A Qualitative Comparison between the Proportional Navigation and Differential Geometry Guidance Algorithms

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Abstract: *This paper discusses and presents an overview of the proportional navigation (PN) guidance law as well as the differential geometry (DG) guidance algorithm that are used to develop the intercept course of a certain target. The intent of this study is to illustrate the advantages of the guidance algorithm generated based on the concepts of differential geometry against the well-known PN guidance law. The basic principles behind the both algorithms are mentioned. Moreover, the different versions of the PN approach is briefly clarified to show the essential improvement from one version to the other. The paper terminated with numerous two-dimension simulation figures to give a great value of visual aids, illustrating the significant relations and main features and properties of both algorithms.*

Key Words*: Homing Guidance; Differential Geometry; Proportional Navigation; Intercept; Engagement; Missile; Latax; Air Defense Systems (ADS); Line-Of-Sight(LOS)*

1. INTRODUCTION

There is variety of techniques that the guided missiles may adopt to home in on their target. Each technique requires particular mathematics laws and may be subject to some constraints. One of the common and modern guidance techniques is the line-of-sight (LOS) base technique, where the LOS and its rate is the essential source of the target information required to guide the interceptor missile [1-4].

The guidance expression stands for the way and the mean by which the guided system is steered to certain target. Both the DG and PN guidance approaches that are taken into account in this study, utilize the idea of the sight line and its rate.

The LOS is known as an imaginary line connecting the missile and the target, where the seeker function is to continuously establish the direction of this line. The basic idea of the PN experimental guidance law is to generate guidance commands as a ratio depends on the angular velocity of the LOS.

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The turning ratio of the missile, “let’s represent it as k_{pn}” is commonly named the proportional gain or constant.

In order to guarantee intercept occurring, the proportional constant should be greater than one, usually ranging between two and six [5-7]. If k_{pn} is equal to one then, the turning rate of the missile is equal to the LOS rate. Furthermore, if k_{pn} is less than one then the interception is impossible to occur, since the missile turning rate will build down a lag angle with respect to the LOS.

On the other hand, the basic idea behind the differential geometry approach is the use of calculus to describe the kinematic properties of a particle moving along continuously differentiable curves (smooth curves) in 3D Euclidean space R^3, or to determine the geometric properties of the curve itself [8-11].

By virtue of the differential geometry concepts in guidance techniques, since it was a functional method of kinematic analysis in the machine theory, we can deal directly with the nonlinearity that exists in the system without need for simplification or linearization. Hence, the convergence and stability are global [12, 13].

2. AN OVERVIEW OF THE PROPORTIONAL NAVIGATION GUIDANCE LAW

Because of its simplicity and reliability, the proportional navigation is one of the well-known guidance laws in homing missiles especially for short and medium ranges between the missile and target. The basic principle behind the PN guidance law is based on the fact that, if the rate of change of the line of sight (LOS) between two approaches bodies is zero this will guarantee that these two bodies eventually will impact on each other [14-16]. In other words, the aim of the PN guidance law (PNGL) “ideally speaking” is to prevent the sightline rotation rate against non-maneuvering targets by making the missile producing a lateral acceleration proportional to the LOS rate. The classical expression of this basic notion is as follows:

$$\boldsymbol{a_m = k_{pn} V_m \dot{\theta}_s} \qquad \text{or} \qquad \boldsymbol{\dot{\theta}_m = k_{pn} \dot{\theta}_s};$$

where:

$\boldsymbol{a_m}$ = Missile lateral acceleration (m/sec^2)
$\boldsymbol{k_{pn}}$= Navigation constant/ “effective” navigation ratio.
$\boldsymbol{V_m}$ = Missile velocity (m / sec)
$\boldsymbol{\dot{\theta}_s}$ = Sightline rate (LOS rate) (rad/sec)
$\boldsymbol{\dot{\theta}_m}$= Rate of change of missile angle (rad/sec)

The necessary missile lateral acceleration decreases as the navigation constant increase as shown in figures 1 and 2; at the same time, practically speaking, there is an upper limit for the navigation ratio due to many factors.

For instance, the potential noise that may contaminate the seeker measurements during tracking maneuvering targets. This error is directly multiplied by the navigation constant as shown in the PN guidance law.

It is worth mentioning that the value of the navigation gain k_{pn} is also subject to the requirements of the missile normal acceleration against the target maneuver capability. An acceptable range of the navigation constant/ratio usually used is from three to five so as to ensure reasonable miss distance and minimize the necessary missile lateral acceleration [2].

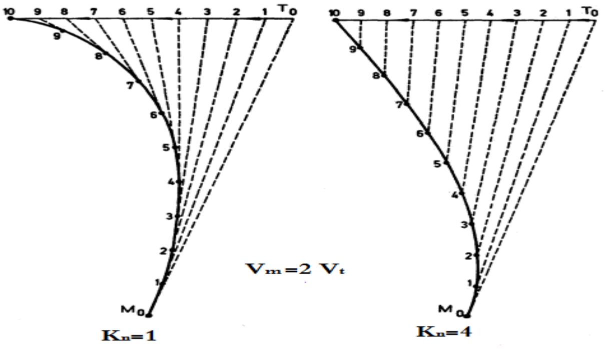

Figure 1: Missile-Target trajectories for PN constant= {1, 4}

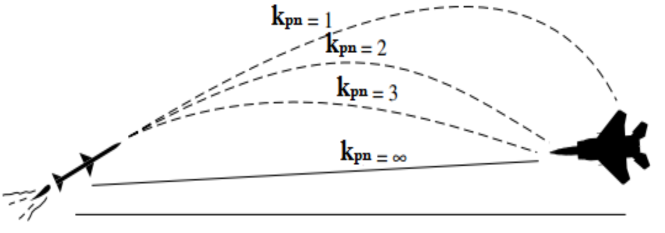

Figure 2: Missile flight/ latax for different $\boldsymbol{k_{pn}}$

Regarding the derivation of the PN approach, an exact closed form analytical solution has been obtained, but for special cases with highly restricted conditions [15].

In spite of that, the PNGL may performs acceptably in relatively wide range of interception conditions but from a practical point of view, its accuracy and capability decrease rapidly in such cases like smart and maneuverable targets and relatively large heading angle-error during the launching.

On the other hand, ideally, the PNGL required missile induced lateral acceleration to be normal to the sightline, but practically it occurs normal to the instantaneous missile unit tangent vector. Which mean that the angle between the unit vectors of the LOS and missile trajectory (θ_{ms}) plays an important role in assessing the applicability of the PN algorithm [3], as shown in figure (3).

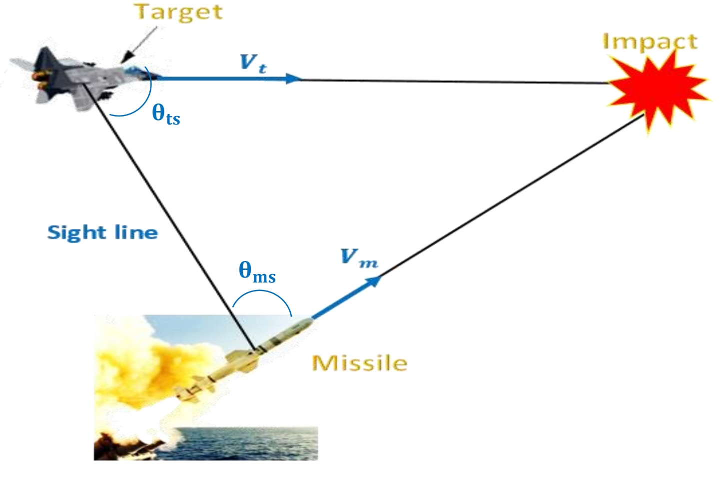

Figure 3: The homing configuration and impcat triangle

A. Parameters Affecting the PN Constant k_{pn}

We mention below some parameters that have a significant effect on the chosen value of PN effective ratio k_{pn} in addition to the missile dynamics consideration:

a) Noise

As the navigation constant k_{pn} increased this tends to increase the effect of the guidance noise associated with LOS rate $\dot{\theta}_s$. The noise tends to hide the exact value of $\dot{\theta}_s$; these noises may occur due to tracker receiver or intended guided noise signals from the target.

b) Missile's heading error angle

This parameter is highly depending on the proportional ratio k_{pn} while treatment of the heading error became greater for large k_{pn}.

c) Target maneuverability

The capture capability of the maneuverable targets increases proportionally with $\mathrm{k_{pn}}$ [1, 3].

B. Proportional Navigation Versions

In seeking to improve the classical PN approach, many modified versions have been developed. This section will briefly indicate the common versions:

a) Pure Proportional Navigation (PPN):

The required lateral acceleration is applied normally to the missile velocity direction; this approach gives a good solution for stationary targets.

b) Biased Proportional Navigation (BPN):

An additional parameter has been included in this improved version i.e. the rate bias of the rate of change of the LOS ($\dot{\theta}_b$). This version may give good performance in operation conditions outside the atmosphere [3].

$$\boldsymbol{a_m = k_{pn}\, v_m(\dot{\theta}_s - \dot{\theta}_b)};$$

c) True Proportional Navigation (TPN):

This modified method considered as good choice for nonmaneuvering targets and it produce great accuracy. However, this version is restricted to some initial conditions.

d) Generalized Proportional Navigation (GPN) and the Ideal Proportional Navigation (IPN):

These two versions are almost similar where the direction of the applied acceleration is within a constant bias angle between the perpendicular to the LOS and the normal to the missile velocity vector.

e) Augmented Proportional Navigation (APN):

This approach added new term proportional to the estimated target acceleration and may give an acceptable performance for certain maneuvering targets [1,7] and have an advantage regarding decreasing the required induced missile acceleration due to target maneuvers.

PN is widely used in tactical missiles areas. However, more relatively advanced guidance laws may have advantages according to the required missile lateral acceleration and miss distance, which mean that both the required normal acceleration and the miss distance are relatively smaller. On the other hand, these advanced approaches need more information such as well estimation of the approximate time-to-go and the target relative positions and motions.

3. DIFFERENTIAL GEOMETRY AND GUIDANCE ALGORITHM

The concepts of differential geometric control theory offer functional tools for modelling, analysis and design of the nonlinear guidance and control systems. The basic concept of differential geometry in dealing with the continuously differentiable curves (smooth curves) in 3D Euclidean space R^3, is to use the calculus to describe the kinematic properties of a particle moving along the mentioned curve or to determine the curve itself geometric properties [17].

On the other side, the differential geometry became an attractive approach to improve and offer useful guidance strategies, since the DG was used in mechanism and machine theory as a functional method of kinematic analysis [18,19].

One of the essential tools from the differential geometry uses in developing the missile guidance algorithm is the Frenet-Serret formula, which describes the derivatives of the triple unit vectors in term of each other as a function of the curvature and the torsion.

The Frenet-Serret Formula

The Frenet-Serret equations describe the curves in the space and extending notion of curvature into the torsion notion. In another word, if we have a particle moving along a manifold “continuous differentiable curve” in the Euclidean 3D space then, the Frenet-Serret formula introducing the kinematic properties of this particle by using the so-called Frenet coordinate frame as shown in figure (4). In addition to the Bi-normal Unit vector this frame includes the Tangent and Normal unit vectors, to complete the right-hand-rule of the coordinate system.

To illustrate the Frenet-Serret formula in two-dimensional space, let's suppose that we have a particle moving along a circle of radius $\boldsymbol{a}$ as depicted in figure (5) where the position and the velocity of this particle is represented by:

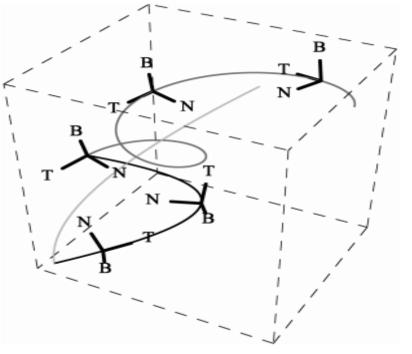

Figure 4: Frenet coordinate frame

$$\alpha(t) = (a\,cos(t), a\,sin(t))$$

$$\alpha'(t) = (-a\,sin(t), a\,cos(t))$$

where, $|\alpha'(t)| = a$ which represents a constant speed curve.

Let $\alpha(t)$ be a regular curve i.e $\alpha'(t) \neq 0$ which means the particle keep moving all the time. If we parameterize the particle equations using the arc length (s) where

$$s = \int |\alpha'(t)| dt \qquad \text{then} \qquad t = \frac{s}{a}$$

then the new curve is represented as follows:

$$\beta(s) = \left(a \cos\left(\frac{s}{a}\right), a \sin\left(\frac{s}{a}\right)\right)$$

$$\beta'(s) = (-\sin(\frac{s}{a}), \cos(\frac{s}{a}))$$

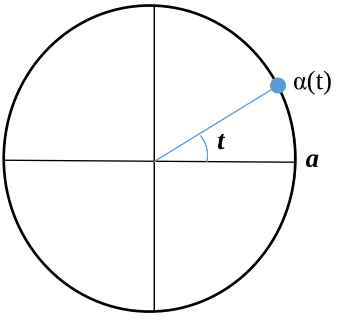

Figure 5: Circle represents particle motion

where $|\beta'(s)| = 1$ which represents a unit speed curve.

The particle acceleration is $\beta''(s) = (-\frac{1}{a}\cos(\frac{s}{a}), -\frac{1}{a}\sin(\frac{s}{a}))$ and its length is $|\beta''(s)| = \frac{1}{a}$ which is the curvature$k$. If we normalized the acceleration vector, we get

$$N(s) = \frac{\beta''(s)}{|\beta''(s)|} \; or \; N(s) = (-\cos(\frac{s}{a}), -\sin(\frac{s}{a}))$$

which is a unit normal vector representing the direction of the acceleration of the moving particle.

$$T(s) = \frac{\beta'(s)}{|\beta'(s)|} = \beta'(s)$$

$T(s)$ the unit tangent vector representing the particle velocity direction. So, we can define the curvature as the length of the derivative of the tangent vector $T(s)$ i.e. $\kappa(s) = |T'(s)|$ where $T'(s) = \beta''(s)$

If we generalize the above analysis, we can conclude that at any point p on the curve β $N(s)$ points towards the center of the curvature “center of the approximate circle at that point p.

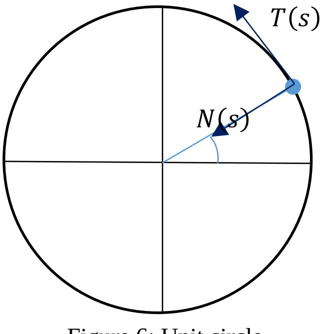

Figure 6: Unit circle

To complete the $T(s)$ and $N(s)$ coordinate system by using the right-hand-rule we defined the Bi-normal vector $B(s) = T(s) \times N(s)$, and then we get $(T(s), N(s), B(s))$ as the Frenet frame of the curve β. The Frenet-Serret equation defines the rate of change of the three-unit vectors in the three directions as follows:

$$\begin{bmatrix} \boldsymbol{T'(s)} \\ \boldsymbol{N'(s)} \\ \boldsymbol{B'(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\kappa(s)} & \boldsymbol{0} \\ \boldsymbol{-\kappa(s)} & \boldsymbol{0} & \boldsymbol{\tau(s)} \\ \boldsymbol{0} & \boldsymbol{-\tau(s)} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{T(s)} \\ \boldsymbol{N(s)} \\ \boldsymbol{B(s)} \end{bmatrix}$$

To derive that, we get the first equation from the definition, where

$$T'(s) = \beta''(s), \qquad N(s) = \frac{\beta''(s)}{|\beta''(s)|}, \kappa(s) = |T'(s)|$$

Then:

$$\boldsymbol{T'(s) = \kappa(s)\, N(s)} \tag{A.1}$$

Let us claim and then prove that, $B'(s)$ is a multiple of $N(s)$

$\because B(s).B(s) = 1$ unit vector everywhere then $B'(s) \perp B(s)$

$\because B(s).T(s) = 0$ by differentiating it, we get:

$$B'(s).T(s) + B(s).T'(s) = 0$$

We know from (A.1) that, $T'(s)$ is a multiple of $N(s)$ and $B(s) \perp N(s)$ then $B(s).T'(\mathrm{s}) = 0$; then $B'(s).T(s) = 0$;

So $B'(s) \perp B(s)$ *and* $B'(s) \perp T(s)$ and we know that there is only one direction perpendicular to $B(s)$ *and* $T(s)$ that is the normal vector $N(s)$ then we conclude that, $B'(s)$ is a multiple of $N(s)$; we can define $\tau(s)$ to be that multiple and we call $\tau(s)$ the torsion of the curve. Thus, we get the Frenet-Serret third equation as follows:

$$\boldsymbol{B}'(\boldsymbol{s}) = -\boldsymbol{\tau}(\boldsymbol{s})\boldsymbol{N}(\boldsymbol{s}) \tag{A.2}$$

The negative sign is for historical reasons. Now to get the second equation which is evaluated as $N'(s)$

$\because N = B \times T$ and its derivative is $N'(s) = B'(s) \times T(s) + B(s) \times T'(s)$

Then: $N'(s) = -\tau(s)\, N(s) \times T(s) + B(s) \times \kappa(s) N(s)$ or

$$N'(s) = \tau(s)\, B(s) + \kappa(s)(-T(s))$$

$\therefore$The second equation is

$$\boldsymbol{N}'(\boldsymbol{s}) = -\boldsymbol{\kappa}(\boldsymbol{s})\, \boldsymbol{T}(\boldsymbol{s}) + \boldsymbol{\tau}(\boldsymbol{s})\boldsymbol{B}(\boldsymbol{s}) \tag{A.3}$$

To this end, by virtue of the Frenet-Serret equations in addition to the Lyapunov theory we derive the DG guidance algorithm. Lyapunov tests whether the system dynamics are stable/asymptotically stable. The resultant DG guidance law introduced in detailed in [19] shows that there is no restriction on the initial conditions of both missile and target.

$$\dot{\theta}_m = f(\eta, \dot{\theta}_s, \frac{r_s}{s_t}, \theta_\varepsilon, K_{DG}, \theta_{\mathrm{ms}}, \theta_{\mathrm{ts}})$$

$$a_m = v_m \dot{\theta}_m;$$

where:

$\dot{\theta}_m$	= Rate of change of actual missile angle.
a_m	= Missile lateral acceleration (LaTax).
v_m	= Missile velocity.
η	= Missile-target velocities ratio.
$\dot{\theta}_s$	= Rate of change of the LOS angle.
$\frac{r_s}{s_t}$	= Range to target arc length ratio.
θ_ε	= The error angle between actual and required missile tangent vector.
θ_{ms} & θ_{ts}	= Relative angle of the missile and target.
K_{DG}	= DG guidance gain.

The limitation will appear only due to physical reasons, for instance, limitation on the missile lateral acceleration and on the accuracy and capability of the tracking sensor and the sensor looking angle range. The DG approach also offers flexibility and a set of possibilities in controlling and choosing the intercept trajectories. Furthermore, it allows us to optimize the engagement trajectory and the guidance algorithm as illustrated in [20, 21].

4. SIMULATION RESULTS OF PN AND DG GUIDANCE ALGORITHMS

The aim of this section is to compare the use of differential geometry approach and PNGL in the homing missile guidance algorithms and to present the major characteristics and properties for the two approaches. At the same time, the simulation also illustrates their advantages against each other. The missile guidance and control loop and the simplified block diagram are illustrated in figures 7 and 8, respectively, where the missile controls the system and the target dynamics controls the missile motion. The terminal sensor and the state estimator determine the relative geometry and feed the guidance law by the missile range, LOS angle and their rates of change. The guidance algorithm then introduces the steering commands such as the required missile lateral acceleration to the flight control system. The flight dynamics and control system force the actuators and aerodynamics surfaces to follow the guidance commands so as to ensure accurately interception the given target.

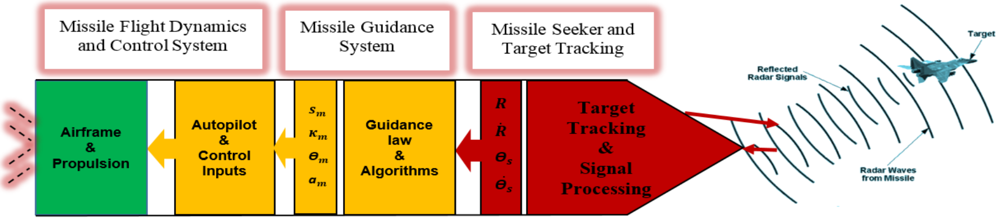

Figure 7: Scheme of missile guidance and control loop

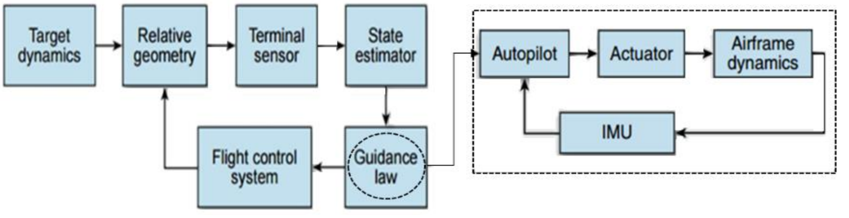

Figure 8: The basic elements of the missile flight control system

The comparison is performed in different stages to show the effect of the different parameter variation on the intercept characteristics and engagement configuration. Three stages will consider observing the interception specifications when:

1) Change the missile velocity.
2) Change the range between the missile and the target.
3) Special case illustrates the DG approach as a generalized guidance law.

A. Frist Stage: Different Missile Velocities

In the first stage, the initial positions and heading angles of both the missile and the target remained fixed. The lateral acceleration of the missile and the target and the guidance gain in the PN and DG algorithms also not changed. We changed only the missile velocity as a ratio of the target velocity. The inputs to the simulation program are as follows:

Table 1: Simulation inputs of the DG vs PN. 1st stage

Target initial position	X=-2000 m, Y=12000 m
Missile initial position	(0,0)
Target initial angle	θ_t=-30°
Target latax	a_{to}= -2g
Missile initial angle	θ_m=0°
Missile latax	a_{mo}=1g
Gain (K_{DG})	0.5
Gain (K_{PN})	5

In figure (9) the missile velocity is 372 m/sec i.e. (η=1.2). The simulation results demonstrate the engagement trajectory in both the DG and the PN guidance laws, the missile lateral acceleration during the flying, the curvature of the missile and the missile’s heading angle as the engagement proceeds also shown in the simulation results to give a complete overview of the two approaches.

The results depict that the required approximate time-to-go in the DG guidance approach is about (20 sec) where in the PN guidance law it's (28 sec). The required missile latax during the flight, is also illustrated. The simulation stated that the benefit of the smaller time-to-go in DG guidance required relatively larger initial normal acceleration in order to correct the heading angle error.

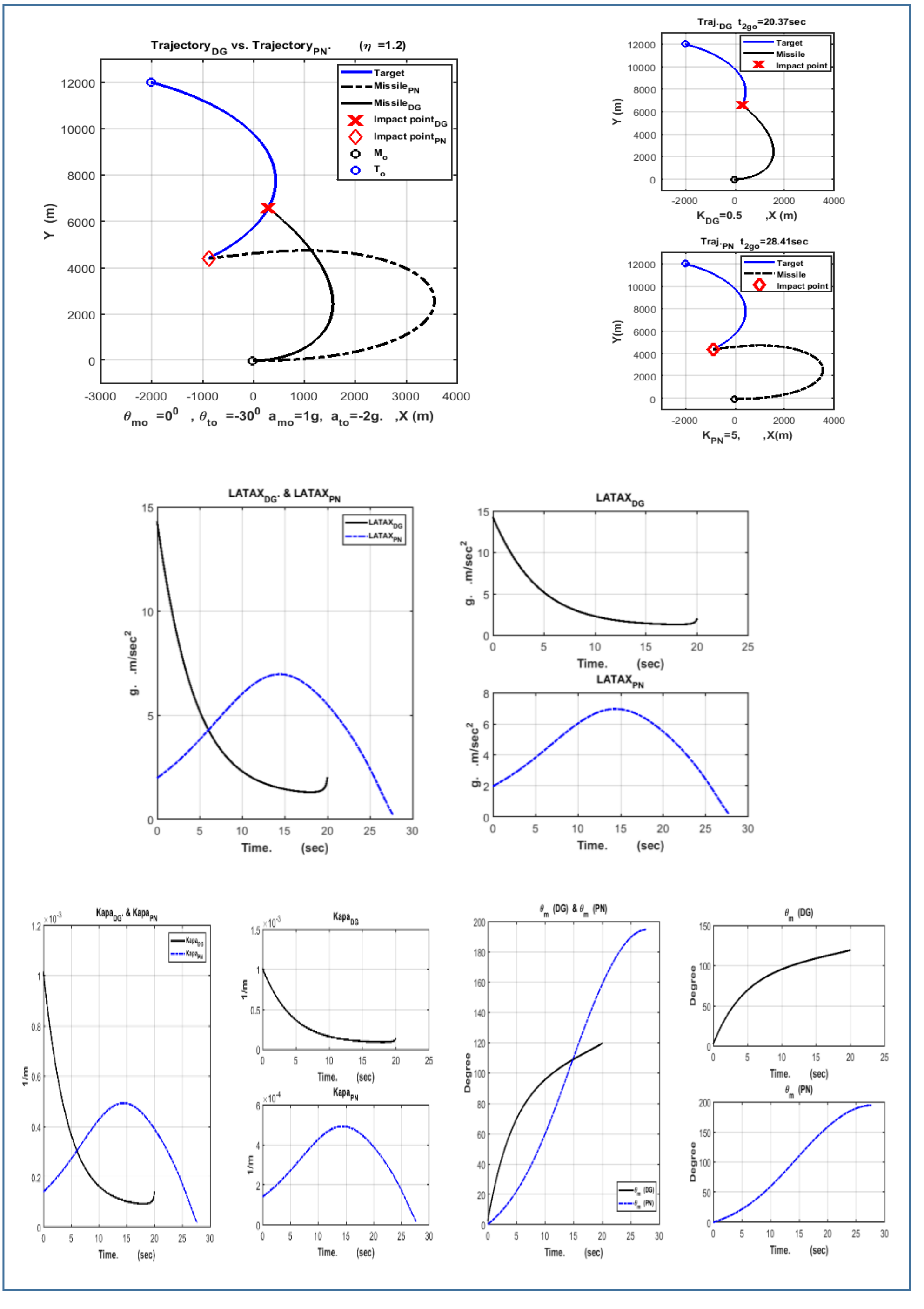

Figure 9: PN vs DG {Trajectories – LaTax – Missile Curvature} at η=1.2

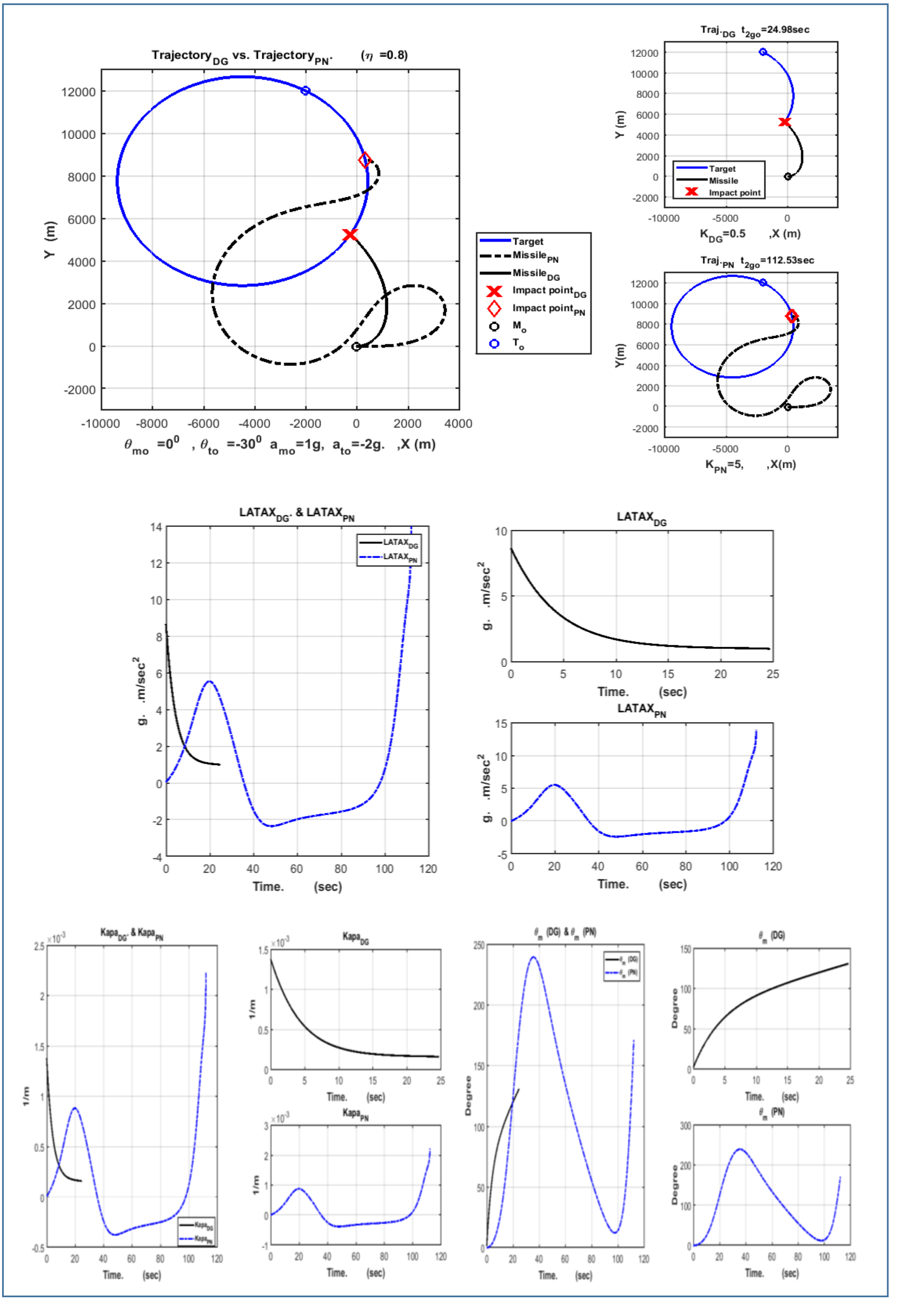

Figure 10: PN vs DG {Trajectories – LaTax – Missile Curvature} at η=0.8

More precisely at the first 5 sec. the missile latax declined from 13g to 5g then keep decreasing to reach the minimum latax a_{mo}=1g as we enter this value to the program as a lower limit in the DG guidance approach. We mention that the lower limit can also be zero or a negative value.

In (PN) law, the missile trajectory is considering only the LOS rate and it is based on the direct interception, this is why the latax performance in the PN approach seems as not controlled to fallow certain pattern.

Now let us decrease the missile velocity and see the new characteristics of the engagement configuration. Figure (10) illustrates the simulation results of the engagement when the missile velocity is equal to (248 m/sec) i.e. (η=0.8).

It is noticeable that the distinction between the engagement flight times in both approaches is obviously large.

In the DG guidance law t_{2go}=25 sec while in PN guidance law t_{2go}=113 sec. Also, the missile lateral acceleration during the flight is clearly shown in figure (10) where the pattern of the DG latax changes smoothly within smaller boundary in comparison with the PN one.

In addition to the previous two cases, the simulation also tests the case when the missile velocity increased to be twice the target velocity which decreased the impact time to 15 sec, 17 sec. in DG and PN guidance laws, respectively.

To this end, in this stage of distinguishing, it seems that as the missile velocity decreases relative to a maneuvering target and the heading angle error is considerably large, the DG guidance law ensures acquiring the impact quicker, and the missile lateral acceleration required to achieve the interception is better and within the design limits.

B. Second Stage: Different Missile-to-Target Ranges

In this stage of comparison, the target and missile velocities for both cases are 372 m/sec i.e. (η=1.2). The initial heading angles of the missile and the target are 300^{o} and 150^{o} respectively.

The target is maneuver in a constant curvature at a latax equal to 1g. The guidance gains in DG approach and PN approach are 0.4 and 4, respectively. We only changed the ranges between the missile and the target.

The simulation results in figure (11) demonstrate the engagement when the range is about 12km where the target's initial position is set to (2000, 12000) m, and figure (12) displays the results when the range is smaller {about 10km}.

Table 2: Simulation inputs of the DG vs PN. 2nd stage

Missile initial position	(0,0)
Target initial angle	θ_{to}=300^{o}
Target latax	a_{to}= 1g
Missile initial angle	θ_{mo}=150^{o}
η	1.2
Gain (K_{DG})	0.4
Gain (K_{PN})	4

Again, as shown in the simulation results, the time-to-go in the DG guidance algorithm is smaller; the difference is about 20 sec. the curvature of the missile during the flight is within the design limits.

The simulation results also illustrate the time history of the missile angle and the change in the lateral acceleration.

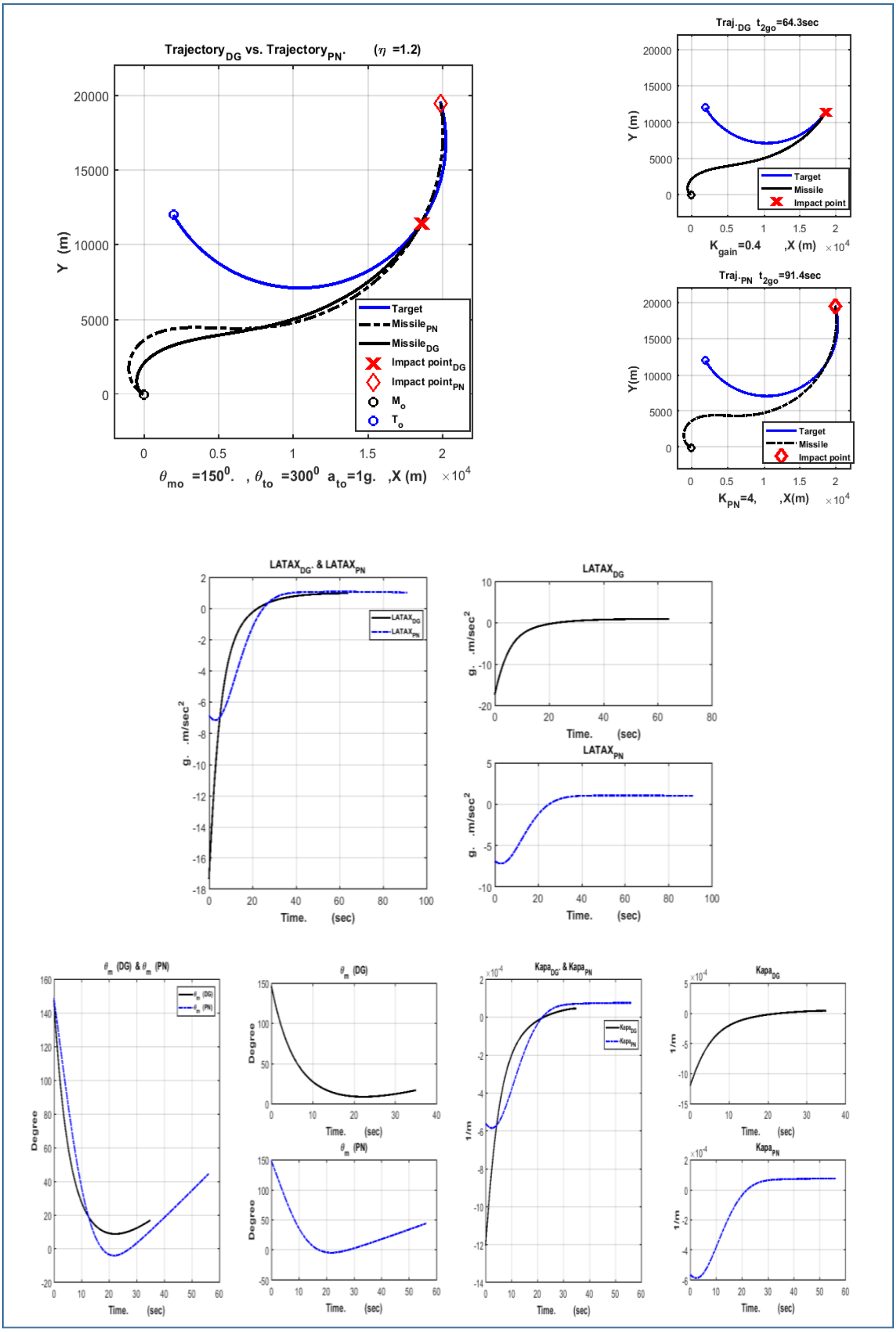

Figure 11: PN vs DG {Trajectories – LaTax – κ_m }, (x_{to}, y_{to})=(2000, 12000)

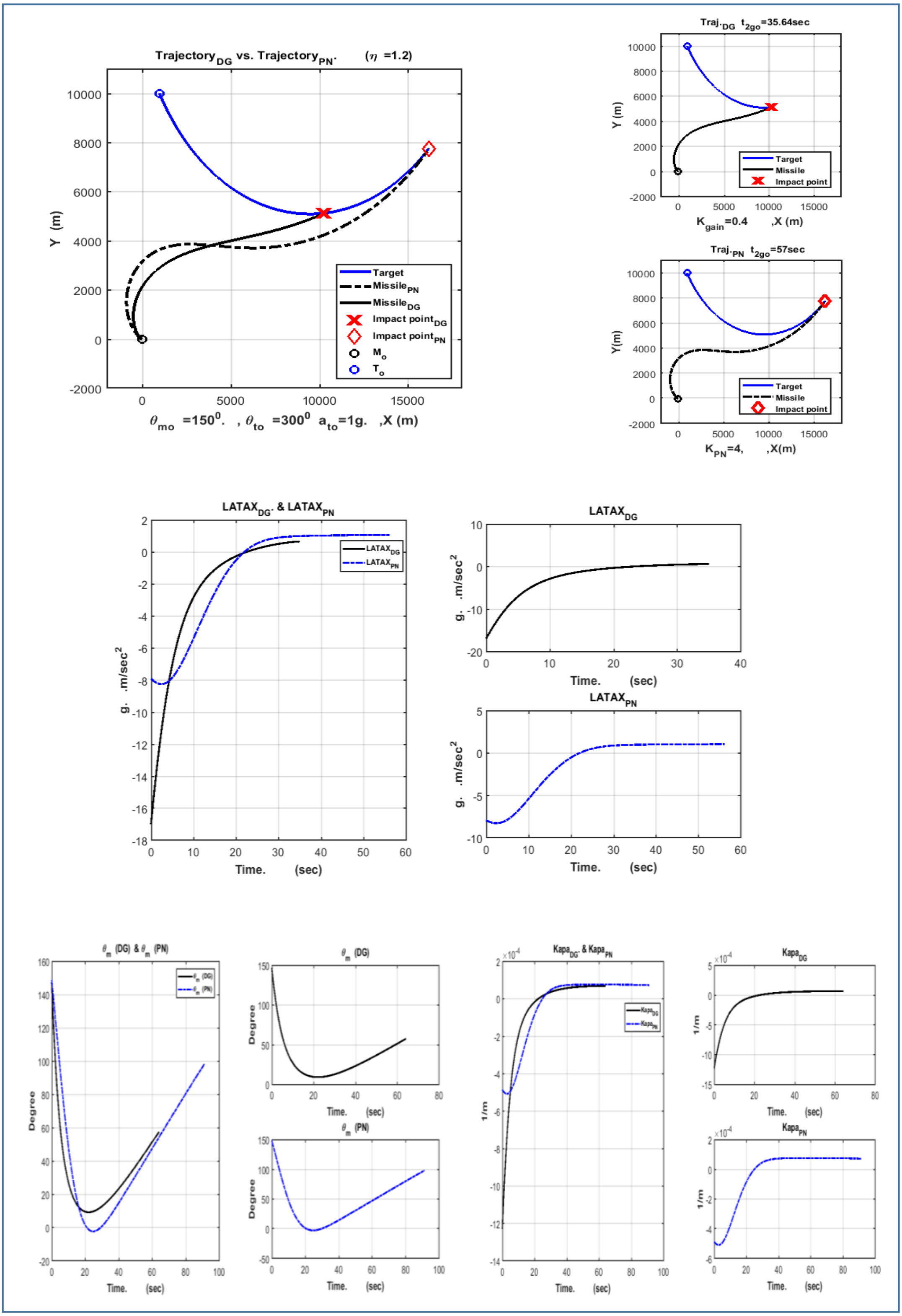

Figure 12: PN vs DG {Trajectories – LaTax – κ_m }, (x_{to}, y_{to})=(1000, 10000)

C. DG Approach as a Generalized Guidance Approach

The following comparison just shows the generality of our method and how the DG guidance algorithm looks like a global approach. The input parameters are:

Table 3: Simulation inputs of the DG vs PN. 3rd stage

Target initial position	(20,20) km
Missile initial position	(10,10) km
Target initial angle	θ_t=45°
Target latax	a_{to}= 0g
Missile initial angle	θ_m=225°
H	2
Gain (K_{DG})	0.3
Gain (K_{PN})	15 or any

In this configuration as shown in figure (14), the location and initial direction of the missile and target are located along the same line and in opposite directions. The target velocity is 310 m/sec and (η=2), the heading angle of the missile is in the opposite direction of the target's heading angle. The simulation results demonstrate that using PN guidance algorithm will not guarantee the anticipation whatsoever the guidance gain $\{k_{\mathrm{PN}}\}$ and the missile velocity are.

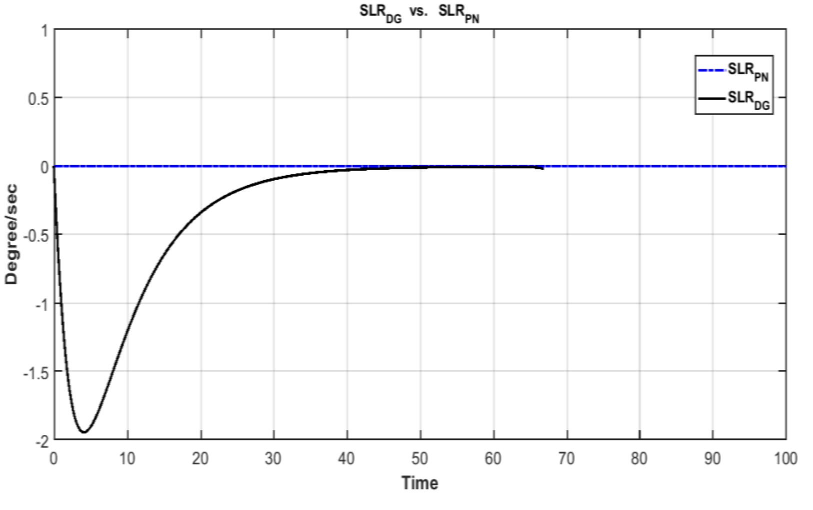

Figure 13: SLR in case of using DG and PN approaches

This is because the SLR will remain zero along all the missile flight as shown in figure (13); this mean the missile's heading angle also will remain constant and equal to the initial heading angle. Conversely, the DG guidance algorithm will ensure the missile-target impact and the required time-to-go is (67 sec).

5. CONCLUSIONS

The study presented in this article deals with well-known proportional navigation guidance law and the guidance algorithm generated base on the differential geometry concepts. The study illustrates the essential idea behind each approach as well as the major properties and features of the PN and DG guidance laws. The qualitative study draws conclusions as follows:

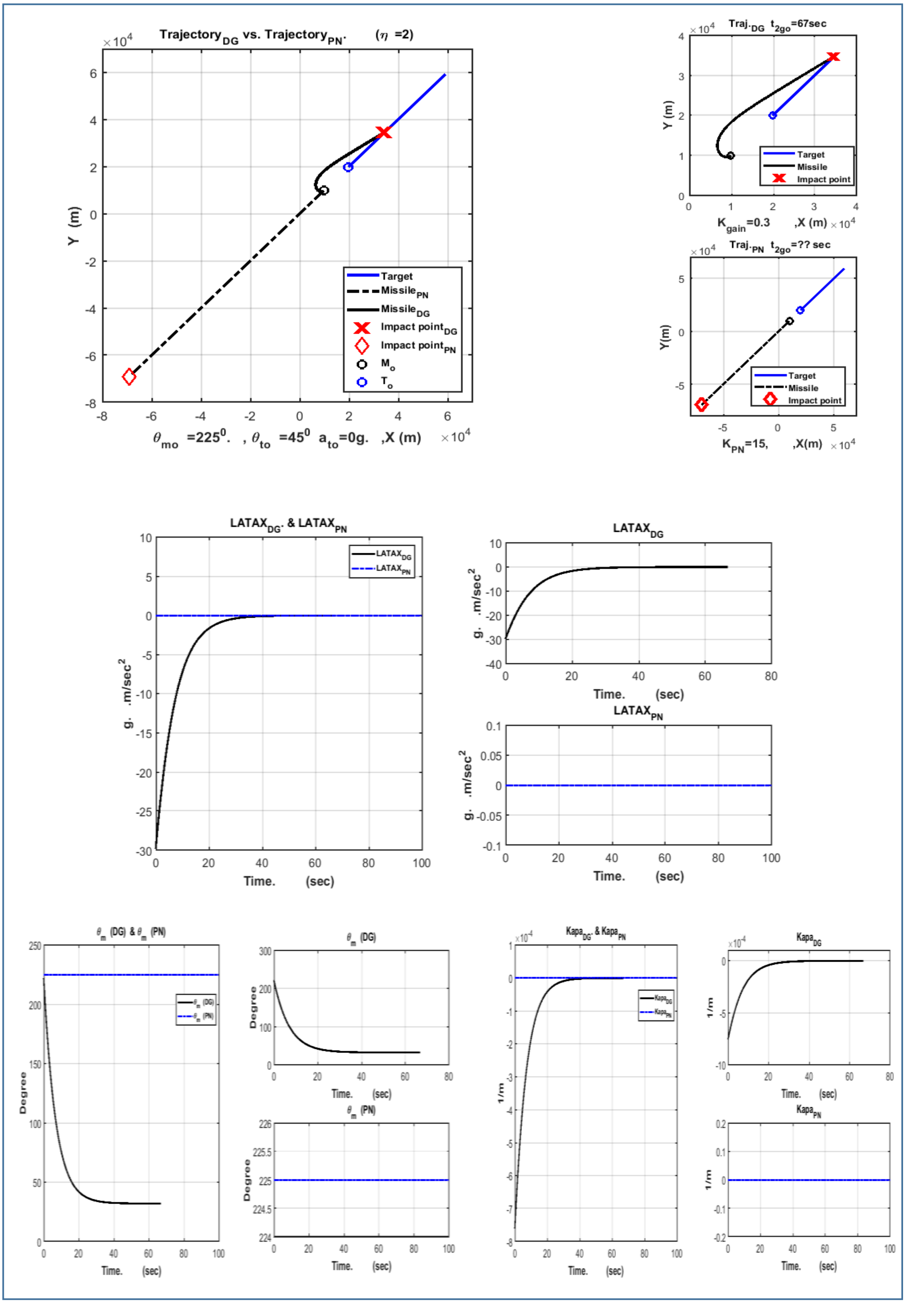

Figure 14: PN vs DG {Trajectories – LaTax – Missile Curvature}, “Theoretical case”

(1) The DG approach considered as more generalized guidance algorithm, which deals with curved and straight-line trajectories and the PN, is a subset of DG,
(2) The DG approach deals with the nonlinear geometry directly hence, the convergence and stability are global,
(3) The DG methods offer flexibility and a set of possibilities in controlling and choosing the intercept trajectories,
(4) Traditional PN is based on straight line and constant velocity, and it does not allow us to control the choice of possible intercept trajectories. From another side, the necessary missile lateral acceleration is initially greater when we use the DG approach to quickly correct the missile initial heading and the pattern of latax is monotonic for the DG approach.

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