Mathematical review of the attitude control mechanism for a spacecraft

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Abstract: *The issue of inertial pointing for a spacecraft with magnetic actuators is considered and a practical global response to this problem is obtained by static attitude and speed feedback methods. A local solution dependent on dynamic attitude feedback is additionally introduced. The simulation results show the practical applicability of the proposed approach. The issue of attitude regulation of rigid spacecraft, i.e., spacecraft demonstrated by the Euler's conditions and by an appropriate parameterization of the attitude, has been broadly concentrated as of late. As a matter of first importance, it is beyond the realm of imagination by methods for magnetic actuators to give three autonomous control torques at each time instant. Moreover, the conduct of these actuators is characteristically time-varying, as the control instrument relies on the varieties of the Earth magnetic field along the spacecraft orbit. In any case, demeanor adjustment is conceivable in light of the fact that on normal the framework has solid controllability properties for a wide range of orbit inclinations. A lot of work has been devoted as of late to the issues of examination and structure of attractive control laws in the straight case, i.e., nominal operation of a satellite near its equilibrium attitude. Specifically, ostensible and vigorous solidness and execution have been contemplated, utilizing either devices from occasional control hypothesis misusing the (quasi) intermittent conduct of the framework close to an equilibrium or other techniques aiming at developing suitable time-varying controllers.*

Key Words: *Euler angle, quaternion, Kalman filtering (EKF, UKF)*

1. INTRODUCTION

A satellite attitude control is depicted in which a single inertia wheel is mounted for pivoting around a fixed axis of rotation corresponding to the pitch axis of the satellite. The dormancy wheel is a piece of a motor-inertia- wheel-tachometer generator unit steadily mounted on the satellite [1]. As the satellite vehicle circles the earth, the vehicle will be balanced out about the roll and yaw (*X* and *Y* axes, respectively) axes by the inactivity wheel. Variety of the speed of turn of the inactivity wheel about the pitch axis balances out the vehicle about the last axis [2-3]. For a typical satellite application, the satellite is settled with the yaw axis along the geocentric pivot of the earth by the turning dormancy wheel [4]. Unsettling the influence torque about the pitch pivot will generally increase or decrease the inactivity wheel speed. An electromagnetic actuation system creates response torques with the Earth magnetic field which will generally keep the wheel speed consistent and expel the precession of the idleness wheel speed steady and evacuate the precession of the inactivity wheel pivot [5-7]. Clearly whether a net unsettling disturbance torque exists around the pitch axis the wheel will persistently change speed to conquer the aggravation torque. In the end, thus, the wheel will achieve its most extreme conceivable speed and won't be capable any more extended to counter the aggravation [8]. The tachometer joined to the wheel detects when this condition is occurring and supplies fitting signs to the loop current PCs to infuse the suitable flows into the proper curls to evacuate the abundance energy, the vehicle must most likely decide the quality and heading of the Earth's field of attraction. This is finished by a three axes magnetometer which estimates the segments of the magnetic field of the Earth. In this paper the methods for attitude control of satellites have been discussed [9-11].

2. FUNDAMENTALS OF ATTITUDE CONTROL

The spacecraft attitude determination and control issue include rotational kinematics. In this segment, the rotational kinematics of an unbending body are considered to depict the introduction of a spacecraft that is in rotational movement [12-15]. All through this area, the introduction of a reference outline fixed in a body is utilized to describe the introduction of the spacecraft body itself and the attitude control loop system as shown in figure 1 [16].

2.1 Direction matrix

Consider a reference outline *A* with a right-handed set of three symmetrical unit vectors $\{\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}\}$ and a reference outline *B* with another right-handed set of three symmetrical unit vectors $\{\overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3}\}$ [17-21]. The basis vectors $\{\overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3}\}$ of *B* are communicated regarding the basis vectors $\{\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}\}$ of *A* as follows [22]

$$\begin{bmatrix}\overrightarrow{b_1}\\ \overrightarrow{b_2}\\ \overrightarrow{b_3}\end{bmatrix} = \begin{bmatrix}c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ c_{31} & c_{32} & c_{33}\end{bmatrix}\begin{bmatrix}\overrightarrow{a_1}\\ \overrightarrow{a_2}\\ \overrightarrow{a_3}\end{bmatrix} = C^{\frac{B}{A}}\begin{bmatrix}\overrightarrow{a_1}\\ \overrightarrow{a_2}\\ \overrightarrow{a_3}\end{bmatrix} \tag{1}$$

Where $C^{\frac{B}{A}}$: $[C_{ij}]$ is known as the heading cosine matrix which depicts the introduction of *B* with respect to *A* [23-25]. The direction cosine matrix $C^{\frac{B}{A}}$ is additionally called the rotation matrix to *B* from *A*. Such a coordinate transformation is emblematically spoken of as

$$C^{\frac{B}{A}} : B \leftarrow A \tag{2}$$

For quickness, *C* for $C^{\frac{B}{A}}$ is frequently utilized. Since each arrangement of premise vectors of *A* and *B* comprises of symmetrical unit vectors, the bearing cosine lattice *C* is an orthogonal matrix [26-28]; accordingly

$$C^{-1} \equiv C^T \tag{3}$$

which is equivalent to

$$CC^T = I = C^T C \tag{4}$$

As a rule, a square matrix A is called a symmetrical matrix if AA^T is a diagonal matrix, and it is called an orthogonal matrix if AA^T is an identity matrix [29-32].

For an orthogonal matrix A, we have $A^{-1} = A^T$ and $|A| = \pm 1$. For an arbitrary vector $\vec{r}$ defined as

$$\vec{r} = y_1\vec{b}_1 + y_2\vec{b}_2 + y_3\vec{b}_3 = x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 \tag{5}$$

The coordination transformation relationship can be deduced as

$$y = Cx \tag{6}$$

where C is the direction cosine matrix of B relative to A and y and x are the two corresponding component vectors defined as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{7}$$

Three elementary rotations about the 1st, 2nd, and 3rd axes, respectively, of the reference frame A are described by the following rotation matrices

$$C_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_1 & sin\theta_1 \\ 0 & -sin\theta_1 & cos\theta_1 \end{bmatrix} \tag{8}$$

$$C_2(\theta_2) = \begin{bmatrix} cos\theta_2 & 0 & -sin\theta_2 \\ 0 & 1 & 0 \\ sin\theta_2 & 0 & cos\theta_2 \end{bmatrix} \tag{9}$$

$$C_3(\theta_3) = \begin{bmatrix} cos\theta_3 & sin\theta_3 & 0 \\ -sin\theta_3 & cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

where $C_i(\theta_i)$ denotes the direction cosine matrix C of an elementary rotation about the itch axis of A with an angle θ_i [33-37].

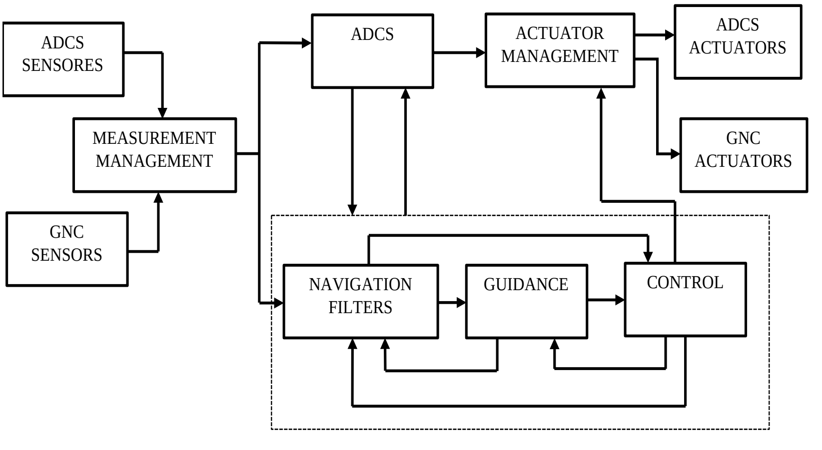

Fig. 1 Schematic diagram of attitude control loop

2.2 Euler Angle

One plan for situating an unbending body to a desired attitude is known as a body-axis rotation; it includes progressively pivoting multiple times about the axes of the turned, body-fixed reference outline [38-40]. The primary revolution is about any pivot. The second pivot is about both of the two axes not utilized for the principal revolution. The third revolution is then about both of the two axes not utilized for the second turn. There are 12 sets of Euler plots for such progressive pivots about the axes fixed in the body [41]. Consider three progressive body-axis pivots to portray the introduction of a reference outline *B* in respect to a reference outline *A* [42-45]. A specific arrangement picked here is emblematically spoken to as

$$C_1(\theta_1) \leftarrow C_2(\theta_2) \leftarrow C_3(\theta_3)$$

where $C_i(\theta_i)$ indicates a rotation about the its axis of the body-fixed frame with an angle θ_1 [46-48]. The rotation matrix to *B* from *A*, or the direction cosine matrix of *B* relative to *A*, is then defined as

$$C^{\frac{B}{A}} \equiv C_1(\theta_1)C_2(\theta_2)C_3(\theta_3) \tag{11}$$

$$C^{\frac{B}{A}} = \begin{bmatrix} c_2c_3 & c_2s_3 & -s_2 \\ s_1s_2c_3 - c_1s_3 & s_1s_2s_3 + c_1c_3 & s_1c_2 \\ c_1s_2c_3 + s_1s_3 & c_1s_2s_3 - s_1c_3 & c_1c_2 \end{bmatrix} \tag{12}$$

where $c_i \equiv cos\theta_i$ and $s_i \equiv sin\theta_i$.

In general, there are 12 sets of Euler angles, each subsequent in an alternate structure for the rotation matrix $C^{\frac{B}{A}}$. For instance, the grouping of $C_1(\theta_1) \leftarrow C_2(\theta_2) \leftarrow C_3(\theta_3)$ to *B* from *A* might be considered [49-52]. For this scenario, the rotation matrix becomes

$$C^{\frac{B}{A}} \equiv C_1(\theta_1)C_3(\theta_3)C_2(\theta_2) \tag{13}$$

$$C^{\frac{B}{A}} = \begin{bmatrix} c_2c_3 & s_3 & -s_2c_3 \\ -c_1c_2s_3 + s_1s_2 & c_1s_3 & c_1s_2s_3 + s_1c_2 \\ s_1c_2s_3 + c_1s_2 & -s_1c_3 & -s_1s_2s_3 + c_1c_2 \end{bmatrix} \tag{14}$$

In general, Euler angles have leverage over direction cosines in that three Euler angles decide an interesting orientation, despite the fact that there is no special arrangement of Euler plots for a given orientation [53-54].

2.3 Quaternion

Consider Euler's Eigen axis revolution around an arbitrary axis fixed both in a body-fixed reference frame *B* and in an inertial reference frame *A*. A unit vector $\vec{e}$ along the Euler axis is characterized as [55],

$$\vec{e} = e_1\vec{a}_1 + e_2\vec{a}_2 + e_3\vec{a}_3 \tag{15}$$

$$\vec{e} = e_1\vec{b}_1 + e_2\vec{b}_2 + e_3\vec{b}_3 \tag{16}$$

where e_i are the direction cosines of the Euler axis relative to both *A* and *B*, and $e_1^2 + e_2^2 + e_3^2$ = 1. Then the four Euler parameters or the quaternion can be defined as follows

$$q_1 = e_1 sin(\theta/2) \tag{17}$$

$$q_2 = e_2 sin(\theta/2) \tag{18}$$

$$q_3 = e_3 sin(\theta/2) \tag{19}$$

$$q_4 = cos(\theta/2) \tag{20}$$

where θ is the rotation angle about the Euler axis. Similar to the Eigen axis vector e = (e_1,e_2,e_3,e_4), a vector $\bar{q}$= (q_1,q_2,q_3) and the quaternion vector q= (q_1,q_2,q_3,q_4) can be defined as,

$$\bar{q} = e\ sin\theta/2 \tag{21}$$

$$q = \begin{bmatrix} \bar{q} \\ q_4 \end{bmatrix} \tag{22}$$

Note the all Euler parameters are not independent to each other, but constraint by the relation

$$q^T q = q^{-T}\bar{q} + q_4^2 = q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{23}$$

The direction cosine matrix can be parameterized as follows

$$C^{B/A} = C(q) \tag{24}$$

$$C^{B/A} = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2+q_3q_4) & 2(q_1q_3-q_2q_4) \\ 2(q_2q_1-q_3q_4) & 1-2(q_1^2+q_3^2) & 2(q_2q_3+q_1q_4) \\ 2(q_3q_1+q_2q_4) & 2(q_3q_2-q_1q_4) & 1-2(q_1^2+q_2^2) \end{bmatrix} \tag{25}$$

which can be written as,

$$C(q) = (q_4^2 - \bar{q}^T\bar{q})I + 2\bar{q}\bar{q}^T - 2q_4Q \tag{26}$$

where,

$$Q \equiv \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{27}$$

Consider 2 successive revolution to A'' from A defined by

$$C(q'): A' \leftarrow Av \tag{28}$$

$$C(q''): A'' \leftarrow A' \tag{29}$$

where (q') is the quaternion associated with the coordinate transformation $A' \leftarrow A$ and q'' is the quaternion associated with the coordinate transformation $A'' \leftarrow A'$ [56-59].

These successive revolutions are represented by single revolution to A'' directly from A, as follows

$$C(q): A'' \leftarrow A$$

where q is the quaternion associated with the coordinate transformation $A'' \leftarrow A$ and

$$C(q):C(q'')C(q')$$

The resulting quaternion transformation relationship can be defined as,

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} q''_4 & q''_3 & -q''_2 & q''_1 \\ -q''_3 & q''_4 & q''_1 & q''_2 \\ q''_2 & -q''_1 & q''_4 & q''_3 \\ -q''_1 & -q''_2 & -q''_3 & q''_4 \end{bmatrix} \begin{bmatrix} q'_1 \\ q'_2 \\ q'_3 \\ q'_4 \end{bmatrix} \tag{30}$$

which is known as the quaternion multiplication rule in matrix form [60].

The 4*4 orthonormal matrix in previous equation is called quaternion matrix, it can be re-written as,

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} q'_4 & q'_3 & -q'_2 & q'_1 \\ -q'_3 & q'_4 & q'_1 & q'_2 \\ q'_2 & -q'_1 & q'_4 & q'_3 \\ -q'_1 & -q'_2 & -q'_3 & q'_4 \end{bmatrix} \begin{bmatrix} q''_1 \\ q''_2 \\ q''_3 \\ q''_4 \end{bmatrix} \tag{31}$$

The 4*4 matrix in above equation is also orthonormal matrix and is known as quaternion transmuted matrix.

2.4 Kinematic differential equation

Consider kinematics in which the relative orientation between two reference frames in time dependent [61]. The time dependent connection between two reference frames is depicted by the supposed kinematic differential conditions. Consider two reference frames *A* and *B*, which are moving with respect to one another [62-65]. The precise velocity vector of a reference frame *B* regarding a reference frame *A* is indicated by $\vec{\omega} \equiv \vec{\omega}^{B/A}$, and it is communicated as far as the basis vectors of *B* as pursues

$$\vec{\omega} = \omega_1\overrightarrow{b_1} + \omega_2\overrightarrow{b_2} + \omega_3\overrightarrow{b_3} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{32}$$

where the $\vec{\omega}$ is time dependent. The kinematic diff [66] equation for the direction cosine matrix *C* is given by,

$$C + \Omega C = 0$$

where,

$$\Omega \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{33}$$

Like the kinematic differential condition for the direction cosine matrix *C*, the orientation of a reference frame *B* with respect to a reference frame *A* can likewise be depicted by presenting the time dependence of Euler angles [67-69].

Consider the rotational succession of $C_1(\theta_1) \leftarrow C_2(\theta_2) \leftarrow C_3(\theta_3)$ to *B* from *A*. The time derivatives of Euler angles, called Euler rates, are signified by $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$. These successive revolutions result in,

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ 0 \\ 0 \end{bmatrix} + C_1(\theta_1) \begin{bmatrix} 0 \\ \dot{\theta}_2 \\ 0 \end{bmatrix} + C_1(\theta_1)C_2(\theta_2) \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} \tag{34}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -sin\theta_2 \\ 0 & cos\theta_1 & sin\theta_1 cos\theta_2 \\ 0 & -sin\theta_1 & cos\theta_1 cos\theta_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{35}$$

Note that the 3*3 matrix in above equation is not an orthogonal matrix because $\overrightarrow{b_1}\overrightarrow{a''_2}$ and $\overrightarrow{a'_3}$ don't constitute a set of orthogonal unit vector [70].

The inverse relation can be found by inverting the matrix which is not an orthogonal matrix, as follows

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = 1/cos\theta_2\begin{bmatrix}cos\theta_2 & sin\theta_1 sin\theta_2 & cos\theta_1 sin\theta_2\\ 0 & cos\theta_1 cos\theta_2 & -sin\theta_1 cos\theta_2\\ 0 & sin\theta_1 & cos\theta_1\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{36}$$

which is the kinematic differential equation for the sequence of $C_1(\theta_1) \leftarrow C_2(\theta_2) \leftarrow C_3(\theta_3)$.

On the off chance that the above equations are known as functions of time, at that point the orientation of *B* with respect to a function of time can be decide by settling the above condition. Numerical integration of the above condition, be that as it may, includes the calculation of trigonometric elements of the points [71]. Additionally, note that the above condition winds up particular when $\theta_2 = \pi/2$.

Such a numerical peculiarity issue for a specific orientation angle can be stayed away from by choosing an alternate sets of Euler angles, yet it is an intrinsic property of every single different arrangement of Euler angles [72]. The kinematic differential conditions for the quaternion are given by,

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = 1/2\begin{bmatrix}q_4 & -q_3 & q_2 & q_1\\ q_3 & q_4 & -q_1 & q_2\\ -q_2 & q_1 & q_4 & q_3\\ -q_1 & -q_2 & -q_3 & q_4\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{bmatrix} \tag{37}$$

which can also be written as,

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = 1/2\begin{bmatrix}0 & \omega_3 & -\omega_2 & \omega_1\\ -\omega_3 & 0 & \omega_1 & \omega_2\\ \omega_2 & -\omega_1 & 0 & \omega_3\\ -\omega_1 & -\omega_2 & -\omega_3 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} \tag{38}$$

In terms of $\bar{q}$ and ω derived as,

$$\bar{q} = \begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} \quad \omega = \begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}$$

The kinematic differential equation can be written as follows,

$$\dot{\bar{q}} = 1/2(q_4\omega - \omega * \bar{q}) \tag{39}$$

$$\dot{q}_4 = -1/2\omega^T\bar{q} \tag{40}$$

where,

$$\omega \times \bar{q} \equiv \begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} \tag{41}$$

In strap down inertial reference systems of aerospace vehicles, the body rates, ω_1, ω_2, and ω_3 are estimated by rate gyros which are 'strapped down' to the vehicles. The kinematic differential condition is then coordinated numerically utilizing an on-board flight PC to decide the orientation of the vehicles as far as the quaternion [73-75]. Inertial sensors, for example, star trackers or Sun sensors are utilized to address state spread blunders brought about by precise rate estimation vulnerabilities (e.g., gyro drift and bias). The quaternion has no characteristic geometrical singularity, dissimilar to Euler angles. In addition, the quaternion is appropriate to on-board ongoing calculation in light of the fact that just items and no trigonometric relations exist in the quaternion kinematic differential equations. Subsequently, spacecraft orientation is currently usually portrayed regarding the quaternion.

3. KALMAN FILTERING

3.1 Extended Kalman filtering

An assortment of recursive attitude estimation algorithms dependent on Kalman filtering, extended Kalman filtering, unscented Kalman filtering, or particle filtering. The Kalman filtering was initially created in 1960 as another way to deal with linear filtering and prediction problems. When it is connected to non-linear dynamical frameworks, it is at that point alluded to as the extended Kalman filtering (EKF) [76]. The standard of the EKF is quickly presented here. Consider a nonlinear dynamical framework portrayed by

$$\dot{x}(t) = f(x,t) + G(t)w(t) \tag{42}$$

where x is the state vector and w is the process noise vector. It is accepted that the process noise is a Gaussian white noise whose mean and covariance are described as

$$E[w(t)] = 0 \tag{43}$$

$$E[w(t)w^T(\tau)] = Q(t)\delta(t-\tau) \tag{44}$$

The initial mean values of the state vector and the initial covariance of the state estimation error vector are given by

$$E[x(t_0)] = \hat{x}(t_0) = \hat{x}_0 \tag{45}$$

$$E\{[x(t_0)-\hat{x}][x(t_0)-\hat{x}]^T\} = P(t_0) = P_0 \tag{46}$$

The estimated state vector satisfies the following equation

$$\dot{\hat{x}} = E[f(x,t)] = \hat{f}(x,t) \approx f(\hat{x},t) \tag{47}$$

and its solution can be derived by

$$\hat{x}(t) = \Phi(t,\hat{x}(t_0),t_0) \tag{48}$$

Let the state estimation error vector and its error covariance matrix be derived by

$$\tilde{x}(t) = x(t) - \hat{x}(t) \tag{49}$$

$$P(t) = E[\tilde{x}(t)\tilde{x}^T(t)] \tag{50}$$

Therefore,

$$\dot{\tilde{x}} \approx F(t)\tilde{x}(t) + G(t)w(t) \tag{51}$$

where,

$$F(t) = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}(t)} \tag{52}$$

Then the solution would be

$$\tilde{x}(t) = \Phi(t,t_0)\tilde{x}(t_0) + \int_{t_0}^{t} \Phi(t,\tau)G(\tau)w(\tau)d\tau \tag{53}$$

where $\Phi(t, t_0)$ is the state transition matrix with the following properties:

$$\frac{\partial}{\partial t}\Phi(t, t_0) = F(t)\Phi(t, t_0) \quad (54)$$

$$\Phi(t_0, t_0) = I \quad (55)$$

The error covariance matrix *P(t)* satisfies the Riccati equation,

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) \quad (56)$$

And the solution of the above equation would be as follows,

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) \quad (57)$$

The estimated state vector and the state estimation error covariance matrix can be generated as,

$$\widehat{x_j^-} = \Phi(t_j, \hat{x}_{j-1}^+, t_{j-1}) \quad (58)$$

$$P_j^- = \Phi(t_j, t_{j-1})P_{j-1}^+\Phi(t_j, t_{j-1})^T + N_{j-1} \quad (59)$$

where,

$$N_{j-1} = \int_{t_{j-1}}^{t_j} \Phi\,(t_j, \tau)G(\tau)Q(\tau)G^T(\tau)\Phi^T(t_j, \tau)d\tau \quad (60)$$

A measurement model can be derived by,

$$y_j = h(x_j) + v_j \quad (61)$$

and,

$$E[v_j] = 0 \quad (62)$$

$$E[v_j v_j^T] = R_j \quad (63)$$

and its measurement sensitivity matrix has been obtained as follows,

$$H_j = \left.\frac{\partial h(x)}{\partial x}\right|_{\hat{x}_j^-} \quad (64)$$

The minimum-variance estimate of x_j using the measurement y_j is as follows,

$$\hat{x}_j^+ = \hat{x}_j^- + K_j[y_j - h(\hat{x}_j^-)] \quad (65)$$

where Kalman filter gain is,

$$K_j = P_j^- H_j^T[H_j P_j^- H_j^T + R_j]^{-1} \quad (66)$$

The error covariance is,

$$P_j^+ = [I - K_j H_j]P_j^- \quad (67)$$

3.2 Unscented Kalman Filtering

The EKF is broadly utilized for the state estimation of nonlinear dynamical frameworks. Be that as it may, the unscented Kalman filtering (UKF) is known to perform superior to the EKF in light of the fact that the UKF lessens the linearization errors of the EKF [77-78]. The UKF calculation is derived as follows,

$$x_{j+1} = f(x_j, j) + w_j \tag{68}$$

$$y_j = h(x_j, j) + v_j \tag{69}$$

where x_j is the state vector, y_j is the measurement vector, w_j is the process noise vector, and v_j is the measurement noise vector.

The UKF is initialized as,

$$\hat{x}_0^+ = E[x_0] \tag{70}$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \tag{71}$$

The next step is to get a set of sigma points using the current best estimate of the mean and covariance as follows,

$$\hat{x}_{j-1}^i = \hat{x}_{j-1}^+ + \tilde{x}_{j-1}^i \tag{72}$$

$$\tilde{x}_{j-1}^i = \left[\sqrt{nP_{j-1}^+}\right]_i^T i = 1, \ldots, \text{n} \tag{73}$$

$$\tilde{x}_{j-1}^{n+i} = -\left[\sqrt{nP_{j-1}^+}\right]_i^T i = 1, \ldots, n \tag{74}$$

Using the propagated sigma point vectors $\hat{x}_j^i$, we obtain a priori state estimate $\hat{x}_j^-$ and error covariance P_j^- as

$$\hat{x}_j^i = f(\hat{x}_{j-1}^i, j) \tag{75}$$

$$\hat{x}_j^- = \frac{1}{2n}\sum_{i-1}^{2n} \hat{x}_j^i \, a_j \tag{76}$$

$$P_j^- = \frac{1}{2n}\sum_{i-1}^{2n}(\hat{x}_j^i - \hat{x}_j^-)(\hat{x}_j^i - \hat{x}_j^-)^T + Q_{j-1} \tag{77}$$

where a_i are weighting coefficients.

Sigma points are recomputed using the current best estimate of the mean and covariance,

$$\hat{x}_j^i = \hat{x}_j^- + \tilde{x}_j^i \tag{78}$$

$$\tilde{x}_j^i = \left[\sqrt{nP_j^-}\right]_i^T i = 1, \ldots\text{n} \tag{79}$$

$$\tilde{x}_j^{n+i} = -\left[\sqrt{nP_j^-}\right]_i^T i = 1, \ldots\text{n} \tag{80}$$

The predicted observation vector y_j and the covariance matrices are calculated,

$$\hat{y}_j^i = h(x_j^i, j) \tag{81}$$

$$\hat{y}_j = \frac{1}{2n}\sum_{i-1}^{2n} \hat{y}_j^i \tag{82}$$

$$P_y(j) = \frac{1}{2n}\sum_{i-1}^{2n}(\hat{y}_j^i - \hat{y}_j^-)(\hat{y}_j^i - \hat{y}_j^-)^T + R_j \tag{83}$$

$$P_y(j) = \frac{1}{2n}\sum_{i-1}^{2n}(\hat{y}_j^i - \hat{y}_j^-)(\hat{y}_j^i - \hat{y}_j^-)^T + R_j \tag{84}$$

Similar to the Kalman filter, the posteriori state vector $\hat{x}_j^+$ is updated using the measurement vector y_j,

$$\hat{x}_j^+ = \hat{x}_j^- + K_j(y_j - \hat{y}_j) \tag{85}$$

$$K_j = P_{xy(j)}P_{y(j)}^{-1} \tag{86}$$

$$P_j^+ = P_j^- - K_j P_{y(j)} K_j^T \tag{87}$$

4. EULER'S ROTATIONAL EQUATIONS OF MOTION

Consider a rigid spacecraft with a body-fixed reference outline *B* that has its origin at the focal point of mass. The angular velocity vector of the reference outline *B* regarding an inertial reference outline *A* is indicated by $\vec{\omega} \equiv \vec{\omega}^{B/A}$, and it is derived as far as the basis vectors of *B* as pursues

$$\vec{\omega} = \omega_1\overrightarrow{b_1} + \omega_2\overrightarrow{b_2} + \omega_3\overrightarrow{b_3} = \begin{bmatrix}\overrightarrow{b_1} & \overrightarrow{b_2} & \overrightarrow{b_3}\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{88}$$

The angular momentum equation of a rigid body about its center of mass is

$$\vec{M} = \dot{\vec{H}} \tag{89}$$

where $\vec{H}$ is the angular momentum vector of a rigid body about its mass center and $\vec{M}$ is the outside moment following up on the body about its mass center, derived as far as body-fixed basis vectors $\{\overrightarrow{b_1} \quad \overrightarrow{b_2} \quad \overrightarrow{b_3}\}$, as pursues

$$\vec{H} = H_1\overrightarrow{b_1} + H_2\overrightarrow{b_2} + H_3\overrightarrow{b_3} \tag{90}$$

$$\vec{M} = M_1\overrightarrow{b_1} + M_2\overrightarrow{b_2} + M_3\overrightarrow{b_3} \tag{91}$$

Therefore,

$$\dot{\vec{H}} \equiv \{d\vec{H}/dt\}_A = \{d\vec{H}/dt\}_B + \vec{\omega} \times \vec{H} \tag{92}$$

where,

$$\{d\vec{H}/dt\}_B = \dot{H}_1\overrightarrow{b_1} + \dot{H}_2\overrightarrow{b_2} + \dot{H}_3\overrightarrow{b_3} \tag{93}$$

The angular momentum vector is described by $\vec{H} = \hat{J}.\vec{\omega}$ where $\hat{J}$ is the inertia dyadic related to the inertia matrix expressed as,

$$\hat{J} = \begin{bmatrix}\overrightarrow{b_1} & \overrightarrow{b_2} & \overrightarrow{b_3}\end{bmatrix}\begin{bmatrix}J_{11} & J_{12} & J_{13}\\ J_{21} & J_{22} & J_{23}\\ J_{31} & J_{32} & J_{33}\end{bmatrix}\begin{bmatrix}\overrightarrow{b_1}\\ \overrightarrow{b_2}\\ \overrightarrow{b_3}\end{bmatrix} \tag{94}$$

The rotational equation of motion of a rigid body about its center of mass is then expressed as

$$\vec{M} = \left\{d\vec{H}/dt\right\}_B + \vec{\omega} \times \vec{H} \tag{95}$$

$$\vec{M}= \left\{d(\vec{J}.\vec{\omega})/dt\right\}_B + \vec{\omega} \times \hat{J}.\vec{\omega} \tag{96}$$

$$\vec{M}= \left\{d\hat{J}/dt\right\}_B.\vec{\omega}+\hat{J}.\{d\vec{\omega}/dt\}_B + \vec{\omega} \times \hat{J}.\vec{\omega} \tag{97}$$

where $\left\{d\hat{J}/dt\right\}_B$=0 and $\{d\vec{\omega}/dt\}_B$=$\{d\vec{\omega}/dt\}_A$= $\dot{\vec{\omega}}$. Then we get,

$$\vec{M}= \hat{J}.\vec{\omega}+\vec{\omega} \times \hat{J}.\vec{\omega} \tag{98}$$

is called Euler's rotational equation of motion in vector form. The rotational equation of motion in matrix form can be expressed as

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \tag{99}$$

As,

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{100}$$

It is absurd that

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{101}$$

Defining a skew-symmetric matrix

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{102}$$

From the conditions, $\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$ can be expressed as

$$J\dot{\omega}+\Omega\ J\dot{\omega}=M \tag{103}$$

where,

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix},\ \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix},\ \text{and } M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

Using cross product notation of two column vectors, ω and $J\omega$, expressed as

$$\omega^*J\omega \equiv \Omega J\omega \tag{104}$$

and it can be rewritten as,

$$J\dot{\omega}+\ \omega^*J\dot{\omega}=M \tag{105}$$

For a principal-axis reference frame with a set of basis vectors $\{\overrightarrow{b_1}\ \ \overrightarrow{b_2}\ \ \overrightarrow{b_3}\}$, Euler's rotational equations of motion of a rigid body become

$$\dot{J}_1\dot{\omega}_1\text{-}(J_2\text{-}J_3)\omega_2\omega_3=M_1 \tag{106}$$

$$\dot{J}_2\dot{\omega}_2\text{-}(J_3\text{-}J_1)\omega_3\omega_1=M_2 \tag{107}$$

$$\dot{J}_3\dot{\omega}_3\text{-}(J_1\text{-}J_2)\omega_1\omega_2=M_3 \tag{108}$$

where J_1, J_2 and J_3 are the principal moments of inertia, these are three coupled, nonlinear ordinary differential equations for state variables $\omega_1\omega_2\omega_3$ of a rigid body.

These dynamical equations and the kinematic differential equations of the preceding sections completely describe the rotational motions of a rigid body with three rotational degrees of freedom [79-80].

5. CONCLUSIONS

We are aware that there is a world farther away than we can get and that is why we believe that space investigation must be uninterrupted. Orbital elements dependent on Newton's standards have served human spaceflight attempts for more than 60 years, making wonders in science, innovation and construction. In the fields we have explored in this article, advances in circle configuration, control and motion will keep on contacting the utmost of future spaceflights, empowering new space missions. In this paper, we have presented a quaternion feedback controller based on a quaternion product and Euler angles algorithms and Kalman filtering with extended and non-centered features that asymptotically stabilize two equilibrium points. And moreover, among these results whether they have been successful or failed, whether they are from the East or the West, the common outcome is that they will light the path in front of us, and bring us new expectation and certainty to widen and extend our comprehension of the universe.

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