Case study of TCAS implementation in modern FMS

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Abstract: *In this paper, uncertainty is included in the extended framework of global air traffic both through the action of the wind and through the incomplete determination of the aerodynamic coefficients and the inevitable imprecision involved in the execution of ATC instructions. In order to study the air conflict detection process in detail, the possibility of potential conflicts must be evaluated, considering the current state of the international airspace, and taking into account the uncertainty in calculating the future position of the aircraft. A mathematical model for predicting potential aerial conflicts is necessary for this objective. In a probabilistic framework, the mathematical model thus created could be either an empirical distribution of future aircraft positions in space or a dynamic model, such as a stochastic differential equation, that describes the movement of the aircraft and implicitly defines a distribution for the position's future of the aircraft/ future aircraft positions. Based on the prediction model, aircraft flight safety matrices can be evaluated. The mathematical methods applied in the Detection and Resolution of Air Traffic Conflicts - CDR involve a wide range of fields, but also the appropriate modeling of physical systems such as airplanes, the codification of mathematical algorithms for conflict detection, as well as the properly applied procedures for resolving potential air conflicts.*

Key Words: *ATC, ATM, FMS, Flight Simulator, TCAS*

1. INTRODUCTION

The detection and resolution of air traffic conflicts, a process synthetically called CDR, represents the process of predictability, perfect communication with the pilot, and the need to resolve the air conflict quickly and correctly in FMS. TCAS information is needed that is later distributed to ATC, which communicates continuously with the pilot to resolve the air conflict immediately and perfectly. Obviously, uncertainty is thus introduced into the air traffic process through the mode of action of the wind speed, but also because of the incomplete knowledge of all the considered aerodynamic coefficients and through the inevitable imprecision in the reaction in carrying out the instructions given by the air traffic controller. To correctly detect air conflicts, the possibility of potential air conflicts must be evaluated, considering the modern state of the international airspace and the uncertainty in determining the future position of the considered aircraft. To fulfill this task, a mathematical model is needed to predict the potential conflicts in the considered airspace.

Obviously, in a probabilistic framework, the mathematical model could be an empirical distribution of future positions of the aircraft, or a dynamic model, such as a stochastic differential equation, which thus describes the dynamics of the aircraft and implicitly defines a correct distribution for future positions of the aircraft, at a later point in time. Based on the prediction model, matrices related to air transport safety can be evaluated [1-5].

2. STATE OF ART FOR TCAS

Most of the CDR problems in real aviation include numerous modeling uncertainties, which appear due to the existence of disturbances, calculation and estimation errors, and aircraft continue their flight, obviously, under the action of various uncertainties, such as, errors due to unforeseen meteorological phenomena, as well as those of air navigation. All these uncertainties have an important influence on the dynamics of the considered aircraft/target, therefore also on the detection and resolution of potential air conflicts. The stochastic optimal control method is combined with the probabilistic conflict detection algorithm to guarantee the correct resolution of potential conflicts between aircraft under the action of atmospheric wind uncertainty. By correctly estimating optimal trajectories without any air conflict, thus starting from any given initial states under wind uncertainty, they can thus be generated, in real-time, without solving stochastic optimization problems and without somehow minimizing the accuracy process [6-9]. Conflicts between aircraft positioned in a two-dimensional horizontal plane, in which the intruding aircraft approaches from different spatial directions and thus merges with the crossing point, are considered.

The aircraft dynamics is given by the following point mass model with three state variables $x = (x, y, \Psi)^T$ and one control variable - u: [10-15]

$$\dot{x} = v cos\Psi + \mathrm{w_x} \tag{1}$$

$$\dot{y} = v sin\Psi + \mathrm{w_y} \tag{2}$$

$$\dot{\Psi} = u \tag{3}$$

$$C\big((x, y), (x', y')\big) = \sigma_w^2 \exp\ (-\mu_x |x - x'|) exp\big(-\mu_y |y - y'|\big) \tag{4}$$

$$w_x(x, y) \approx \sum_{i=1}^{N_{KL}} \left(\sqrt{\lambda_i}\ \ g_i\ (x, y)\ \theta_{xi}\right) \tag{5}$$

$$w_y(x, y) \approx \sum_{i=1}^{N_{KL}} \left(\sqrt{\lambda_i}\ \ g_i\ (x, y)\ \theta_{yi}\right) \tag{6}$$

$$\lambda_i g_i(x, y) = \int_D C\big((x, y), (x', y')\big)\ g_i\ (x', y')\ dx' dy' \tag{7}$$

$$d_{H\ min} \le L_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}\ \ (\forall\, i,j \in \{1,\dots,s\}: i \ne j) \tag{8}$$

$$J = E\left[g_M\ \left(x(0),\ \ x\left(t_f\right)\right) + \int_0^{t_f} g_L(x(t),u(t),t)\ dt\right] \tag{9}$$

$$\dot{x}(t) = f(x(t),u(t),t) \tag{10}$$

$$b_{min} \le E\left[b\left(x(0),x\left(t_f\right)\right)\right] \le b_{max} \tag{11}$$

$$\eta_{min} \le P_r[c_{min} \le c(x(t),u(t),t) \le c_{max}] \le \eta_{max} \tag{12}$$

$$CP_{i-j} = \int_{-\Delta T_{sep}}^{\Delta T_{sep}} P_{i-j}\ (\tau)\, d\tau \tag{13}$$

$$P_{i-j}(\tau) = \int_{-\infty}^{\infty} P_i\ (t)\ \ P_j\ (t+\tau)d\tau \tag{14}$$

$$-\Delta T_{sep} \le \tau \le \Delta T_{sep} \tag{15}$$

$$\Theta = \bigcap_{j=1}^{m_k} \theta_{jt}\,,\ \ j = 1,\dots,m_k\quad ;\ t = 0,1,\dots,T \tag{16}$$

$$\widehat{\mathbf{\Omega}} = \left|\widehat{\omega_{Jt}}(\Theta)\right|;\ \ \widehat{\omega_{Jt}}(\Theta)\ \ \begin{cases} 1\ \ if \quad \theta_{jt} \in \Theta \\ 0\ \ \text{otherwise} \end{cases} \tag{17}$$

$$\phi_i = \psi_i - u\,;\ \ \bar{a} = \ v_i\, cos\phi_i - v_j\, cos\psi_j\,;\ \bar{b} = \ r_i\, cos\theta_i - r_j\, cos\theta_j \tag{18}$$

$$\bar{c} = \ v_i\, sin\phi_i - v_j\, sin\psi_j\,;\ \bar{e} = \ r_i\, sin\theta_i - r_j\, sin\theta_j\,;\ \ i,j = 1,2,\dots,N \tag{19}$$

$$\begin{aligned} S_{ij}^1(t) = &\left(x_i(t) - x_j\ (t)\right)^2 + \left(y_i(t) - y_j\ (t)\right)^2 = \\ &= \left[\left(v_i\, cos\phi_i - v_j\, cos\psi_j\right)t + \left(r_i\, cos\theta_i - r_j\, cos\theta_j\right)\right]^2 + \\ &+\left[\left(v_i\, sin\phi_i - v_j\, sin\,\psi_j\right)t + \left(r_i\, sin\theta_i - r_j\, sin\theta_j\right)\right]^2 = \\ &= \left(\bar{a}\, t + \bar{b}\right)^2 + \ (\bar{c}\, t + \bar{e})^2 \end{aligned} \tag{20}$$

$$\bar{b}^2 + \bar{e}^2 - \frac{\left(\bar{a}\,\bar{b} + \ \bar{c}\,\bar{e}\right)^2}{\bar{a}^2 + \bar{c}^2} \ge R^2 \tag{21}$$

$$w_{max} = \max_{\mathrm{i}}\ \ |w_i|\ \cdot sign(w_i),\ \ i = 1,2,\dots,N. \tag{22}$$

$$u = \begin{cases} u_{exact\ conflict}\ \ \cdot sign\ (w_{max})\ \ if\ \ u_{exact\ conflict} > |w_{max}| \\ \qquad w_{max}\ \ \ if\ \ \ u_{exact\ conflict} \le |w_{max}| \end{cases} \tag{23}$$

3. EXPERIMENTAL RESULTS AND CONCLUSIONS

At the time when a conflict is detected, all aircraft involved in it prepare to quickly initiate a maneuver to resolve the conflict. It is assumed that the aircraft involved in this conflict flies at a constant speed in the resolution maneuver. The disorder of the considered system is represented in the measurements determined from non-target factors, such as nearby objects, but also the weather, electromagnetic interference, which are generally random quantities in number, location, and intensity. From the measurements with included uncertainties, we cannot correctly determine the number of aircraft included in the surveillance region and how the aircraft trajectory is presented. To develop a conflict detection and resolution algorithm for the air traffic controller, a mathematical algorithm for state estimation is needed, which can track the trajectories of the multitudes of existing aircraft within the surveillance region of the sensors used. A finite partition of the airspace positioned in the vicinity of the conflict is made in real-time according to the minimum relative angle between the aircraft. The results from the exact and inaccurate conflict cases are considered, from which the protocol is derived to resolve even the worst case of conflict within each considered partition. The change in direction from the original trajectory is the primary control input and is used exclusively when the conflict is mathematically determined with precision. The change in velocity is also used as a control input parameter when an initially thought to be perfectly accurate assumption of conflict fails.

Fig. 1 Case Study # 1 - intruder at 15 NM

Fig. 2 Autopilot

Fig. 3 Case Study # 2 – potential conflict at 5 NM

Fig. 4 CD– null resolution – quickly detection

Fig. 5 Zone Clear of Conflict

Fig. 6 Area of potential conflict avoided

Fig. 7 TCAS display

Fig. 8 TCAS is activated by distance < 5 NM

Fig. 9 TCAS is activated

Fig. 10. Aircraft Collision Avoided

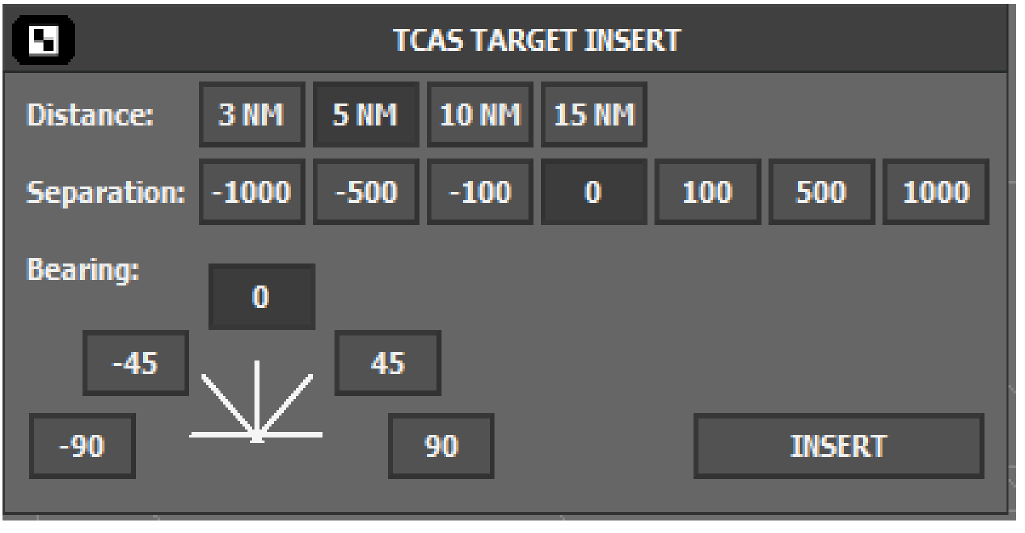

Fig. 11 TCAS target insert # Phase 1

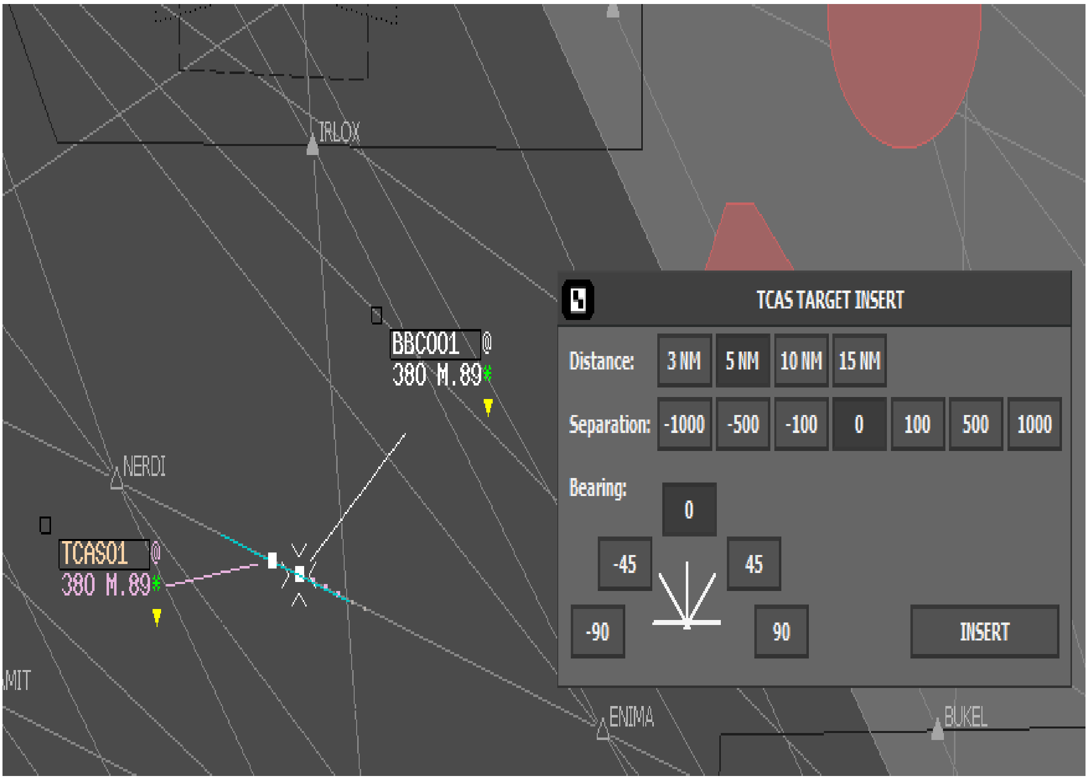

Fig. 12 TCAS target insert # Phase 2

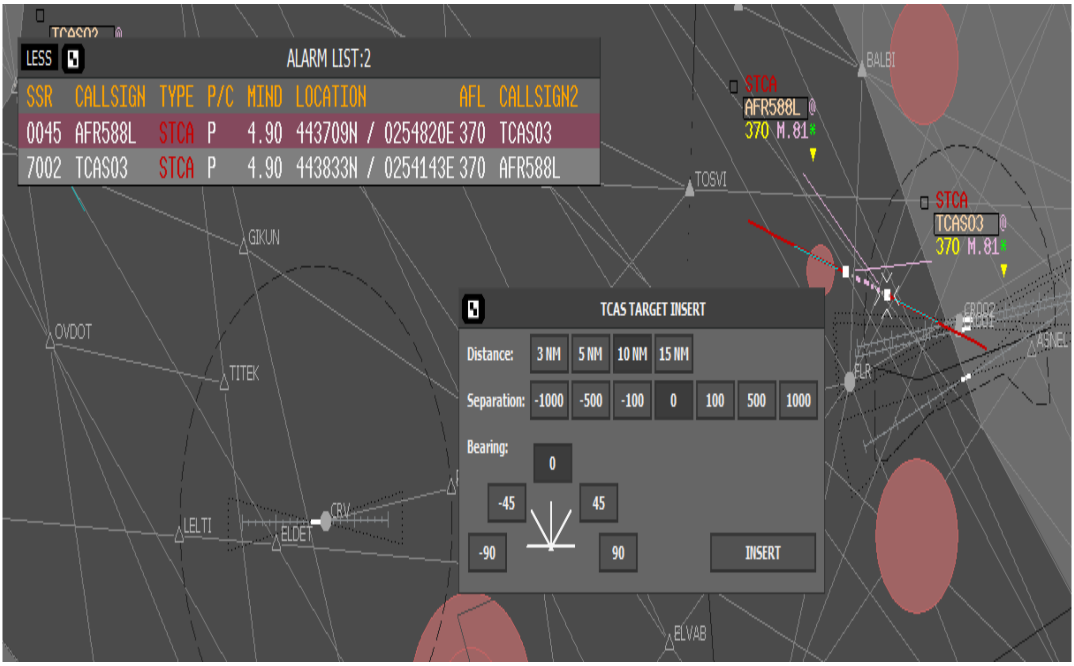

Fig. 13 TCAS target insert # Phase 3

This paper presents how the stochastic optimal control method is combined with the mathematical algorithm used to detect air conflicts, thus generating a new proposed algorithm for solving the conflict resolution problem.

The model used for the wind vector will only consider its stochastic component, i.e., the wind prediction error, which is thus the uncertainty in deterministic weather predictions.

Wind velocities w_x and w_y represent the wind prediction errors. When the intruder violates its own airship, the PAZ - Protected Airspace Zone, the CR - Conflict Resolution determines a new heading and resolves this air traffic conflict.

When the Safe Separation Zone is re-established by CR- Conflict Resolution it is instantly deactivated and the flight plan is recalculated.

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