Wing in Ground Effect over a Wavy Surface

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Abstract: *A vortex method has been used to investigate the effect of a wavy ground on the aerodynamic forces acting on a wing that flies in its proximity. The air is considered inviscid and incompressible. The problem is obviously unsteady, and the solutions were found numerically.*

Key Words: *aerodynamic interference, wing in ground effect, unsteady flow, waves in sea water.*

1. INTRODUCTION

The wing in **g**round **e**ffect **v**ehicle (GEV) [1] is a vehicle that is designed to fly over the surface of a sea or even ground with almost horizontal surface, using ground effect. They are also known as WIG (wing in ground effect) or, in russian literature, ekranoplans.
Although this kind of vehicle concept has been criticized by many [2], there are a large number of such crafts that had been designed and built for all kind of applications. The main advantage of them is that in ground proximity, the lift induced drag decreases, [3], [4], [5].
Referring to the disadvantages of GEV, we will focus on the unfavorable effect of the waves, [1]. It can be said that although it flies over the water level, the influence of the waves is felt by the structure and passengers. This issue is the subject of our study. In this article however, we will limit ourselves to the two dimensional flow of the fluid about the aerofoil in the presence of a “wavy ground”. Although the wind and the wave speeds are neglected, they can easily be taken into account for this 2D case. They simply add or subtract from the flight speed. The real problem occurs in 3D flows. That’s why we neglected here the wind and wave speeds which are small when compared to the craft speed. Instead, we built a theory that will allow us to investigate the effect of different types of wave shapes on the main aerodynamic coefficients and derivatives.

2. INFLUENCE OF A WAVY SURFACE ON A VORTEX

2.1 Wavy Surface Vorticity in the Presence of a Vortex

The usual way to consider the ground effect is to use the so-called ”image method”. Consider for example the two dimensional case of a vortex at (x_0, z_0) in the vecinity of the wall represented here as Ox axis (fig. 1). The fluid ocuppies the domain of all points (x, z), $x \in R, z > 0$. The vortex at (x_0, z_0) having the intensity Γ_0 induces a perturbation whose potential and velocity fields are given by [6], [7]:

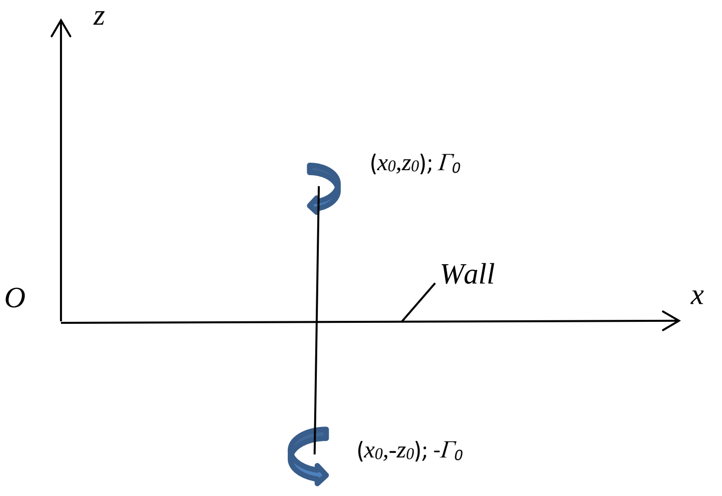


Fig. 1 A vortex and its image with respect to *Ox* axis

$$
\begin{aligned}
\varphi(x,z;x_0,z_0,\Gamma_0) &= -\frac{\Gamma_0}{2\pi}\tan^{-1}\frac{z-z_0}{x-x_0};(a)\\
u(x,z;x_0,z_0,\Gamma_0) &= \frac{\Gamma_0}{2\pi}\frac{z-z_0}{(x-x_0)^2+(z-z_0)^2};(b)\\
w(x,z;x_0,z_0,\Gamma_0) &= -\frac{\Gamma_0}{2\pi}\frac{x-x_0}{(x-x_0)^2+(z-z_0)^2}.(c)
\end{aligned}
\tag{1}
$$

It is known that the presence of the wall is simulated by considering an auxiliary flow extended over the entire 2D space with a second vortex with intensity $-\Gamma_0$ at $(x_0,-z_0)$. The velocity field of the two vortices is

$$
\begin{aligned}
\varphi &= -\frac{\Gamma_0}{2\pi}\left[\tan^{-1}\frac{z-z_0}{x-x_0}-\tan^{-1}\frac{z+z_0}{x-x_0}\right]\\
u &= \frac{\Gamma_0}{2\pi}\left[\frac{z-z_0}{(x-x_0)^2+(z-z_0)^2}-\frac{z+z_0}{(x-x_0)^2+(z+z_0)^2}\right]\\
w &= -\frac{\Gamma_0}{2\pi}\left[\frac{x-x_0}{(x-x_0)^2+(z-z_0)^2}-\frac{x-x_0}{(x-x_0)^2+(z+z_0)^2}\right]
\end{aligned}
\tag{2}
$$

So, for $z=0$ we get

$$
\begin{aligned}
\varphi &= 0\\
u &= \frac{\Gamma_0}{2\pi}\left[\frac{-z_0}{(x-x_0)^2+(z_0)^2}-\frac{+z_0}{(x-x_0)^2+(z_0)^2}\right] = -\frac{\Gamma_0}{2\pi}\frac{z_0}{(x-x_0)^2+{z_0}^2}\\
w &= -\frac{\Gamma_0}{2\pi}\left[\frac{x-x_0}{(x-x_0)^2+(z_0)^2}-\frac{x-x_0}{(x-x_0)^2+(z_0)^2}\right] = 0
\end{aligned}
\tag{3}
$$

Then (2) represents the flow in the proximity of the infinite wall, for $z>0$. However, we are interested to study the flow over a ground surface as represented in fig. 2. To do this, we can use a conformal map and transform the hilly surface into a plane. Unfortunately, this method cannot be extended to the 3D case, as we would like to do in a future work. Instead, we can consider the ground surface as the surface of a thin body. This body can be represented as a vortex surface.

Suppose the ground surface (*G*) is given by the equation $z = z_G(x)$.

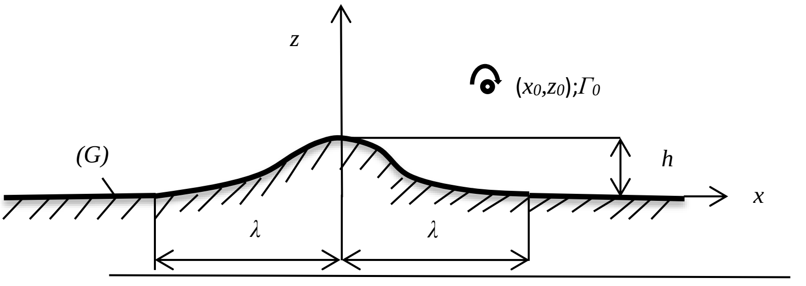


Fig. 2 Vortex in the proximity of a hilly ground

Let us consider the unit normal vector to the ground:

$$\boldsymbol{n}_G(x) = \left(-\frac{z_G{}'}{\sqrt{1+(z_G{}')^2}};\quad \frac{1}{\sqrt{1+(z_G{}')^2}}\right)^T \tag{4}$$

The velocity field induced by the surface of the ground is

$$\boldsymbol{V}_G(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma(\xi)\left[\frac{z-z_G(\xi)}{(x-\xi)^2+[z-z_G(\xi)]^2};\quad -\frac{x-\xi}{(x-\xi)^2+[z-z_G(\xi)]^2}\right]^T ds \tag{5}$$

In the above equation $\gamma(\xi) = \frac{d\Gamma}{d\xi}$ is the vortex sheet strength, and $ds = \sqrt{1+(z_G{}')^2}d\xi$. The velocity field will be then:

$$\boldsymbol{V}(x,z) = \boldsymbol{V}_G(x,z) + \boldsymbol{V}_0(x,z)$$

Here $\boldsymbol{V}_0(x,z)$ is the velocity field induced by the vortex at (x_0,z_0) and given by (1b,c). If $(x,z) \in G$ or $z = z_G(x)$, the boundary condition is expressed as

$$\boldsymbol{V}(x,z)\boldsymbol{n}_G(x) = 0$$

so that we obtain the integral equation

$$\int_{-\infty}^{\infty} K(x,\xi)\gamma(\xi)\,d\xi = 2\pi W_G(x;x_0,z_0;\Gamma_0) \tag{6}$$

The above improper integral should be evaluated using the Cauchy principal value concept:

$$\int_{-\infty}^{\infty} K(x,\xi)\gamma(\xi)\,d\xi = \lim_{L\to\infty}\left\{\lim_{\varepsilon\to 0}\left(\int_{-L}^{x-\varepsilon} K(x,\xi)\gamma(\xi)d\xi + \int_{x+\varepsilon}^{L} K(x,\xi)\gamma(\xi)d\xi\right)\right\}, \tag{7a}$$

where ε>0.

The kernel of the previous integral equation can be written as

$$K(x,\xi) = \frac{1}{x-\xi}\frac{1+\dfrac{z_G(x)-z_G(\xi)}{x-\xi}z_G{}'(\xi)}{1+\left[\dfrac{z_G(x)-z_G(\xi)}{x-\xi}\right]^2}\sqrt{1+[z_G{}'(\xi)]^2} \tag{7b}$$

So it is singular for $\xi = x$. The right hand term is given by

$$W(x) = -\frac{\Gamma_0}{2\pi}\frac{1}{x-x_0}\frac{1+\dfrac{z_G(x)-z_0}{x-x_0}z_G{}'(x)}{1+\left[\dfrac{z_G(x)-z_0}{x-x_0}\right]^2} = -\frac{\Gamma_0}{2\pi}\frac{x-x_0+[z_G(x)-z_0]z_G{}'(x)}{(x-x_0)^2+[z_G(x)-z_0]^2} \tag{8}$$

2.2 Particular cases

The integral equation (6) has not a simple solution. However, we can easily get simple solutions in three particular cases:

1. In case of $z_G(x) \equiv 0$, $z_G{}'(x) \equiv 0$ (flat ground) and the integral equation (6) takes the simpler form

$$\int_{-\infty}^{\infty} \frac{1}{x-\xi}\gamma(\xi)\,d\xi = -\Gamma_0 \frac{x-x_0}{(x-x_0)^2+{z_0}^2} \tag{9}$$

One can verify (see Appendix A) that (9) has a solution:

$$\gamma(x) = -\frac{\Gamma_0}{\pi}\frac{z_0}{(x-x_0)^2+{z_0}^2} \tag{10}$$

In the same Appendix, we proved that:

$$\int_{-\infty}^{\infty} \gamma(x)dx = -\Gamma_0 \tag{11}$$

It is not by chance that in this particular case $\gamma(x)$ looks like the first formula in (3). The reason is that $\gamma(x) = 2u$, and that is how we found this solution.

2. The case of the flow induced by a vortex Γ_0 located at (x_0,z_0) above a plane surface with a semicircular "wave", i.e fluid domain $z \geq 0$. Consider the real and auxiliary flows as represented in fig. 3.

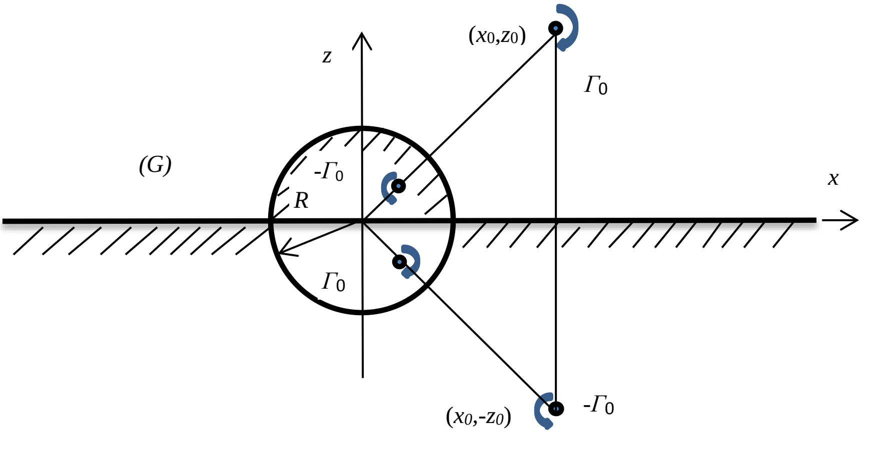


Fig. 3 The real and auxiliary flow; the auxiliary flow extends for $(x,z) \in (-\infty,\infty)^2$ while the real flow only beyond the shaded area

One can write the potential of the auxiliary flow as:

$$\varphi(x,z)=\varphi(x,z;x_0,z_0,\Gamma_0)+\varphi(x,z;x_0,-z_0,-\Gamma_0)+\varphi\left(x,z;x_0^i,z_0^i,-\Gamma_0\right)+\varphi\left(x,z;x_0^i,-z_0^i,\Gamma_0\right) \quad (12)$$

Here the functions φ are are calculated using (1a) and the "image vortices" are placed at $\left(x_0^i,z_0^i\right)$ and $\left(x_0^i,-z_0^i\right)$

$$x_0^i=\frac{R^2}{x_0^2+z_0^2}x_0;\ \ z_0^i=\frac{R^2}{x_0^2+z_0^2}z_0 \quad (13)$$

Knowing the potential, one can write the velocity field and so the fluid velocity which is tangential to the shaded border in fig. 3. Let us denote its magnitude by $v_t(x_G,z_G),(x_G,z_G)\in(G)$:

$$v_t(x_G,z_G)=grad\ \varphi(x,z)]_{x=x_G,z=z_G}\boldsymbol{\tau}(x_G,z_G)$$

In the above equation, $\boldsymbol{\tau}(x_G,z_G)$ is the unit tangent vector to the (G) border. Now as $\gamma(\xi)=2v_t[\xi,z_G(\xi)]$, we found a solution of (6).
This can be verified it in a similar way as in the first case, Appendix A. However, the calculations are cumbersome.

3. Two or more semicircular "waves" that have their centres on *Ox* and do not intersect with each other, can be treated in a similar way.

We can extend the complexity of the studied cases *superimposing a uniform flow at infinity*. For the simple three cases presented above, when we have exact solutions, the new flows are obtained as follows:

1. Case of the plane ground- simply add $\varphi=U_\infty x$, no aditional flow required.
2. Case of a single semicircular wave- as usually, to $\varphi=U_\infty x$ add a proper doublet at the centre of the circle, [7].
3. Case many semicircular waves - to $\varphi=U_\infty x$ add proper doublets at the centres of the circles and "images" of each dublet with respect to the other circles.

These exact solutions will allow us to build solutions that are not entirely numerical, as are those presented at Chapter 3.

2.3 The General Cases

Returning to the general case, a numerical solution is given in Appendix B. The next figure depicts a symmetrical "hilly ground" or a "flood wave" and a vortex at (x_0, z_0). Figures (5) presents the bound vorticities on the wall as a result of the influence of this vortex (γ).

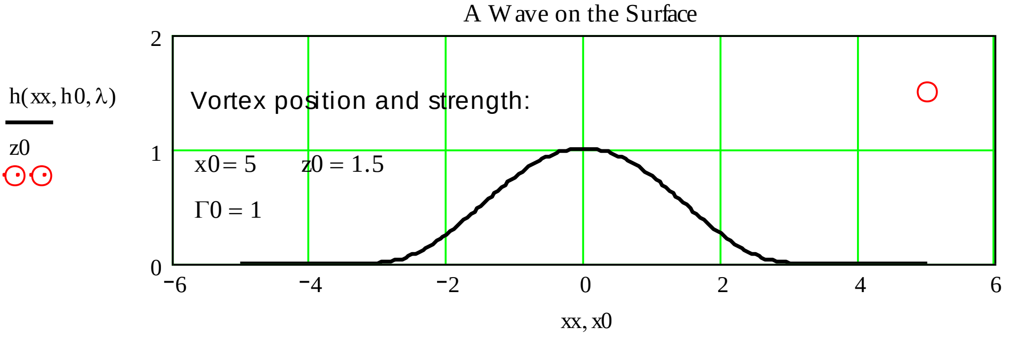
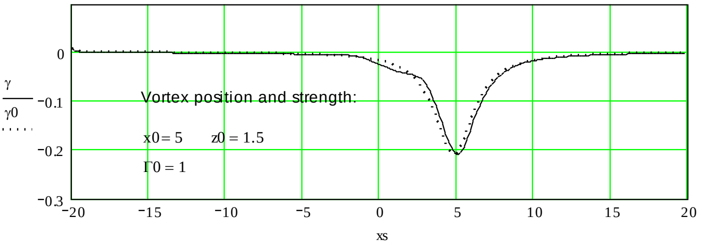
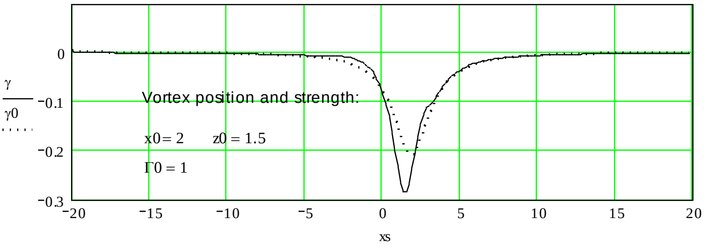


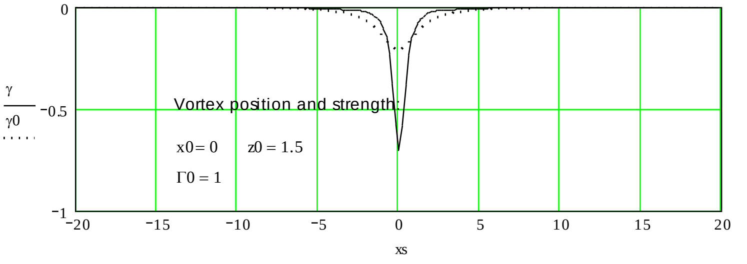
Fig. 4 A vortex at (x_0,z_0) in the presence of a "flood wave" on the plane surface



(*a*)



(*b*)



(*c*)

Fig. 5 Vorticity distribution at three different positions of the vortex. γ_0 represents the vorticity on a plane surface, while γ represents the vorticity on the wavy surface in fig. 3.

3. THIN WING IN THE NEIGHBOURHOOD OF A WAVY SURFACE

3.1 Coordinate systems

Suppose we have a certain wing in level flight at a given height H_0 over the mean surface of the ground or sea level. This wing has a characteristic aerofoil, (*A*). The ground surface is not plane, but it has a certain distribution of bumps or waves on its mean surface. Considering the fixed frame *XOZ*, we can write the ground surface equation.

$$Z = H(X) \tag{14}$$

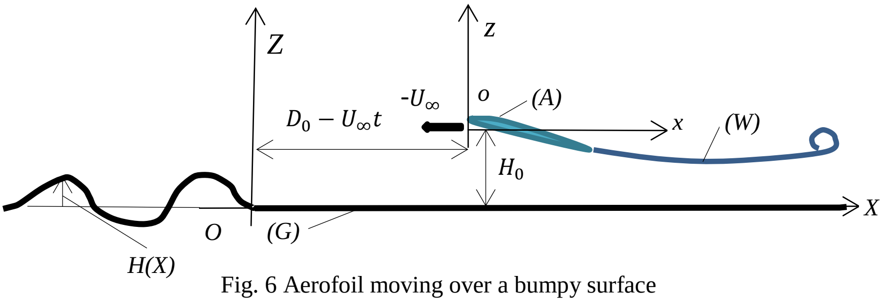


Fig. 6 Aerofoil moving over a bumpy surface

On the other hand, the aerofoil frame is *xoz*, and it moves with the velocity $-U_\infty$ with respect to the fixed frame (fig. 6). We can write the following transformation that links the two coordinate systems

$$\begin{cases} X = x + D_0 - U_\infty t \\ \quad Z = z + H_0 \end{cases} \tag{15}$$

In the above system, D_0 is the abscissa of the origin *o* at t=*0*. In the next calculations, we will use both (X, Z) and (*x*,*z*) as it is convenient for the approach.

3.2 Wing contribution

In this stage of approach, *we use thin aerofoil theory*. So, the aerofoil is replaced using the well-known procedure (fig. 7) with its mean line

$$\zeta = \zeta(\xi), 0 \leq \xi \leq 1 \tag{16}$$

One can see that the aerofoil camber line equation is given in its non-dimensional form (i.e for chord *c*=1).

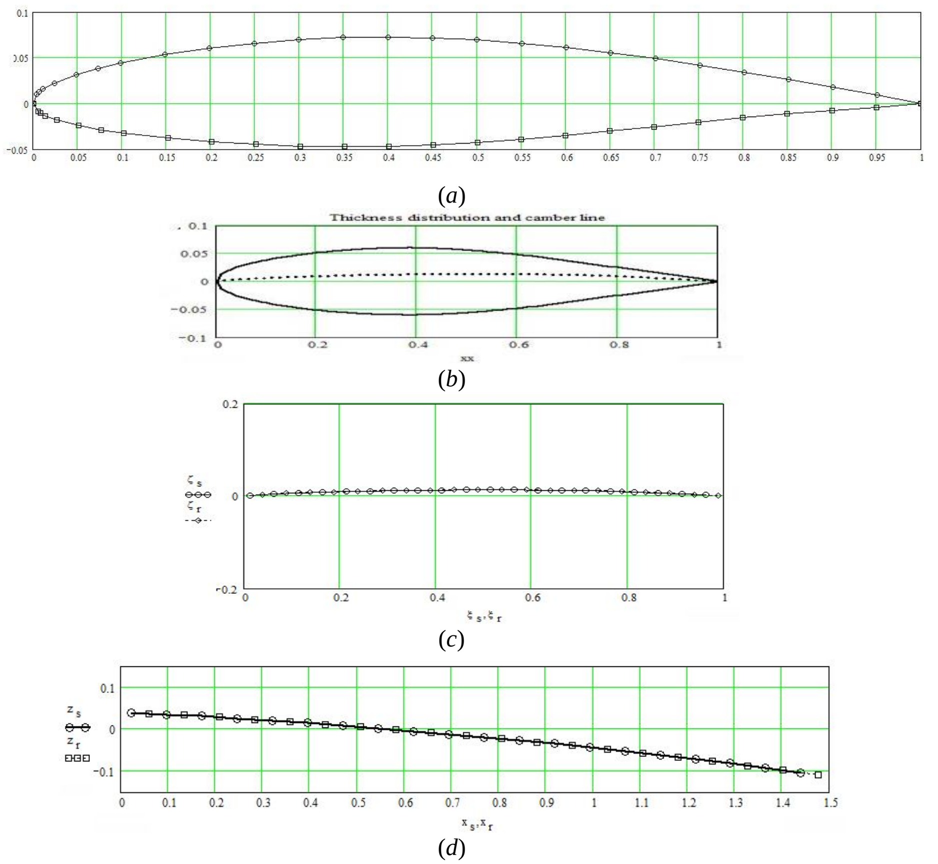


Fig. 7 *(a)*The aerofoil in non dimensional coordinates, *(b)* its thickness and camber line *ζ(ξ)*. *(c)* The camber line *ζ(ξ)* is divided ito a number of "boxes", in each box (ξ_s,ζ_s) represents the ¼ point, while (ξ_r,ζ_r) is the ¾ point; *(d)* The camber line is brought to its dimensional form by multiplying by *c* (its chord); then it is rotated with angle of attack *α* about (say) (1/4 *c*,0) point

The camber line $\zeta = \zeta(\xi)$ is brought to its dimensional form and rotated with the angle of attack α about a certain point, say *x*=0.25*c*, *z*=0. We obtain now the true camber line at the proper angle of attack (fig. 7, *(d)*). It has the equation

$$z = z_A(x), 0 < x < c \tag{17}$$

The unit normal vector on (W) is given by

$$\boldsymbol{n}_A(x) = \left(-\frac{z_A{}'}{\sqrt{1+(z_A{}')^2}}; \quad \frac{1}{\sqrt{1+(z_A{}')^2}}\right)^T \tag{18}$$

If we consider that there is a vorticity distribution on the camber line, say $\gamma_A(x,t)$, then it induces into the surrounding atmosphere the velocity field,

$$\boldsymbol{v}_A(x,z;t) = \int_{(W)} \gamma_A(\xi,t)\left[\frac{z-z_A(\xi)}{(x-\xi)^2+[z-z_A(\xi)]^2}; \quad -\frac{x-\xi}{(x-\xi)^2+[z-z_A(\xi)]^2}\right]^T ds \tag{19}$$

The above formula gives the value of velocity field in *xoz* frame.

3.3 Wake contribution

On the other hand, one must keep in mind that although α and U_∞ are constant, we expect that the aerofoil global circulation

$$\Gamma_A(t) = \int_0^c \gamma_A(x,t)\,dx \tag{20}$$

varies with time due to the wavy surface influence. Consequently, there is always a vortex wake (W) whose global vorticity

$$\Gamma_W(t) = \int_{(W)} \gamma_W(x,t)\,ds \tag{21}$$

satisfies the Kelvin's circulation theorem,

$$\Gamma_A(t) + \Gamma_W(t) = const. \ \text{ or,} \qquad d\Gamma_A(t) + d\Gamma_W\,(t) = 0 \tag{22}$$

The wake equation is

$$\begin{cases} x = x_W(\lambda,t) \\ z = z_W(\lambda,t) \end{cases} \tag{23}$$

This wake contributes to the generation of an induced velocity field,

$$\boldsymbol{v}_w(x,z;t) = \int_{(W)} \gamma_w(\xi,t)\left[\frac{z-z_w(\lambda,t)}{[x-x_W(\lambda,t)]^2+[z-z_w(\lambda,t)]^2}; \quad -\frac{x-x_W(\lambda,t)}{[x-x_W(\lambda,t)]^2+[z-z_w(\lambda,t)]^2}\right]^T ds \tag{24}$$

In the first approximation, the wake could be considered as a line $x = \lambda + U_\infty t, z_W = z_{TE} = const$. Here the letters TE signify trailing edge.

3.4 Wavy ground/sea contribution

In the ”fixed” ground system we have already found that

$$\boldsymbol{V}_G(X,Z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \gamma_G(\Xi,t)\left[\frac{Z-H(\Xi)}{(X-\Xi)^2+[Z-H(\Xi)]^2}; \quad -\frac{X-\Xi}{(X-\Xi)^2+[Z-H(\Xi)]^2}\right]^T ds \tag{25}$$

Taking into account (15), we find the velocity in (x,z):

$$\boldsymbol{V}_G(X,Z,t\,)=\boldsymbol{V}_G(x+D_0-U_\infty t,z+H_0)=\boldsymbol{v}_G(x,z,t) \tag{26}$$

So we have

$$\boldsymbol{v}_G(x,z,t)=\boldsymbol{V}_G(X,Z,t\,)$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma_G(\Xi,t)\left[\frac{Z-H(\Xi)}{(X-\Xi)^2+[Z-H(\Xi)]^2};\quad -\frac{X-\Xi}{(X-\Xi)^2+[Z-H(\Xi)]^2}\right]^T ds \tag{27}$$

where X and Z are calculated as functions of x, z and t using (15).

3.5 Numerical approach

a) Discretizations Essentially, the processing of the wing aerofoil geometry (A) presented in fig 3.2 (a)-(d) poses no numerical problem. So finally, for N_A "boxes" having equal lengths we get the points $(x_S,z_S)_i^A$ and $(x_R,z_R)_i^A$, i=1,2...N. The vortex points are located at $S_i^A=(x_S,z_S)_i^A$ while the collocation points are placed at $R_i^A=(x_R,z_R)_i^A$. The length of each box of the aerofoil is Δs^A. Similarly, the wavy boudary surface of the sea/ground is divided into N_G points $(X_S,Z_S)_j^G$, j=1,2...N_G, (see Appendix B). On the other hand, the wake is generated in time. Let us consider M equally spaced time steps, $t_k=(k-1)\Delta t,$ where $\Delta t=\frac{T}{M}$. Here, T is the entire period of time considered. At each moment t=t_k, we have a wake approximation as a raw of point vortices $(x_{l,k}^W,z_{l,k}^W)$, each having a strength γ_{Wl}, l=1,2...k. At the same moment, the aerofoil has a vorticity distribution $\gamma_{Ai,k}$, while the wavy ground/sea has the vorticity distribution is $\gamma_{Gj,k}$. The table below summarizes the notations:

Object	Box length	Vortex points	Collocation points	Number of boxes
Aerofoil	Δs^A	$S_i^A=(x_S,z_S)_i^A$	$R_i^A=(x_R,z_R)_i^A$	i=1,2...N_A
Waved surface	Δs^G	$S_j^G=(X_S,Z_S)_j^G$	$R_j^G=(X_R,Z_R)_j^G$	j=1,2...N_G
Wake	Δs^W	$S_l^W=(x_S,z_S)_l^W$	$-$	l=1,2...N_W (N_W=k)

The unknowns γ_A and γ_G are matices with M lines and N_A or N_G columns respectively. Each line represents the the vorticity distribution along the wing chord or the wavy ground/sea, respectively at the moment t_k. The unknown γ_W is a vector of M components representing the values of the vorticity issued at the trailing edge of the aerofoil at each moment t_k. Similarly, the matrices x^W,z^Wgive the position of the wake, each line of the matrices representing their position at t_k. These matrices are triangular matrices, since the wake is generated at each moment, t_k. However, at the biginning it is a good practice to consider the wake fixed in XOZ system, as shown in 3.3. In this approximation, points $\left(X_{Sl}^W\,,Z_{Sl}^W\,\right)$ *are fixed*..

b) Velocity fields As regards the ***velocity field***, we consider the vorticity concentrated at $(x_s,z_s)_i$ points. So we have $\Delta\Gamma_{i,k}\;,i=1,2\,...\,N;k=1,2\,...\,M$ concentrated vortices which are related to the distributed circulation by

$$\gamma(x_{si},t_k)=\frac{\Delta\Gamma_{i,k}}{\Delta s} \tag{28}$$

available for the aerofoil (A), wake (W) and for the ground (G).

<u>The velocty field induced by the aerofoil</u> The equation (19) becomes:

$$\boldsymbol{v}_A(x,z,t_k)=\sum_{i=1}^{N}\Delta\Gamma_{i,k}^A\left[\frac{z-z_{Si}^A}{\left(x-x_{Si}^A\right)^2+\left[z-z_{Si}^A\right]^2};\quad -\frac{x-x_{Si}^A}{(x-x_{si})^2+\left[z-z_{Si}^A\right]^2}\right]^T \tag{29}$$

<u>The velocity field induced by the wavy boundary</u> is given by:

$$\boldsymbol{v}_G(x,z,t_k)=\boldsymbol{V}_G(X,Z,t_k\,) =\sum_{j=1}^{k}\Delta\Gamma_{j,k}^{G}\left[\frac{Z-Z_{Sj}^{G}}{\left(X-X_{Sj}^{G}\right)^2+\left[Z-Z_{Sj}^{G}\right]^2};\quad -\frac{X-X_{Sj}^{G}}{\left(X-X_{Sj}^{G}\right)^2+\left[Z-Z_{Sj}^{G}\right]^2}\right]^{\boldsymbol{T}} \tag{30}$$

In both equations (29) and (30) X and Z can be calculated as functions of x,z and t using (15). As regards X_{Sl}^{G} and Z_{Sl}^{G}, they are fixed points chosen as shown in the Appendix B.
The velocity field induced by the wake is given by the equation:

$$\boldsymbol{V}_W(X,Z,t_k) =\sum_{l=1}^{k}\Delta\Gamma_{l,k}^{W}\left[\frac{Z-Z_{Sl}^{W}}{(X-X_{Sl}^{W})^2+[Z-Z_{Sl}^{W}]^2};\quad -\frac{X-X_{Sl}^{W}}{(X-X_{Sl}^{W})^2+[Z-Z_{Sl}^{W}]^2}\right]^{T} \tag{31}$$

At $t=t_k$, *all the quantities in (31) are known and the velocity field* $\boldsymbol{v}_W(x,z,t_k)$ *is consequently computable.*
The total velocity field in xoz frame is then

$$\boldsymbol{v}(x,z,t_k)=U_\infty\boldsymbol{i}+\boldsymbol{v}_A(x,z,t_k)+\boldsymbol{v}_G(x,z,t_k)+\boldsymbol{v}_W(x,z,t_k) \tag{32}$$

c) Boundary condition The relative fluid flow velocity must remain tangent to the wing and the wavy surface. Then:

- On the wing aerofoil, at points $(x_R,z_R)_m^A=\left(x_{Rm}^A,z_{Rm}^A\right)$, the velocity is given by

$$\boldsymbol{v}\left(x_{Rm}^A,z_{Rm}^A,t_k\right)=U_\infty\boldsymbol{i}+\boldsymbol{v}_A\left(x_{Rm}^A,z_{Rm}^A,t_k\right)+\boldsymbol{v}_G\left(x_{Rm}^A,z_{Rm}^A,t_k\right)+\boldsymbol{v}_W\left(x_{Rm}^A,z_{Rm}^A,t_k\right) \tag{33}$$

Then

$$\boldsymbol{v}\left(x_{Rm}^A,z_{Rm}^A,t_k\right)\boldsymbol{n}_A\left(x_{Rm}^A\right)=0,\qquad m=1,2\ldots N_A \tag{34}$$

This is a system of N_A linear equations with $\Delta\Gamma_{Ai,k}$, $i=(1,2\ldots N_A)$ and $\Delta\Gamma_{Gl,k}$,$l=1,2\ldots N_G$ unknowns.

- On the wavy ground/sea, at points $(X_R,Z_R)_n^G=\left(X_{Rn}^G,Z_{Rn}^G\right)$, the *relative velocity* is:

$$\boldsymbol{v}\left(x_{Rn}^G,z_{Rn}^G,t_k\right)-U_\infty\boldsymbol{i}=\boldsymbol{V}_A\left(x_{Rn}^G,z_{Rn}^G,t_k\right)+\boldsymbol{V}_G\left(x_{Rn}^G,z_{Rn}^G,t_k\right)+\boldsymbol{V}_W\left(x_{Rn}^G,z_{Rn}^G,t_k\right) \tag{35}$$

The unit normal vector to the ground/sea surface being denoted by $\boldsymbol{n}_G\left(X_{Rn}^G\right)$**,** the boundary condition is

$$\left[\boldsymbol{V}_A\left(x_{Rn}^G,z_{Rn}^G,t_k\right)+\boldsymbol{V}_G\left(x_{Rn}^G,z_{Rn}^G,t_k\right)+\boldsymbol{V}_W\left(x_{Rn}^G,z_{Rn}^G,t_k\right)\right]\boldsymbol{n}_G\left(X_{Rj}^G\right)=0 \qquad n=1,2\ldots N_G \tag{36}$$

This system of linear equations contains again the unknowns $\Delta\Gamma_{Ai,k}$ and $\Delta\Gamma_{Gj,k}$ with $i=(1,2\ldots N_A)$ and $j=1,2\ldots N_G$. It is better to use a single index say $i=(1,2\ldots N_A+N_G)$ and consider that the first N_A indices refer to the aerofoil, while the last $N_A+1,N_A+2\ldots N_A+N_G$ refer to the ground. So the systems (34) and (36) form a system of N_A+N_G equations with N_A+N_G unknowns.

Numerical modeling of the aerodynamic phenomenon We give here *a very simplified method* and present the main steps to get a solution. At *t=0* i.e. k=1, the aerofoil is at $X=x+D_0$ from the front of the wave, a distace at which its influence is negligible (since $d_0>>c$). Then, for k=1 (time index) there is no wake. The ground influence is practically appoximated by a flat horizontal wall. One can use the Green function method to calculate the vorticity on the aerofoil. On the other hand, $\Gamma_A(t_1)$can be calculated with (20). For k=2, admit that the wake occurs. Use the previous numerical method to find $\Delta\Gamma_{Ai,k}$ and

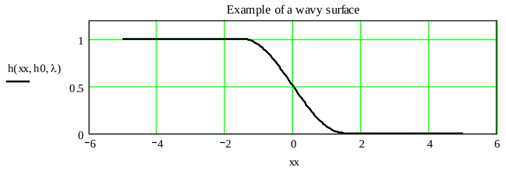
$\Delta\Gamma_{Gi,k}$, then (28) to find γ_A and γ_G. Calculate again $\Gamma_A(t_2)$ using (20). Then, $\Delta\Gamma_{W1,1} = -[\Gamma_A(t_2) - \Gamma_A(t_1)]$; $X^W_{S1} = (c + 0.25\Delta s) + D_0 - U_\infty \Delta t$, $Z^W_{S1} = Z_{TE}$. Here, the simple trailing edge locus was used as the wake, but more sophisticated methods are available. Now (t=t_2) and we defined a wake approximated by only one point. The process can be continued to k=M+1, which is the last moment for which t=T. Obviously, one must calculate the *unsteady* pressure coefficient, lift, vortex drag and pitching moment coefficients. These can be done using the classical methods.

3.6 Results

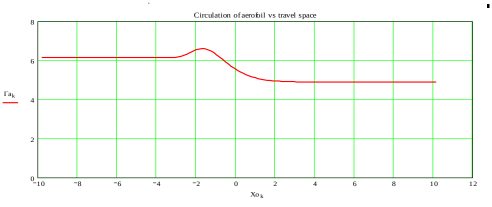
We present here two cases of waves: a ”tidal wave” or a quasi-Heaviside wave and a sinusoidal wave. In the first case (fig. 8) we can see that the circulation Γ_A and the C_L jumps need a time to stabilize to their final levels.

The pitching moment coefficient C_m about the 1/4c point has also an important variation. The sinusoidal case of waves is presented in fig. 9.

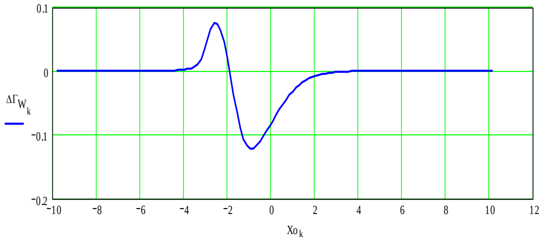
We have here a wave with two peaks and a valey between them (1.5 periods). We find again that both the aerofoil circulation and lift coefficient need a time to stabilize. The pitching moment coefficient C_m has a relative strong variation.



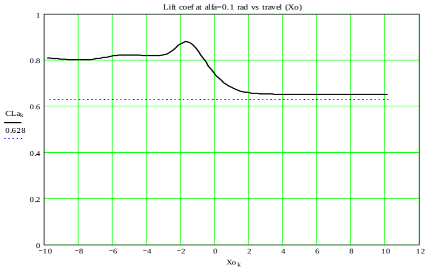
(*a*) Quasi-Heaviside (or tidal) wave



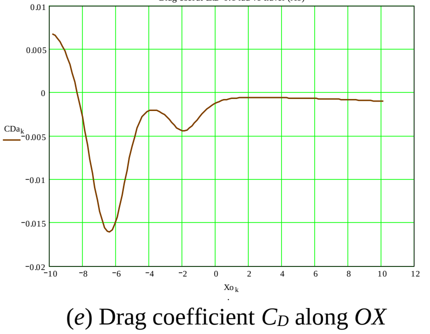
(*b*) Aerofoil circulation along *OX* axis



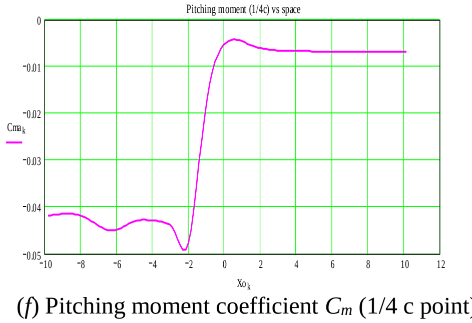
(*c*) Concentrated vortices in the wake



(*d*) Lift coefficient C_L long *OX*

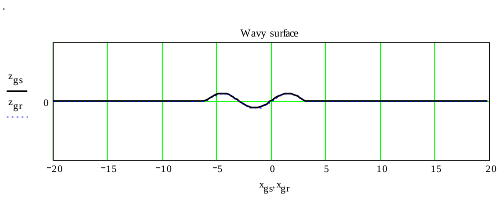


(*e*) Drag coefficient C_D along *OX*

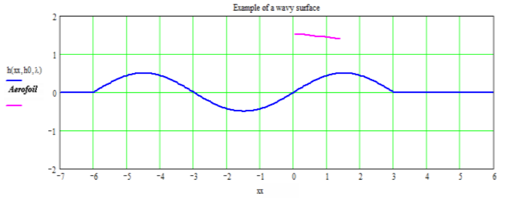


(*f*) Pitching moment coefficient C_m (1/4 c point)

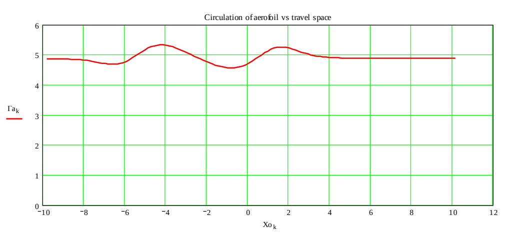
Fig. 8 *(a)*The case of a quasi-Heaviside wave, or “tidal wave” whose velocity is very small when compared with the aerofoil velocity, U_∞. The aerofoil comes from the right and goes to the left (from X=12 to X=-10). c=1.5l_{ref}, H=1.55l_{ref},$U_\infty = 10 l_{ref}/s$



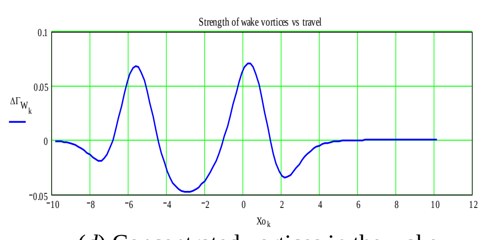
(*a*) The waves and the domain



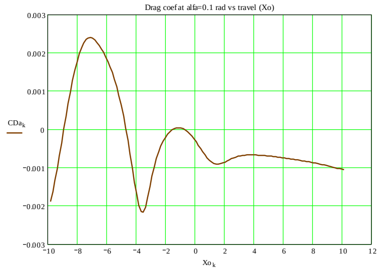
(*b*) The aerofoil over the waves, represented at scale; it moves from right to left



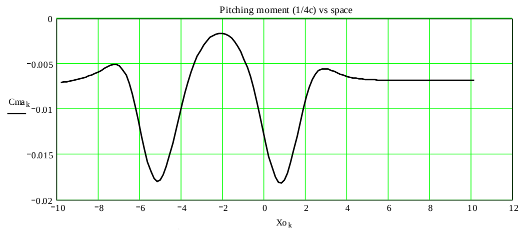
(*c*) Aerofoil circulation along *OX* axis



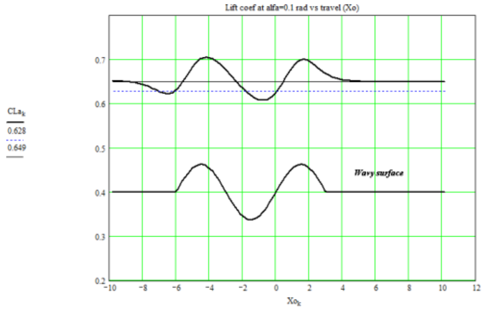
(*d*) Concentrated vortices in the wake



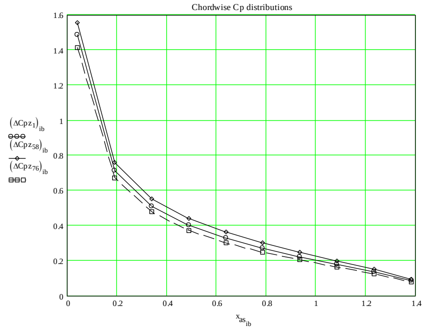
(*e*) Drag coefficient C_D along *OX*



(*f*) Pitching moment coefficient C_m (1/4 c point)



(*g*) Lift coefficient C_L long *OX;* upper curve C_L 0.628 is the value of C_L in unlimited atmosphere; 0.649 is the value of C_L with flat ground influence. Lower curve, the waves



(*h*) Pressure coefficient ΔC_p at different times, having indices k=1;58;76

Fig. 9 *(a),(b)* The case of a sinusoidal wave whose velocity is very small when compared with the aerofoil velocity. The aerofoil comes from the right and goes to the left (from *X*=12 to *X*=-10).

4. CONCLUSIONS

In the article we presented a method of calculating the influence of a wavy ground/sea surface on the aerodynamic characteristics of an aerofoil moving in its proximity. The wavy surface was represented as a vortex layer having a finite length. At the present stage of the study, the aerofoil was considered thin, and so the thin aerofoil theory was applied. The method has led to an unsteady model that conducted to realistic results. The influence of the wavy surface on some aerodynamic coefficients and other quantities seems to be important. The waves affect both the aerodynamic coefficients and the lift and pitch derivatives. The method should be extended to the 3D case, that shows most interest for practical applications and the aerodynamic phenomenon is much more complex.

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APPENDIX A

A Solution of Equation (9)

Let us prove that the integral equation

$$\int_{-\infty}^{\infty} \frac{1}{x-\xi}\gamma(\xi)\,d\xi = -\Gamma_0 \frac{x-x_0}{(x-x_0)^2+{z_0}^2} \tag{A-1}$$

has the next solution

$$\gamma(x) = -\frac{\Gamma_0}{\pi}\frac{z_0}{(x-x_0)^2+{z_0}^2} \tag{A-2}$$

where $z_0 > 0$. We remind that the improper integral from (A-1) is evaluated as:

$$\int_{-\infty}^{\infty} \frac{1}{x-\xi}\gamma(\xi)\,d\xi = \lim_{L\to\infty}\left\{\lim_{\varepsilon\to 0}\left(\int_{-L}^{x-\varepsilon}\frac{1}{x-\xi}\gamma(\xi)d\xi + \int_{x+\varepsilon}^{L}\frac{1}{x-\xi}\gamma(\xi)d\xi\right)\right\}, \tag{A-3}$$

First we have to calculate

$$\int_{-L}^{L}\frac{1}{x-\xi}\left[-\frac{\Gamma_0}{\pi}\frac{z_0}{(\xi-x_0)^2+{z_0}^2}\right]d\xi = -\frac{\Gamma_0 z_0}{\pi}\int_{-L}^{L}\frac{1}{x-\xi}\frac{d\xi}{(\xi-x_0)^2+{z_0}^2} \tag{A-4}$$

Making a change of the variable,

$$\xi - x_0 = u; d\xi = du; -L-x_0 = L^*; L-x_0 = L^{**}$$

$$\int_{-L}^{L}\frac{1}{x-\xi}\frac{d\xi}{(\xi-x_0)^2+{z_0}^2} = -\int_{L^*}^{L^{**}}\frac{1}{u-(x-x_0)}\frac{du}{u^2+{z_0}^2}$$

But

$$\frac{1}{u-(x-x_0)}\frac{1}{u^2+{z_0}^2} = \frac{A}{u-(x-x_0)} + \frac{Bu+C}{u^2+{z_0}^2}$$

where

$$A = \frac{1}{(x-x_0)^2+{z_0}^2}; B = -\frac{1}{(x-x_0)^2+{z_0}^2}; C = \frac{x-x_0}{(x-x_0)^2+{z_0}^2}.$$

Then,

$$\int_{L^*}^{L^{**}}\frac{du}{u-(x-x_0)} = \lim_{\varepsilon\to 0}\left[\int_{L^*}^{(x-x_0)-\varepsilon}\frac{du}{u-(x-x_0)} + \int_{(x-x_0)+\varepsilon}^{L^{**}}\frac{du}{u-(x-x_0)}\right]$$

$$= \lim_{\varepsilon\to 0}\left\{\ln\left|\frac{L^{**}-(x-x_0)}{L^*-(x-x_0)}\right| + \ln\left|\frac{(x-x_0)-\varepsilon}{(x-x_0)+\varepsilon}\right|\right\}$$

$$= \lim_{\varepsilon\to 0}\left\{\ln\left|\frac{L-x}{-L-x}\right| + \ln\left|\frac{(x-x_0)-\varepsilon}{(x-x_0)+\varepsilon}\right|\right\} = \ln\frac{L-x}{L+x}.$$

Clearly, for $L\to\infty$ this integral tends to 0. The second integral splits in two parts:

$$\int_{L^*}^{L^{**}}\frac{Bu+C}{u^2+{z_0}^2}du = B\int_{L^*}^{L^{**}}\frac{u}{u^2+{z_0}^2}du + C\int_{L^*}^{L^{**}}\frac{1}{u^2+{z_0}^2}du$$

Then,

$$\int_{L^*}^{L^{**}} \frac{u}{u^2+{z_0}^2}du = \frac{1}{2}\int_{L^*}^{L^{**}} \frac{d(u^2+{z_0}^2)}{u^2+{z_0}^2} = \frac{1}{2}\ln\frac{(L^{**})^2+{z_0}^2}{(L^*)^2+{z_0}^2} = \frac{1}{2}\ln\frac{(L-x_0)^2+{z_0}^2}{(L+x_0)^2+{z_0}^2}.$$

Again this integral tends to 0 when $L \to \infty$. The last integral is

$$\int_{L^*}^{L^{**}} \frac{1}{u^2+{z_0}^2}du = \frac{1}{z_0}\int_{L^*}^{L^{**}} \frac{d\left(\frac{u}{z_0}\right)}{1+\left(\frac{u}{z_0}\right)^2} = \frac{1}{z_0}\left|\tan^{-1}\frac{u}{z_0}\right|_{L^*}^{L^{**}} = \frac{1}{z_0}\left[\tan^{-1}\frac{L^{**}}{z_0} - \tan^{-1}\frac{L^*}{z_0}\right]$$

$$= \frac{1}{z_0}\left[\tan^{-1}\frac{L-x_0}{z_0} - \tan^{-1}\frac{-L-x_0}{z_0}\right].$$

As $L \to \infty$, this last integral gives $\frac{\pi}{z_0}$. We finally get

$$\int_{-\infty}^{\infty} \frac{1}{x-\xi}\gamma(\xi)\,d\xi = -\frac{\Gamma_0 z_0}{\pi}\frac{\pi}{z_0}C = -\Gamma_0\frac{x-x_0}{(x-x_0)^2+{z_0}^2}, \tag{A-5}$$

QED.

Moreover, we can easily see that

$$\int_{-\infty}^{\infty} \gamma(x)dx = -\frac{\Gamma_0}{\pi}\int_{-\infty}^{\infty} \frac{z_0}{(x-x_0)^2+{z_0}^2}dx = -\Gamma_0 \tag{A-6}$$

APPENDIX B

A Numerical Solution to the Integral Equation (6)

In order to give a solution to the general case of the integral equation (6), we consider a certain surface of the ground or sea (G) which is given as a function

$$z = h(x, h_0), \qquad x \in (\lambda_1, \lambda_2) \tag{B-1}$$

In the above formula the parameter h_0 gives the height of the wave.

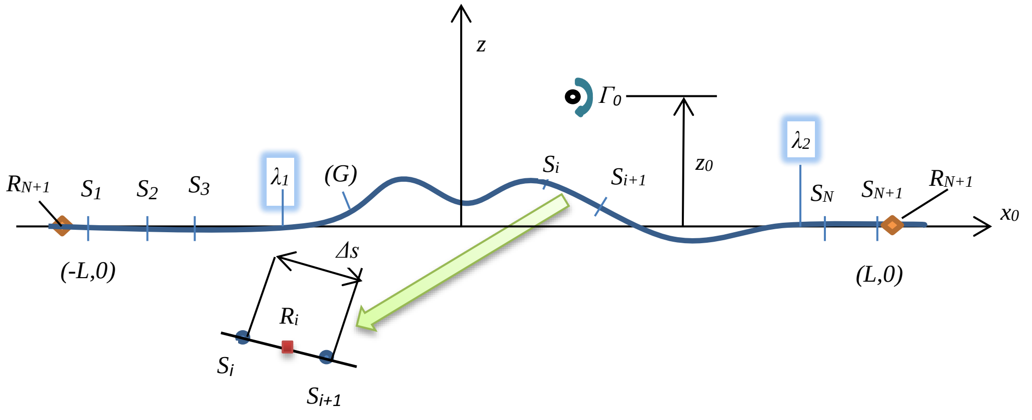

Fig. B-1 Wavy boundary discretization into *n* elements; $h_0 < z_0 \ll L$

Although the function (B-1) is defined on the support (λ_1, λ_2), we will extend it for $x \in (-\infty, \infty)$ with zero values.

For physical reasons, clearly $\gamma(x)$ must vanish at infinity:

$$\lim_{x \to \pm\infty} \gamma(x) = 0 \tag{B-2}$$

As we want to adress to a pure numerical method, the first step is to limit ourselves to a *finite* domain, $x \in (-L, L), L \gg \max\big(abs(\lambda_1), abs(\lambda_2)\big), L \gg z_0$. The second step is to rectify the curve over the entire domain $x \in (-L, L)$ and divide the curve length into N segments $(S_i, S_{i+1}), i = 1,2 \dots N$ such that the length of each curve segment is $s_{i+1} - s_i = \Delta s = \frac{s_{N+1} - s_N}{N}$. We have to distinguish between the point S_i and its intrisec coordinate s_i on (G). The same point has on the other hand its cartesian coordinates (x_i, z_i). Considering again the segments $(S_i, S_{i+1}), i = 1,2 \dots N$ we define the middle points $R_i = \frac{1}{2}(S_i + S_{i+1})$, where $S_i = (x_i, z_i)^T$ and $R_j = \left(x'_j, z'_j\right)^T$ etc. The distributed vorticity $\gamma(\xi)$ is considered concentrated in each point S_i So we have N+1 unknowns $\Gamma_i = \gamma(\xi_i)\Delta s = \gamma_i \Delta s$. The boundary condition (the flow is tangent to the curve (G)) is imposed at the receiving points $R_j\left(x'_j, z'_j\right), j = 1,2 \dots N$. We need one more equation which will be

$$\sum_{i=1}^{N+1} \Gamma_i = -\Gamma_0 \tag{B-3}$$

available for $z_0 \ll L$. This last equation maintains constant the flow circulation, as we observed that happens in the case of a plane surface (Appendix A). Alternatively, we can choose another point R_{N+1} located at a distance Δs at right of R_N, **or** at the left of R_1 at the same distance Δs, fig. B-1. All these three methods give close results. However, the last two methods do not verify exactly the previous equation but respect better (B-2).