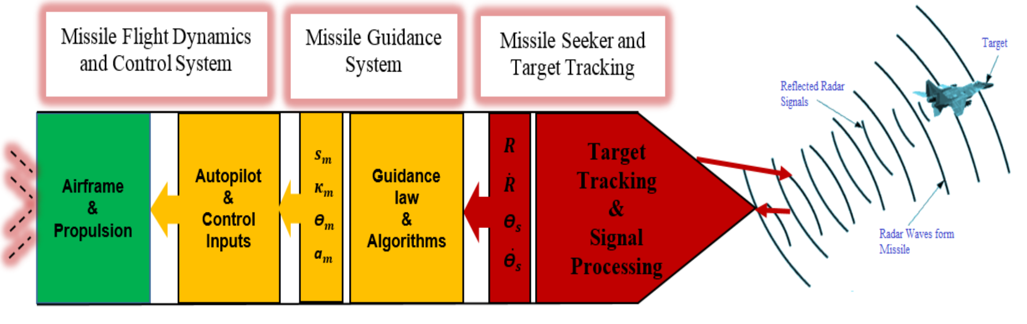

Figure 5: Scheme of missile guidance and control loop

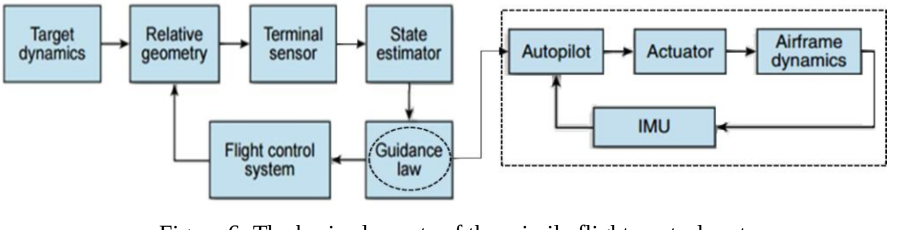

Figure 6: The basic elements of the missile flight control system

The flight dynamics and control system force the actuators and aerodynamics surfaces to follow the guidance commands so as to ensure accurately intercept the target. Diverse scenarios of missile-target engagement are 2D simulated in this section, in order to illustrate the convergence of the differential geometric guidance approach and display the properties of this approach. Different cases of homing missiles intercepting were tested either surface-to-air, air-to-surface or air-to-air missiles. Five different engagement scenarios simulated as follows:

A. Non-maneuvering missile against non-maneuvering target.
B. Maneuvering missile against non-maneuvering target.
C. Non-maneuvering missile against maneuvering target.
D. Maneuvering missile against "constant curve" maneuvering target.
E. Maneuvering missile against "variable curve" maneuvering target (General Maneuverability).

The inputs to the guidance section of the simulation program allow testing the different parameters that may affect the engagement and intercept geometry as follows:

- For the target

The tracker sensor is assumed to be able to measure and/or estimate the target position and the sight light angle, and then the program inputs are:

  - Target's locked initial position
  - Target's locked initial heading angle
  - Target's velocity
  - Target's acceleration

- For The missile
  - Missile's initial position
  - Missile's initial heading angle
  - Missile's velocity as a ratio of the target velocity i.e. enter (η)
  - Missile's acceleration

A. Non-Maneuvering Missile Against Non-Maneuvering Target

In this scenario, we let the target velocity as 310 m/sec. and its acceleration is zero "nonmaneuvering". Intentionally we enlarge the missile's heading angle error, so as to show the global convergence of the DG approach.

Table 1: Input parameters to the guidance algorithm, case (A)

Target initial position	X= -1000 m , Y=10000 m
Target initial angle	θ_{to}=30°
Missile initial angle	θ_{mo}=150°
η	1.2
Gain (K)	0.5

The missile velocity is relatively slaw, about 1.2 of the target velocity (372 m/sec) and the heading angle is within error greater than 100° as shown in figure (7). The guidance gain is set to 0.5, which gives a reasonable convergence within the allowable latax range.

The results represent that, the approximate time-to-go is about (102 sec). In figure (8) the necessary missile lateral acceleration required to intercept the given target is depicted.

Figure (9) shows the Lyapunov function, $\frac{r_s}{s_t}$, LOS_rate, and the missile curvature κ_m. Lyapunov function converges rapidly, and the approach gives a good convergence.

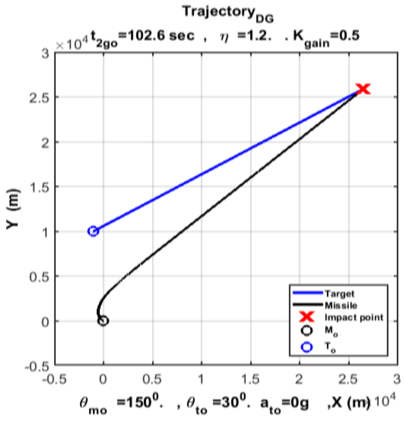

Figure 7: Missile-Target intercept trajectory (η=1.2), case (A)

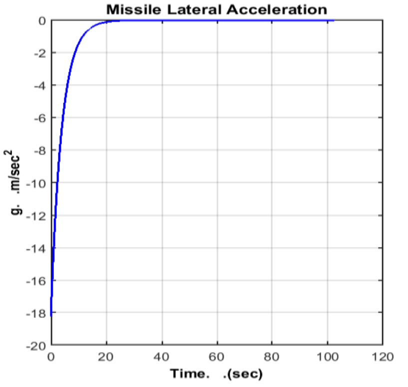

Figure 8: Missile Lateral Acceleration

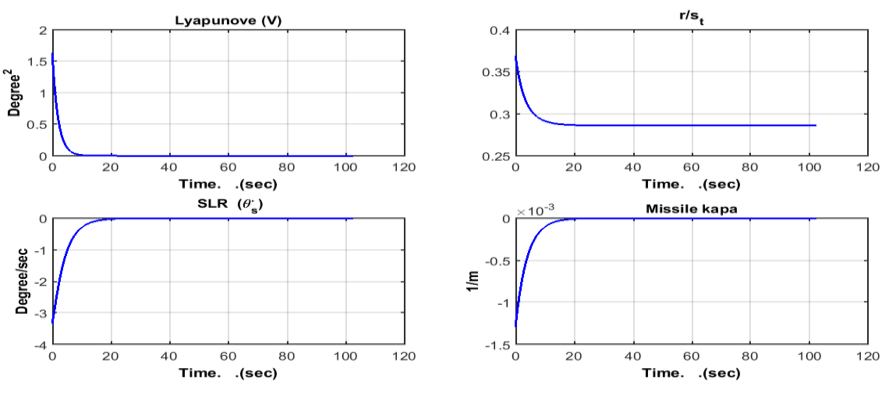

Figure 9: {Lyapunov Fn., r/st, SLR, missile curvature}

B. Non-Maneuvering Missile Against Maneuvering Target

A surface-to-air (STA) engagement for maneuvering target is depicted in the scenario below. In this case, the target velocity is set to 310 m/sec and the target is considered as being able to maneuver in a constant curve. Figure (10) shows the results of a target with latax equal to 1g. The missile velocity is twice the target. The results show that the missile will maneuver by normal acceleration shown in figure (11), and the impact will approximately occur after 26 sec. Lyapunov also converges within 8 seconds. In this case, as the missile velocity is greater this increase the aerodynamics capability of the missile thus the required time-to-go is smaller about 26 sec. while the missile required latax range will increase as shown in figure (11). For the two cases, the guidance gain K is also set to 0.5 and shows a reasonable convergence.

Table 2: Input parameters to the guidance algorithm, case (B-1)

Target initial position	X= -1000 m, Y=10000 m
Target initial angle	θ_{to}= 0°
Target normal acceleration	a_t= 1g
Missile initial angle	θ_{mo}=170°
η	2
Gain (K)	0.5

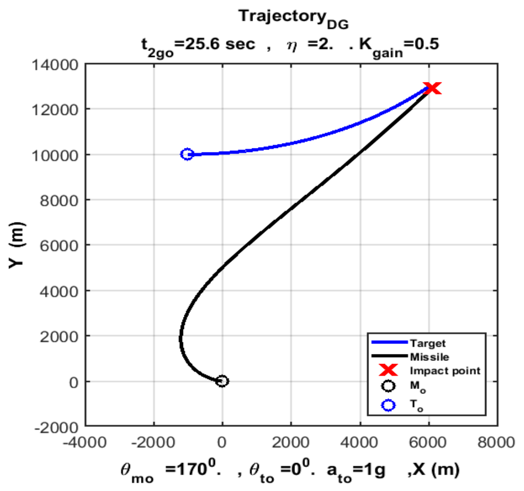

Figure 10: Missile-Target intercept trajectory (η=2), case (B-1)

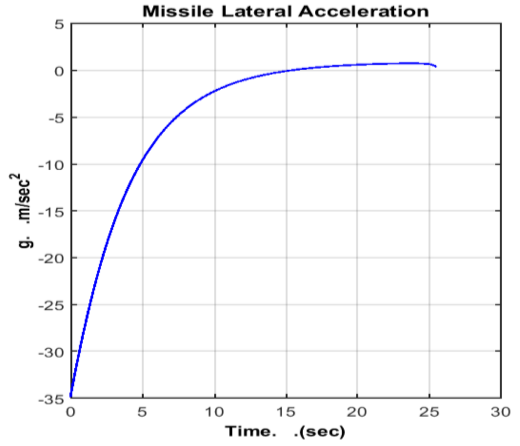

Figure 11: Missile lateral acceleration

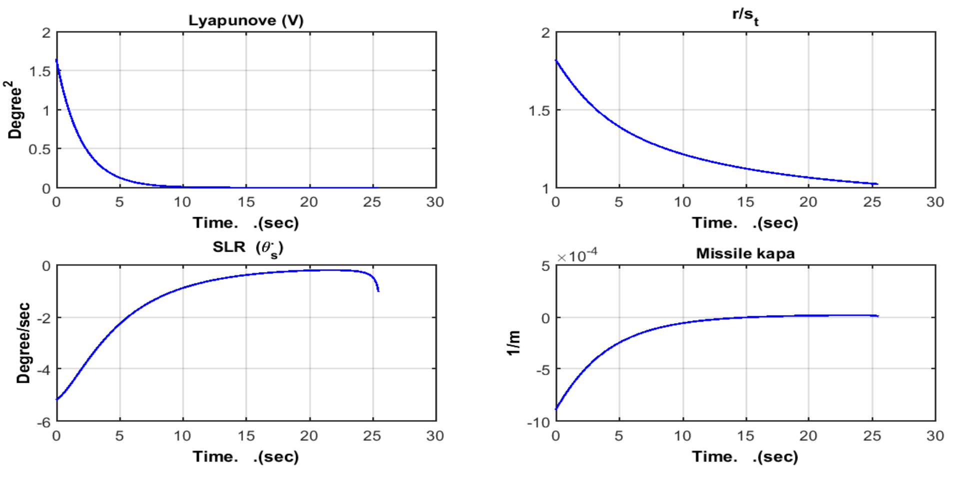

Figure 12: {Lyapunov Fn., r/s_t, SLR, missile curvature}

An air-to-air engagement is shown in figure (13). In this case, the velocity of the missile is smaller than of the target with η=0.6 and the guidance gain is set to one to give a good convergence.

Table 3: Input parameters to the guidance algorithm, case (B-2)

Target initial position	On the ground (0,0)
Missile initial position	X=- 1000 , Y=10000 m
Target initial angle	θ_{to}=60°
Target latax	a_t= 1g
Missile initial angle	θ_m=0°
η	0.6
Gain (K)	1

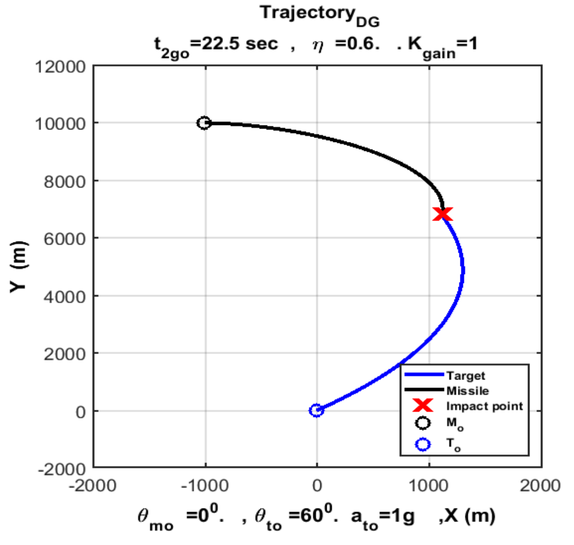

Figure 13: Missile-Target intercept trajectory (η=0.6), case (B-2)

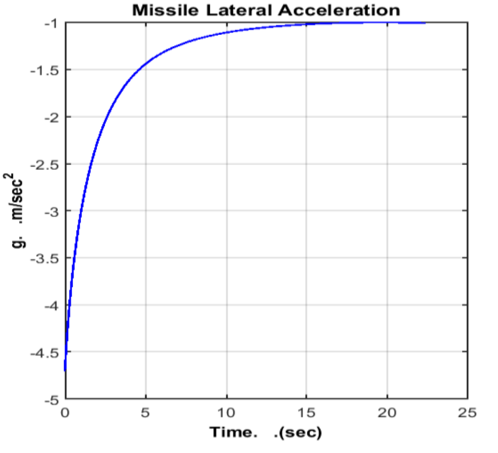

Figure 14: Missile lateral acceleration

The target maneuvers with lateral acceleration equal to 1g. The simulation results of this case stated that even if the missile velocity is less than the target velocity the impact will occur.

It is important here to mention that, if the chaser velocity is less than the target then the impact occurring will impose restrictions on the range and the heading angles at the same time on the target maneuverability.

C. Maneuvering Missile Against Non-Maneuvering Target

In this case, the target is flying in a constant curvature i.e. its latax is set to zero with a constant velocity equal to 310 m/sec and 30º initial heading angle.

The missile is maneuvering by two different latax (±3 g) and initially flying away from the target with relatively large heading error to test the convergence.

The simulation results represent the missile trajectory and its latax during the engagement as shown in figure (15).

Table 4: Input parameters to the guidance algorithm, case (C)

Target initial position	X=1000 m, Y=10000 m
Target initial angle	θ_{to}=30º
Missile initial angle	θ_{mo}=150º
Missile latax	$a_m = \pm 3$ g
η	2
Gain (K)	0.5

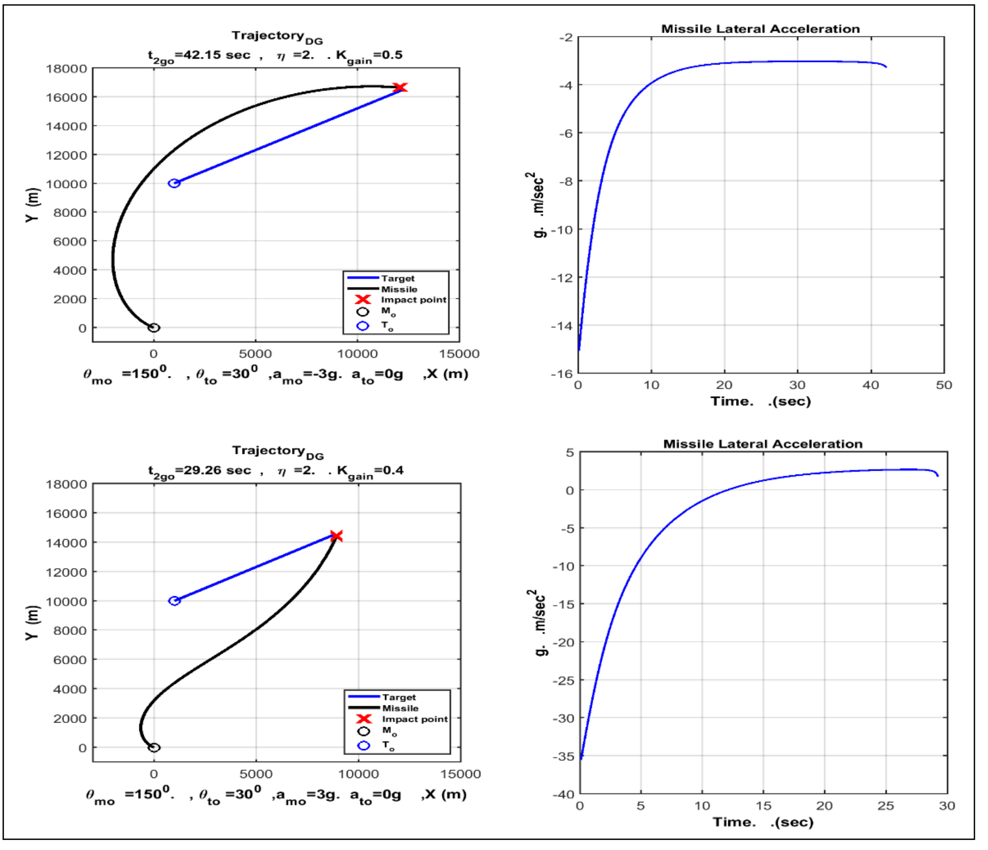

Figure 15: Missile-Target intercept trajectory (η=2), case (C) and the missile LaTax

D. Maneuvering Missile Against Maneuvering Target

In this configuration, the target is maneuver along constant curve as shown in figure (16). The DG guidance algorithm is testing the guidance of air-to-air maneuverable missile flying in an initial heading angle equal to 0°, with 2g normal acceleration. The target latax is (-2g) and its initial heading angle is 60 deg. the missile velocity is about 465 m/sec i.e. 1.5 of the target speed.

Table 5: Input parameters to the guidance algorithm, case (D)

Target initial position	On the ground (0,0)
Missile initial position	X= 1000, Y=12000 m
Target initial angle	θ_{to}=60°
Target latax	a_t= -2g
Missile initial angle	θ_{mo}=0°
Missile latax	a_m=2g
η	1.5
Gain (K)	0.5

The approximated simulation time-to-go is 27 sec. and the missile lateral acceleration during the engagement is within the limits. The results again show good convergence with the guidance gain K=0.5. The missile and target internal arc angles θ_{m_arc} ,θ_{t_arc} approach to zero as the impact point become closer which confirms the analysis. Again, the results show a good convergence and the missile lateral acceleration is within the limits.

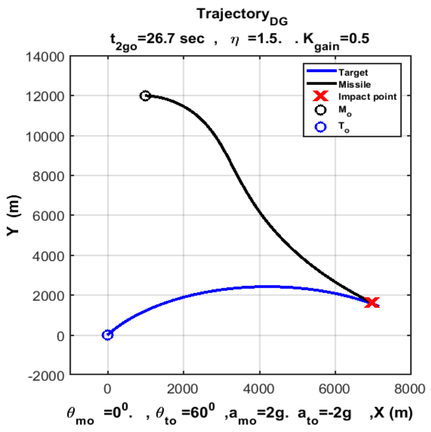

Figure 16: Missile-Target intercept trajectory (η=1.5), case (D)

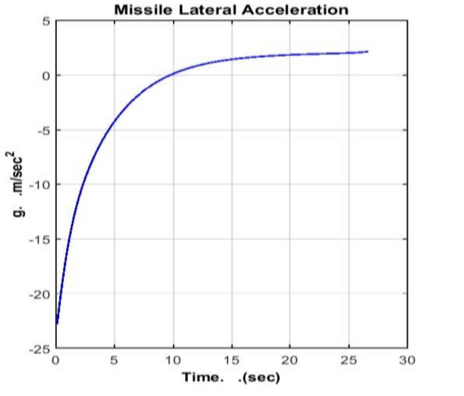

Figure 17: Missile Lateral Acceleration

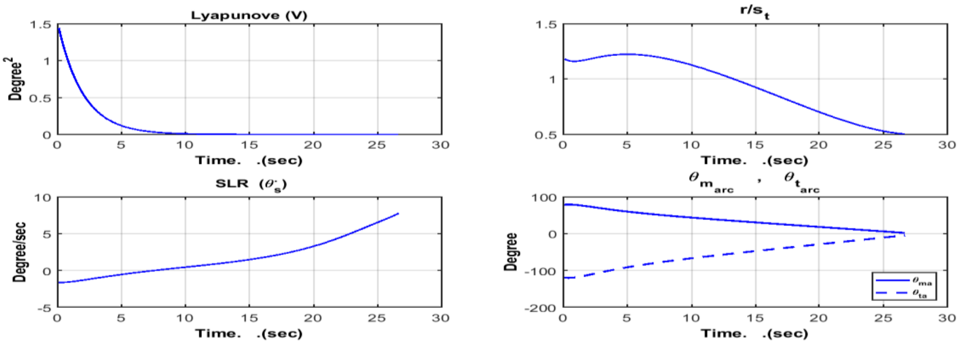

Figure 18: {Lyapunov Fn., r/s_t, SLR, θ_{m_arc} ,θ_{t_arc} }

E. General Maneuvering of The Missile and The Target

In this engagement scenario, we will consider a target with high maneuverability. The target lateral acceleration is variable thus; the target maneuver is changing over time. Let us choose the target normal acceleration function over time as follows:

$$a_t = 0.5 * cos\,(0.01 * t) + 10 * sin\,(0.1 * t) - \; 0.5 * a_{to}$$
$$a_{to} = 1 * g \text{ m/sec}^2.$$

Table 6: Input parameters to the guidance algorithm, case (E)

Target initial position	X= 1000 m , Y=12000 m
Target initial angle	θ_{to}=30°
Target velocity	310 m/sec
Missile initial angle	θ_{mo}=150°
η	1.5
Gain (K)	0.5

The interception considered in this example, examines the capability of the DG guidance algorithm to efficiently intercept the target of variable maneuverability. The time history of the target's lateral acceleration that illustrated in figure (19) varies between (-1.5g) and (+0.5g). The missile-target engagement configuration is illustrated in figure (20). The target arc angle θ_{t_arc} ends up at zero to confirm the analysis. Again, the differential geometric concepts guarantee the missile-target interception; the approximate time-to-hit is about 60 sec.

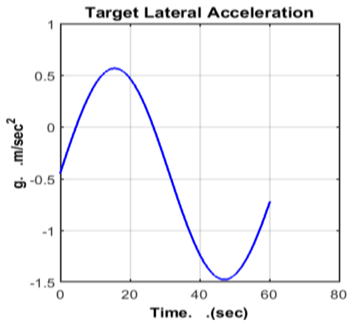

Figure 19: Target lateral Acceleration

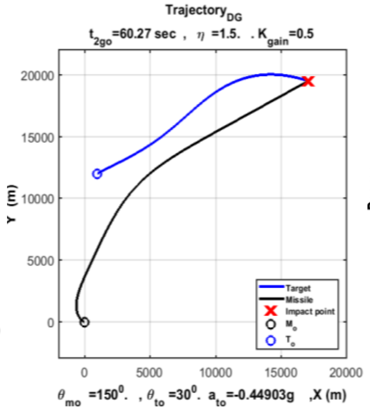

Figure 20: Missile-Target intercept trajectory (η=1.5), case (E)

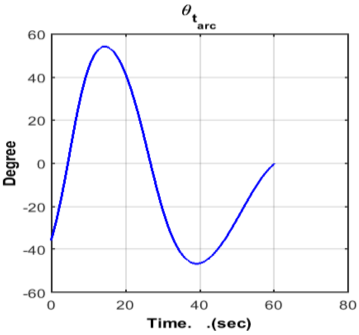

Figure 21: Target internal arc angle θ_(t_arc)

8. CONCLUSIONS

The study presented in this paper used the differential geometric concepts to develop a novel guidance algorithm for the homing missiles and air defense systems (ADS) in order to efficiently intercept the highly maneuverable targets. The study illustrated the essential idea behind the DG approach as well as the major properties and features of this guidance algorithm. The intercept geometry and kinematics of the engagement are developed and expressed in differential geometric terms established upon the direct interception to develop a generalized guidance law. After examining diverse scenarios of the missile-target interception, the DG approach is considered as more generalized guidance algorithm, which deals with curved and straight-line trajectories and the PN is a subset of it. In addition, DG techniques deal with the nonlinear geometry directly hence, the convergence and stability are global. From another aspect, by virtue of the differential geometry techniques we gain more flexibility and set of possibilities in controlling and choosing the interceptor trajectories.

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