Guidance Optimization for Tactical Homing Missiles and Air Defense Systems

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Abstract: *The aim of this paper is to develop a functional approach to optimize the engagement effectiveness of the tactical homing missiles and air defense systems by utilizing the differential geometric concepts. In this paper the engagement geometry of the interceptor and the target is developed and expressed in differential geometric terms in order to demonstrate the possibilities of the impact triangles and specify the earliest interception based on the direct intercept geometry. Optimizing the missile heading angle and suitable missile velocity against the target velocity is then examined to achieve minimum missile latax, minimum time-to-go (time-to-hit) and minimum appropriate missile velocity that is guaranteed a quick and precise interception for the given target. The study terminates with different scenarios of engagement optimization with two-dimensional simulation to demonstrate the applicability of the DG approach and to show its properties.*

Key Words: *Homing Guidance, Guidance Optimization, Differential Geometry, Proportional Navigation (PN), Intercept, Engagement, Latax, Air Defense Systems (ADS), Line-Of-Sight(LOS)*

NOMENCLATURE

Basic Latin Letters

a_m , a_t	=	Missile and Target lateral accelerations (m/sec^2).
I	=	Impact point.
r_s	=	Sight line range (m).
r_t	=	Target range (m).
r_m	=	Missile range (m).
s_m , s_t	=	Length of missile and target trajectories.
t, n, b	=	Tangent, normal and bi-normal unit vectors.
t_s , n_s	=	Tangent and normal unit vectors of the LOS.

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t_m , n_m	=	Tangent and normal unit vectors of the missile trajectory.
t_t , n_t	=	Tangent and normal unit vectors of the target trajectory.
v_m , v_t	=	Missile and target velocities (m/sec).

Greek Letters

κ_m ,κ_t	=	Missile and target curvatures (m^{-1}).
θ_m	=	Missile heading angle.
θ_s	=	Sight line angle.
θ_{mo} , θ_{to}	=	Initial heading angle of the missile and the target.
η	=	Missile to target velocity's ratio.

List of Abbreviations

ADS	=	Air Defense Systems.
DG.	=	Differential Geometry.
LOS.	=	Line of sight.
PN.	=	Proportional Navigation.
SLR.	=	Sight line rate of change.
t_{2go}.	=	Missile time-to-go.

1. INTRODUCTION

Nowadays, seeking for an effective air defense system (ADS) starting with the launching phase and up to the precise interception, became an essential concern for the developers of the guidance algorithms aiming to generate an accurate and carefully chosen trajectories for the interceptor homing missile against highly maneuverable targets [1-3]. For the short and medium ranges, homing guidance is considered as one of the effective approaches for tactical missiles to intercept smart and stealthy targets.

Homing guidance systems or two-points guidance systems "missile and target" as the two reference points, is a common expression referred to missile that steers and directs its motion according to the commands of the missile's onboard seeker which generates its commands based on the reflected or emanating signals from the targets [4-9]. Since the concepts of differential geometric control theory provide useful tools for modelling, analysis and design for nonlinear guidance and control systems [10-16], we will use the differential geometry approach to develop an optimization algorithm for optimally steering the homing missile to the collision point and achieving certain and/or given requirements such as produce the fast collision course which guarantees the minimum required time-to-go. From another aspect, maybe for design reasons, the maximum allowable lateral acceleration for the interceptor missile is limited within a given range, in such case the guidance algorithm should take into consideration the missile latax limits [17,18].

The objective is to develop optimized and better guidance algorithm to ensure intercepting smaller and highly maneuverable targets with greater flexibility in controlling and choosing the engagement trajectories in contrast with the Proportional Navigation (PN) guidance law [19-21]. Furthermore, the study dealt with the challenges such as the complexity and nonlinearity of the missile system, the relative velocity between the missile and target, the imposed restrictions on the missile normal acceleration, and the maximum flying time for the missile based on the burned fuel of the missile.

2. ENGAGEMENT KINEMATIC EQUATIONS

In homing guidance, the sight line or line-of-sight (LOS) is an essential measure of the target-missile relative geometry as depicted in figure (1), where t and n represent the tangent and normal unit vectors, respectively. According to the postulate stating that the smallest distance between two points in the space is the direct path i.e. the straight line, we will examine the kinematics of the direct interception for a target flying at a constant velocity.

The relative position and motion of the target-to-missile in addition to estimate the target LaTax are determined using accurate sensors, of which mostly are located in the nose of the homing missile. For now, to derive the engagement kinematic equations, let's consider the LOS connecting the target c.g. with the missile c.g. and from figure (3.1) we find that:

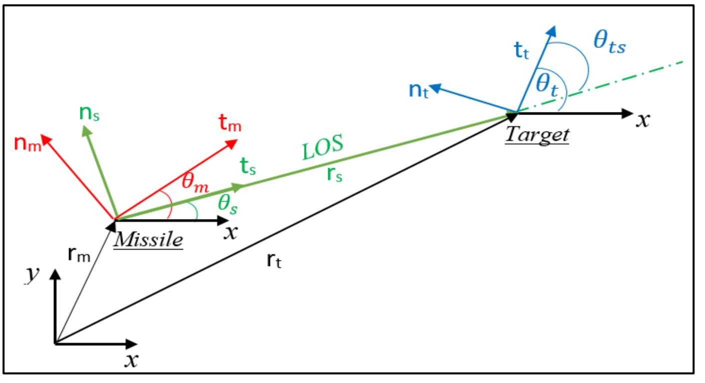

Figure 1: Homing guidance and kinematic geometry

$$r_s = r_t - r_m; \qquad r_s = r_s t_s;$$
$$\dot{r}_s = \dot{r}_s t_s + r_s \dot{t}_s; \quad \dot{t}_s = \dot{\theta}_s n_s; \quad \dot{n}_s = -\dot{\theta}_s t_s;$$
$$\dot{r}_s t_s + r_s \dot{\theta}_s n_s = v_t t_t - v_m t_m; \tag{1}$$

Equation (1) illustrates the relative velocities of the missile as well as the target. By projection onto the basis, t_s, n_s individually we then get the components of the relative velocities along and normal to the LOS.

<u>Along the LOS</u>:

$$\dot{r}_s = v_t t_s . t_t - v_m t_s . t_m;$$

Or
$$\dot{r}_s = v_t \cos(\theta_{\mathrm{ts}}) - v_m \cos(\theta_{\mathrm{ms}}); \tag{2}$$

<u>Normal to the LOS</u>:

$$r_s \dot{\theta}_s = v_t n_s . t_t - v_m n_s . t_m; \quad \text{or} \quad r_s\, \dot{\theta}_s = v_t \sin(\theta_{\mathrm{ts}}) - v_m \sin(\theta_{\mathrm{ms}}); \tag{3}$$

where $\theta_{\mathrm{ts}} = \theta_t - \theta_s$ and $\theta_{\mathrm{ms}} = \theta_m - \theta_s$

By differentiating (1) and substituting, $\dot{t}_s$, $\dot{n}_s$ we get:

$$\left(\ddot{r}_s - r_s \dot{\theta}_s^{\,2}\right) t_s + \left(r_s \ddot{\theta}_s + 2\dot{r}_s \dot{\theta}_s\right) n_s = v_t^{\,2} \kappa_t n_t - v_m^{\,2} \kappa_m n_m \tag{4}$$

Noticing that: From Frenet-Serret formula [11,19],

$$\dot{t} = \kappa\, v\, n = \dot{\theta}\, n; \qquad \text{and} \qquad \dot{n} = -\kappa\, v\, t = -\dot{\theta}\, t;$$

Equation (4) shows the relative acceleration between the missile and the target. The missile and target lateral accelerations are:

$$a_m = {v_m}^2 \kappa_m = v_m \dot{\theta}_m;$$
$$a_t = {v_t}^2 \kappa_t = v_t \dot{\theta}_t;$$

The components of the target-missile relative accelerations, along and normal to the sightline are:

Along the LOS:

$$\ddot{r}_s - r_s \dot{\theta}_s^{\,2} = a_t t_s . n_t - a_m t_m . n_m;$$

or
$$\ddot{r}_s - r_s \dot{\theta}_s^{\,2} = -a_t \sin(\theta_{\mathrm{ts}}) + a_m \sin(\theta_{\mathrm{ms}}) \tag{5}$$

Norma to the LOS:

$$r_s \ddot{\theta}_s + 2\dot{r}_s \dot{\theta}_s = a_t n_s . n_t - a_m n_m . n_m;$$

or
$$r_s \ddot{\theta}_s + 2\dot{r}_s \dot{\theta}_s = a_t \cos(\theta_{\mathrm{ts}}) + a_m \cos(\theta_{\mathrm{ms}}); \tag{6}$$

Easily, we can refer equations (2-6) to the inertial coordinate system x and y.

3.THE ENGAGEMENT GEOMETRY OF HOMING GUIDANCE

As the fact states that, the smallest distance between two points in the space is the direct path i.e. the straight line, thus we will consider the direct intercept geometry for both the missile and target where they are flying at a constant velocity. Let's consider the sightline and the two courses connecting the missile and the target with the anticipated impact point *I* respectively, these three sides establish the so-called impact triangle (MIT) as illustrated in figure (2).

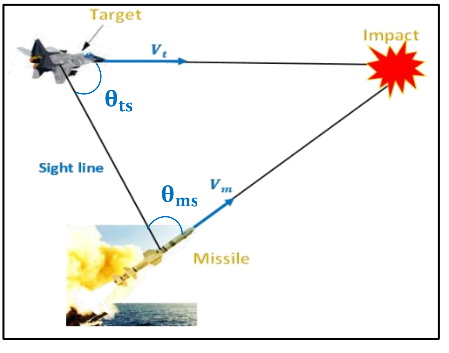

Figure 2: Homing Guidance Configuration

In order to determine the intercepting condition, consider the time $\boldsymbol{T}$, whereas both the target and missile travel form their initial positions to the impact point during this time. Thus, the target path length $\boldsymbol{s_t}$ is equal to

$$\boldsymbol{s_t = v_t T} \tag{7}$$

Which mean that to guarantee the impact occurrence, the missile should travel a distance $\boldsymbol{s_m}$ during the identical time period $\boldsymbol{T}$ where

$$\boldsymbol{s_m = v_m T} \tag{8}$$

In other words, the missile should maneuver until the missile-to-target trajectories-length-ratio is equal to the missile-to-target velocities ratio as follows:

$$\boldsymbol{\frac{s_m}{s_t} = \frac{v_m}{v_t} = \eta} \tag{9}$$

Equation (9) shows the essential impact condition from the geometric point of view.

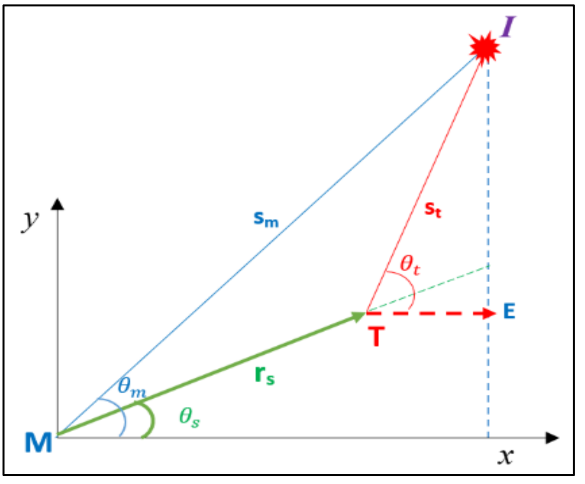

Figure 3: Intercept Geometry

By considering the intercept geometry shown in figure (3), to estimate the expected positions of the impact point, let us consider the two triangles MxI and TEI**;** utilizing Pythagoras's theorem then we can find that

$$s_t{}^2 = (x - r_s\cos(\theta_s))^2 + (y - r_s\sin(\theta_s))^2 \tag{10}$$

$$s_m{}^2 = x^2 + y^2 \tag{11}$$

$$\frac{s_m}{s_t} = \eta = \frac{\cos(\theta_{\mathrm{ts}})}{\cos(\theta_{\mathrm{ms}})} \tag{12}$$

Then: $$\eta^2 = \frac{x^2+y^2}{(x-r_s\cos(\theta_s))^2+(y-r_s\sin(\theta_s))^2}$$

After rearrangement and simplifications of (10), (11), and (12) we get

$$x^2 + y^2 - 2C\cos(\theta_s)\,x - 2C\sin(\theta_s)\,y + r_sC = 0$$

or $$(\,x - C\cos(\theta_s)\,)^2 + (\,y - C\sin(\theta_s)\,)^2 = C^2\left(1 - \frac{r_s}{C}\right) \tag{13}$$

where: $C = \frac{\eta^2\, r_s}{1-\eta^2}$;

Equation (13) shows that the anticipation positions of the impact points can be represented by a circle equation with radius , where $r = C\sqrt{1 - \frac{r_s}{C}}$ and the circle is centered at $(C\,cos\theta_s\,, C\,sin\theta_s)$. In case of $\eta = 1$ the equation yields to

$$x\,cos(\theta_s) + y\,sin(\theta_s) = \frac{r_s}{2} \tag{14}$$

As shown in figure (4), where the circles represent the loci of the anticipated impact points such that, if $\eta > 1$, the circle encloses the target, which indicates that the missile will intercept the target whatsoever the target's approach direction. In contrast with the case of $\eta < 1$, the interception will occur only if the target velocity vector intersects with the anticipated impact circle; in such case the locus of the anticipated impact points encloses the missile.

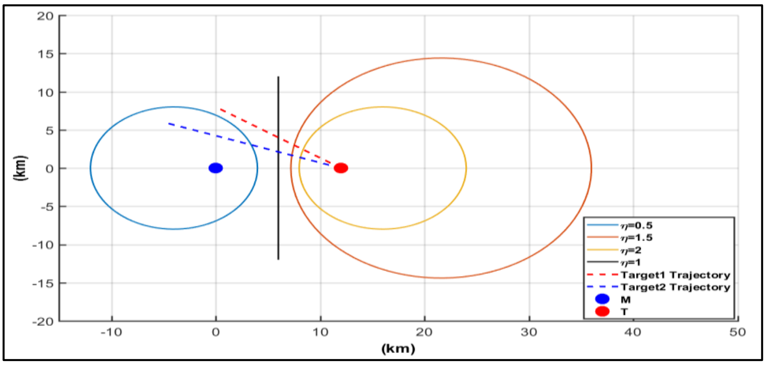

Figure 4: Positions of the expected impact points for different η

Figure (4) also illustrates the earliest interception that can be achieved by the missile, known its heading angle and pursuit velocity according to our treatment, which is implicitly based on the shortest engagement trajectory utilizing the mentioned fact. Moreover, the intercepting geometry is useful also for the targets in order to attempt avoiding the anticipated area of interception. Now, let us demonstrate the possibilities of the impact triangles for certain engagement scenarios, if we have a target flying in a straight line and a constant velocity from the initial position at (0, 0) and constant heading angle θ_{to}=0°, to the impact point I. Further, to illustrate the possibilities for different missile-to-target ranges, the impact points will be located at different ranges from the target, starting form 2 km until 12 km with step=2 km. Thus, we will figure out the third point which represents the missile's positions for each range to complete the impact triangle. Then figure (5) shows the missile position possibilities or the possibilities of the impact triangle for each range and for different missile-target velocity ratio $\eta = \{0.5\ , 1\ ,1.5\ ,2\}$. Again, figure (5) where the circles represent the missile position possibilities also shows that if the velocities ratio is greater than one then the interception will occur whatsoever the missile position is, since the circle encloses the target. This appears to be in contrast with the case in which the missile-target velocities ratio is less than one, where the circle does not enclose the target; this indicates that the missile position possibilities are restricted, so the missile should be in front of the target.

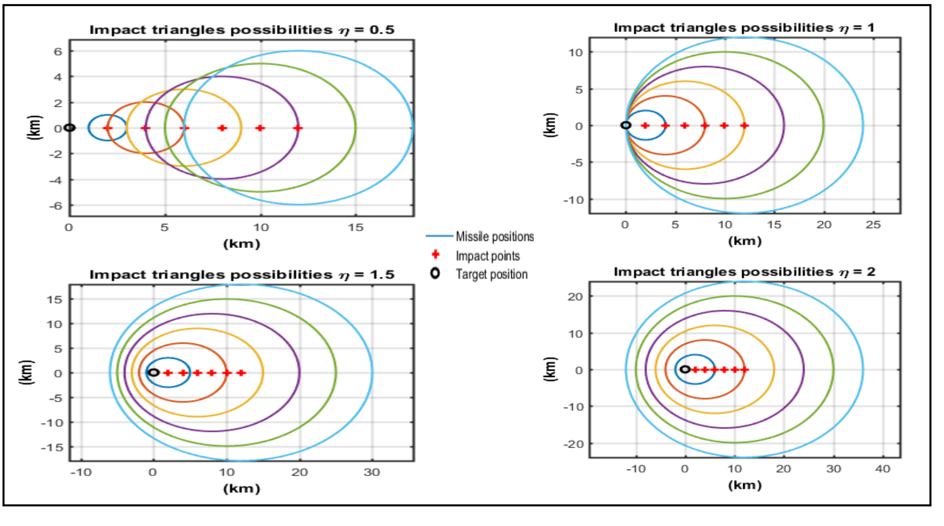

Figure 5: Impact Triangles possibilities for different η

4. OPTIMIZATION OF THE MISSILE'S HEADING ANGLE AND THE VELOCITY RATIO

In this section, we will use the differential geometric principles so as to optimize the engagement configuration seeking for the best trajectory, which guarantees the missile-target intercept within a given requirement such as:

(a) The minimum missile latax. (b) The minimum time to go.

(c) The minimum missile velocity.

Now, let us consider real situations of missile-target intercept scenarios, whereas the inputs to the section of the optimization in our guidance program include:

(a) Target initial position. (b) Target flying velocity. (c) Target heading angle.

As we need to determine the missile parameters to acquire the best interception, we will discuss and examine four different scenarios separately and will illustrate their specific properties. In these scenarios we will take into account the effect of changing the missile velocity, target attacking angle, and the target-missile range on the missile-target intercept geometry as well as the approximate time-to-go and the missile's heading angle or LaTax.

A. Constant Velocity Ratio with Variable Target's Angle

The target initial position is (12 km, 12 km) and the missile initial position is at the origin point, where the target velocity is set to 310 m/sec, i.e. (η=1.5). This scenario observes the effect of the target's angle of attack variation on the engagement configuration. Figure (6) shows different cases of the intercept scenarios for different target's heading angles, whereas the circles represent the expected impact points for the given range and velocity ratio. The optimization algorithm chooses the best missile's heading angle to intercept the given target within a minimum approximated time-to-hit.

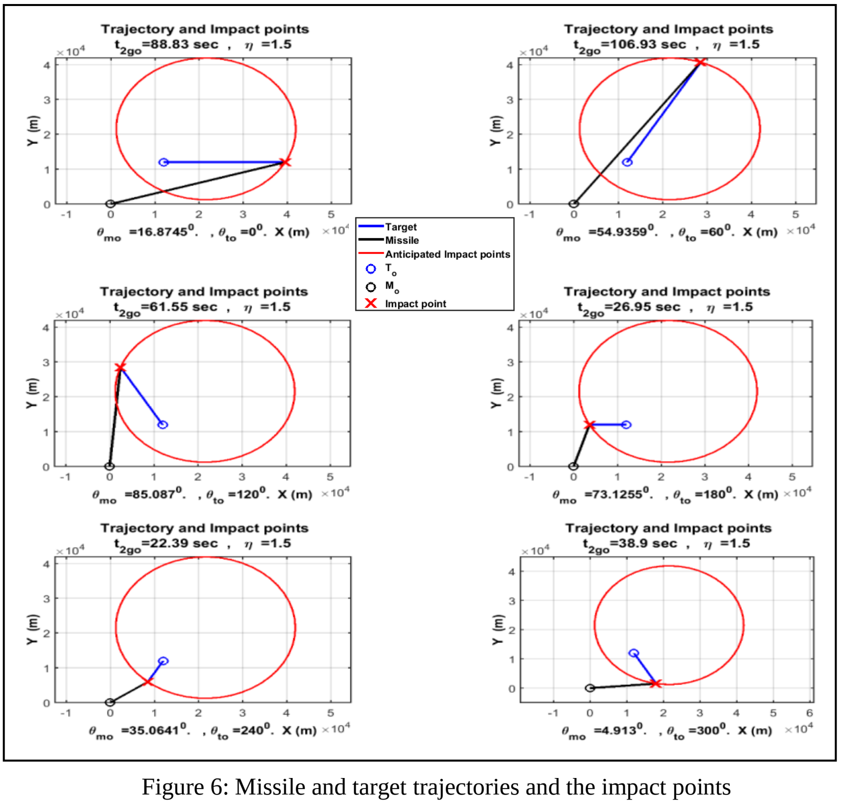

Figure 6: Missile and target trajectories and the impact points

B. Constant Target's Angle with Variable Velocity Ratio (η)

Let the target's heading angle be equal to 30° and its initial position be (0, 12 km); the missile is launched from the origin point, and the target velocity is set to 310 m/sec. We will demonstrate the engagement configuration where the ratios of the missile-to-target velocities are equal to $\{2, 1.5, 1, 0.5\}$. This scenario examines the effect of the velocity ratio η on the engagement possibility. As the two cases of $\eta = \{1, 0.5\}$ are critical, since they rise the question of whether it is possible for a missile to intercept a target even if its velocity equal or is less than the target velocity? To figure out that, let us change the target's heading angle to 290° with the velocities ratios $\eta = \{1, 0.5\}$ and see the properties of the engagement configuration.

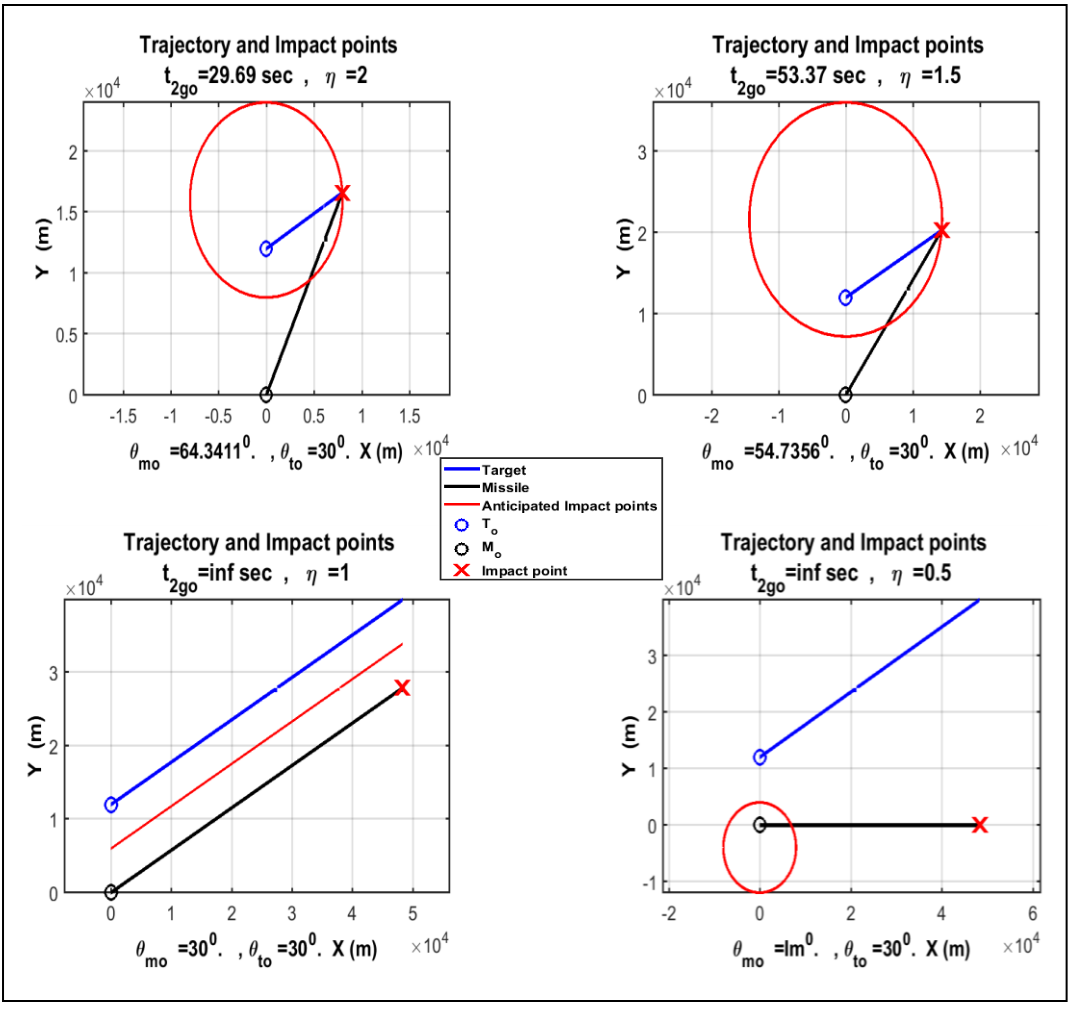

Figure 7: Engagement configuration of different missile velocity ratios at θ_{t0}=30°

Figure (7) displays the engagement configuration of different missile velocity ratios where the simulation results determine the best missile's heading angle for each missile velocity. When $\eta = \{1, 0.5\}$ the results show that no interception will occur for a target's heading angle of 30°. However, if the target's heading angle was as shown in figure (8) then the interception possibly occurs.

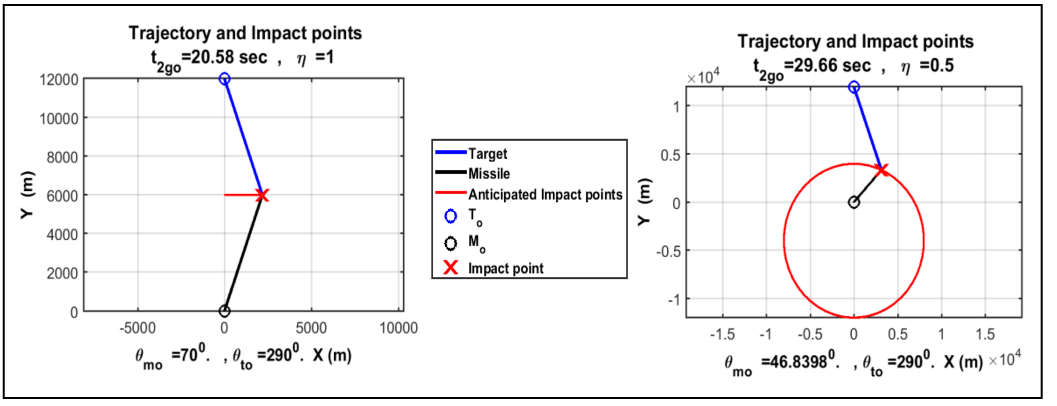

Figure 8: Engagement configuration of $\eta = \{1,\ 0.5\}$ at θ_{to}=290°

C. Variable Range with a Constant Target's Heading Angle

In this scenario we will determine the engagement parameters for different missile velocities i.e. for $\eta = \{2, 1.5, 1,\ 0.5\}$ separately and the target's angle and velocity remaining constant, where v_t =310 m/sec. The range between the missile and the target will change from 12 km to 2 km with step=2 km, to examine the effect on the position of the impact points due to changing the missile velocity and the range between the missile and the target.

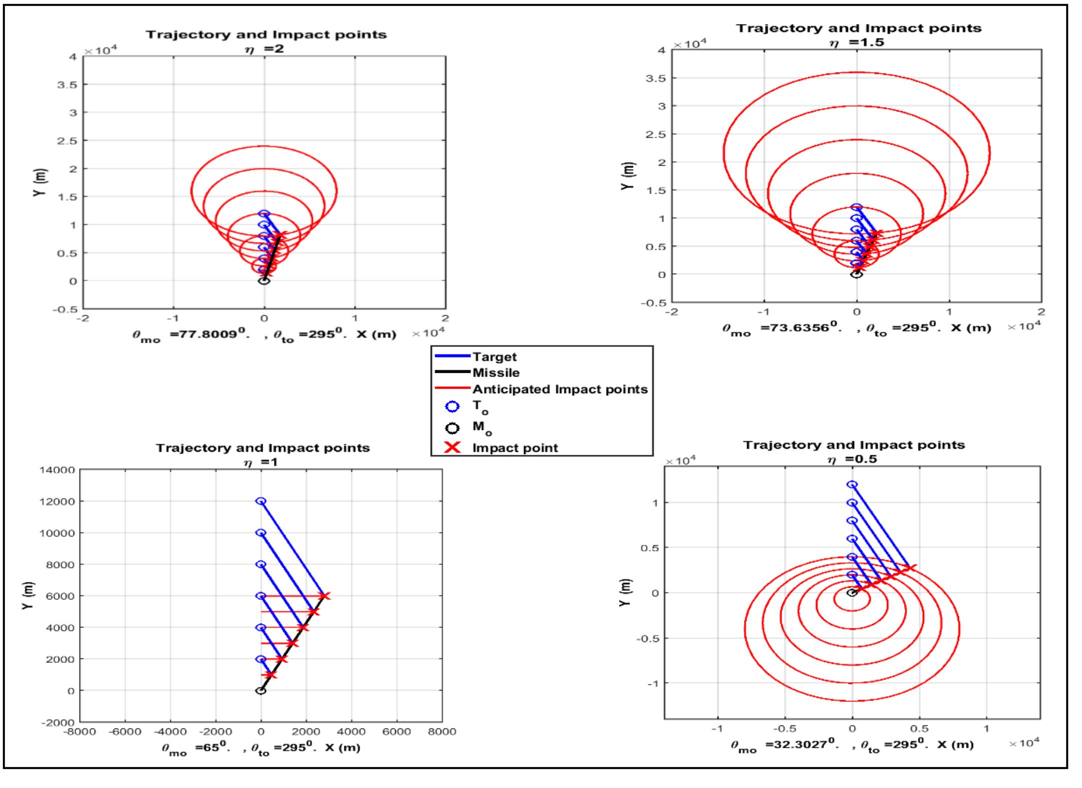

Figure 9: Intercepting possibilities for different missile-target ranges and η

Figure (9) obviously states that the smaller the range, the lower the area of collision points; the figure also illustrates that, when the missile velocity is greater than the target velocity, the circle that represents the expected positions of the impact points encloses the target initial position which mean the missile will hit the target whatever the target angle. On the other hand, when the missile velocity equals to or is less than the target velocity, then the interception will occur if and only if the target direction intersects the circle of the impact points. Which mean, if we need hitting a target with velocity greater or equal to the missile velocity, the target heading angle shall be precisely considered.

D. The Optimum Missile’s Heading and Suitable Velocity Ratio

Now we will consider different scenarios with a variable target’s heading angle and a constant missile-to-target range. Let the target initial position be at (12, 12) km from the origin point and the target velocity equal to 310 m/sec. The guidance optimization algorithm “program” will determine the best missile heading angle that guarantees the minimum missile lateral acceleration required to intercept the target as well as choosing the reasonable missile’s velocity according to design parameters or restrictions on either the time-to-go or the height of the impact point, and so on.

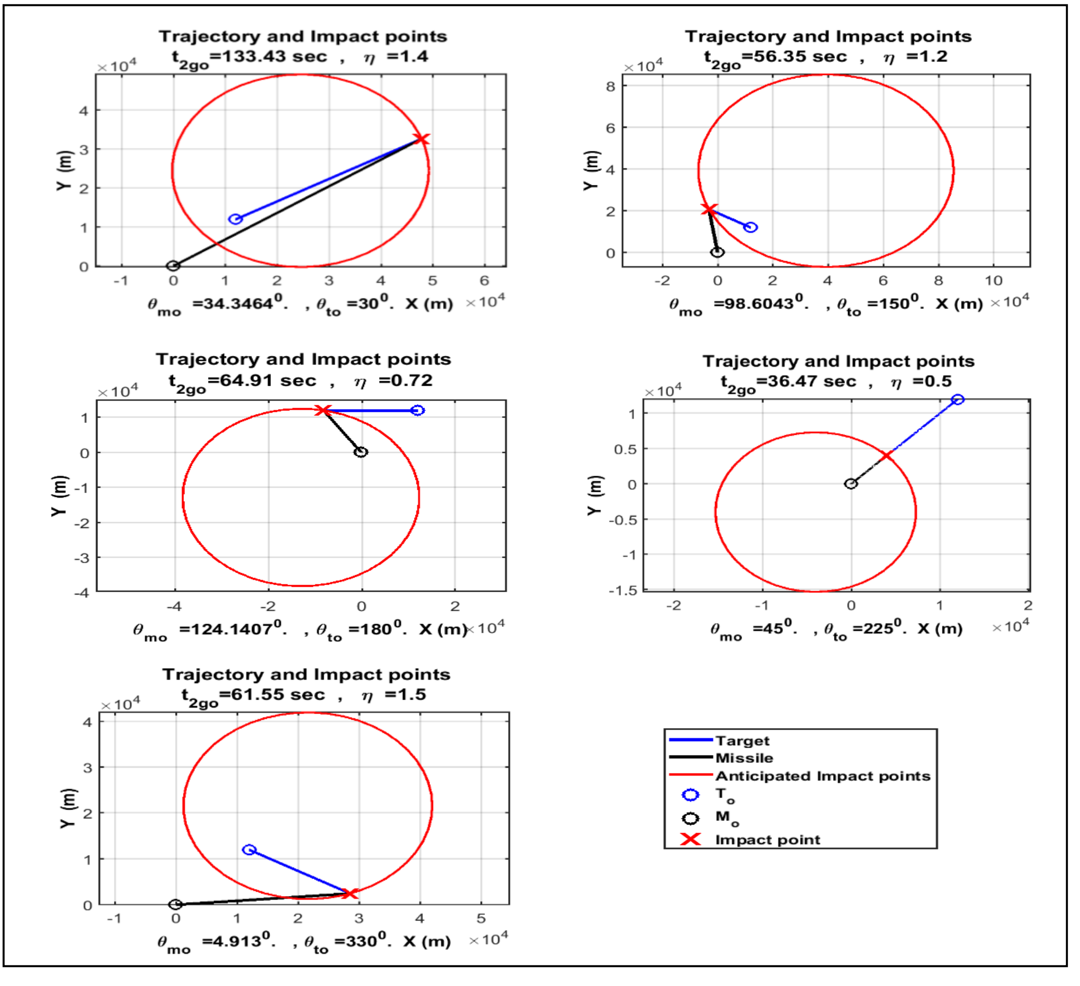

Figure 10: Optimum missile’s velocity and heading angle

The results of these scenarios are illustrated in figure (10) and their summary is shown in table (1).

Table 1: Target and missile angles & time-to-go for different η

θ_t	30^o	150^o	180^o	225^o	330^o
θ_m	34.35^o	98.6^o	124.14^o	45^o	5^o
η	1.4	1.2	0.72	0.5	1.5
t_{2g}	133 sec.	65 sec.	65 sec.	36.5 sec.	61.55 sec

5. SIMULATION EXAMPLE OF OPTIMIZED ENGAGEMENT SCENARIO

In this section, a missile-target intercepting scenario will be demonstrated to show the optimization properties. In this scenario a target flies at an initial angle equal to 200^o with a velocity =310 m/sec and maneuvering by 1g.

A. Before the Optimization Algorithm

In order to intercept the target within 22 sec., the missile requires producing normal acceleration starting from 24g and decreasing gradually as shown in figure (11). The missile initial heading angle is equal to 30^o and its velocity equal one and half of the target velocity. The simulation program displays the convergence of Lyapunov function as well as the rate of change of the LOS, and the missile curvature.

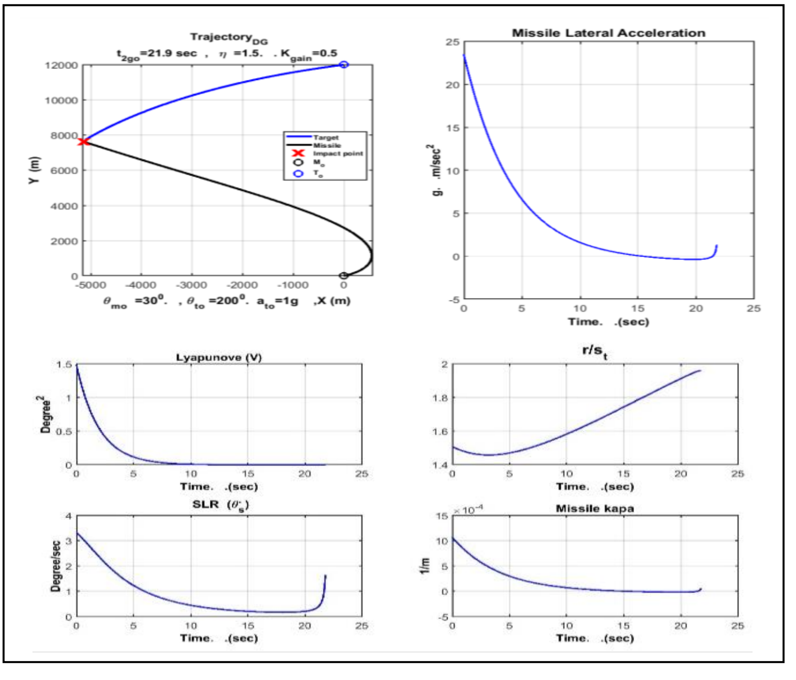

Figure 11: Non-Optimized missile velocity and heading angle {Trajectory, Latax, Lyapunov Fn. , r/s_t, SLR, missile curvature }

B. After the Optimization Algorithm

The optimization algorithm offers the possibility to choose the minimum "suitable" missile velocity, the best initial heading angle to guarantee minimum required latax. As shown in figure (12), the required missile latax has been declined to begin at -2.2g with an initial heading angle "based on the given initial missile-target positions" equal to 171.55^o. What is worth mentioning here is that, the missile can intercept the target even if its velocity is less than the target velocity ($0.9v_t$) according to the direct interception.

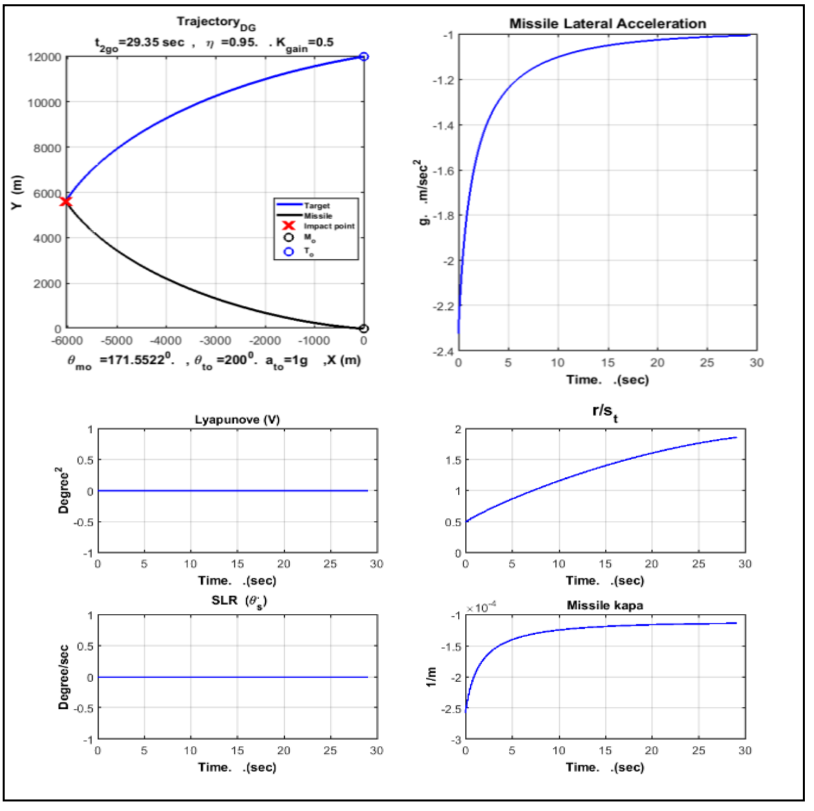

Figure 12: Optimized missile velocity and heading angle {Trajectory, Latax, Lyapunov Fn. , r/s_t, SLR, missile curvature }

6. CONCLUSIONS

The study presented in this paper used the differential geometric concepts to develop an optimization guidance algorithm for homing missiles and air defense systems. The engagement geometry and available positions of the missile against the target have been determined and illustrated based on the direct interception.

Distinct scenarios of the missile-target interception were presented and optimized from different aspects, such as the best missile heading angle to minimize the required missile normal acceleration. Furthermore, the missile velocity compared to the target was also optimized so as to guarantee the target interception by using missiles of which the maximum velocity is as small as possible according to the prior design requirements; this means that to intercept certain targets high speed missiles are not always necessary.

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