PENETRATION OF A SHOCK WAVE IN A FLAME FRONT

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DOI: 10.13111/2066-8201.2009.1.1.13

Abstract
The present paper deals with the interactions between a fully supersonic flame front, situated in a supersonic two-dimensional flow of an ideal homogeneous combustible gas mixture, and an incident shock wawe, which is penetrating in the space of the hot burnt gases. A possible configuration, which was named „simple penetration" is examined. For the anlysis of the interference phenomena, shock polar and shock-combustion polar are used. At the same time, the paper shows the possibility to produce similar but more complicated configurations, which may contain expansion fans and reflected shock waves.

Notation

$_i$ or $_j$ the – subscrips indicating the i or j areas of flow,
$_{ij}$ – subscript for function or variable at the boundary among the two jointed areas,
M_i – Mach number
T_i – absolute temperature,
γ – specific heat ratio,
Q – heat liberated by chemical reactions per unit mass of the gas mixture,
a_i – sound velocity in the i area,
p_i – pressure
ρ_i – density
$\lambda_{ij} = \frac{\rho_i}{\rho_j}$ – density ratio,
$\xi_{ij} = \frac{p_i}{p_j}$ – pressure ratio,
m_i – flame Mach number,
δ_{ij} – deflection angle of the flow behind the flame front (or shock wave) between the i and j areas,
R – gas constant,

1. Introduction

Considering the two-dimensional supersonic flow of an ideal homogeneous combustible gas mixture which can be ignited in a point P (Fig. 1), J.F. Clarke [2] shows, for the first time, the possibility of formation of a *fully supersonic* flame front (combustion wave), PF, after an oblique shock wave, PS, attached in the same point, P (Fig. 1).

The flame front and the shock wave are treated as surfaces of discontinuity. The term *fully supersonic* means that all Mach numbers M_∞, M_1 and M_2 are greater than unity, and so the flow is supersonic in all areas.

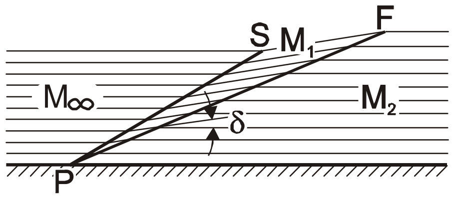

Fig. 1

To realize such a configuration, some conditions are necessary [2]:
- The flame propagation velocity must be high enough, so that the combustion wave inclination (respective to the direction of the initial flow) shall be sufficient to avoid the stability loss
- The burnt gas temperature behind the flame front must not be too high, because it is possible that the sound speed should became greater than the flow speed, and the flow in this area shold become subsonic.
- In P, the ignition must be obtained by heat addition (by pilot burner or by an other heat sourse), because the temperature of the unburned gas must not be too high, so that the temperature behind the shock wave (PS) remains under the limit of the ignition temperature. This means that the combustible gas mixture can not be ignited without external heat addition. The shock wave PS will necessarly appear, in front of the combustion wave to deviate the flow with an angle δ, bringing it to the initial direction.The flow will be redressed to the initial direction by the combustion wave PF.
- We shall asume, as in [2], that the flame front remains separated from the combustion wave, so that a detonation wave is avoided. This means that the flame propagation speed do not exceeds a certain limit.

2. The analysed shock-combustion configuration

The shock wave PS may be reflected by a superior wall parallel to the direction of the undisturbed flow, and the reflected wave will penetrate through the flame front PF in the region of the hot burnt gases. In Fig. 2 such a configuration is presented, which is, at least, theoretically posible.

3. The computation relations

We consider a premixed homogeneous gas mixture, which is characterized by the heat, Q liberated by chemical reaction per unit mass. In the computation programs we will use the notation:

$$q = \frac{2Q(\gamma^2 - 1)}{a_1^2}. \tag{1}$$

The variation of the Mach number with the temperature of the flame will be denoted by:

$$m = m(T) \tag{2}$$

Using the equations of the comservation of matter, momentum and energy, we will obtain the folowing relations:

$$\xi = \frac{p_2}{p_1} = 1 - (\lambda - 1)\cdot\gamma_1\cdot m^2, \tag{3}$$

in which λ is the density ratio:

$$\lambda = \frac{\rho_1}{\rho_2} = \frac{v_2}{v_1}, \tag{4}$$

and γ_1, the specific heat ratio for the cold unburnt gas mixture.

The density ratio will be obtained from the expression [6]:

$$\lambda = \frac{\gamma_2}{\gamma_2+1}\left(1+\frac{1}{\gamma_1 m^2}\right) - \sqrt{\left(\frac{\gamma_2-\gamma_1 m^2}{\gamma_1 m^2(\gamma_2+1)}\right)^2 - \frac{2}{m^2(\gamma_2+1)}\left[(\gamma_2-1)\frac{Q}{a_1^2} - \frac{\gamma_1-\gamma_2}{\gamma_1(\gamma_1-1)}\right]}, \tag{5}$$

where γ_2 is the specific heat ratio for the burnt gas mixture.

The equation for the pressure variation trough the flame front is:

$$\xi = \frac{p_2}{p_1} = 1 - \gamma_1\cdot m^2\cdot\left\{\frac{\gamma_2-\gamma_1\cdot m^2}{\gamma_1\cdot m^2\cdot(\gamma_2+1)} - \sqrt{\left[\frac{\gamma_2-\gamma_1\cdot m^2}{\gamma_1\cdot m^2\cdot(\gamma_2+1)}\right] - \frac{2N}{m^2\cdot(\gamma_2+1)}}\right\} \tag{6}$$

where

$$N = (\gamma_2 - 1)\cdot\frac{Q}{a_1^2} - \frac{\gamma_1-\gamma_2}{\gamma_1\cdot(\gamma_1-1)}. \tag{6a}$$

From the same conservation laws and by using certain geometric considerations, we obtain [26]:

$$tg\delta_F = \frac{m(\lambda-1)\cdot\sqrt{M_1^2 - m^2}}{M_1^2 + (\lambda-1)\cdot m^2}, \tag{7}$$

where M_1 is the Mach number of the unburnt gas mixture flow.

Also, we can compute the Mach number, M_2, behind the flame front:

$$M_2^2 = \frac{\gamma_1\cdot R_1\cdot T_1\cdot\left[M_1^2 + (\lambda^2-1)\cdot m^2\right]}{\gamma_2\cdot R_2\cdot T_2}. \tag{8}$$

It is possible to eliminate the m parameter from the Eq. (2), (5) and (6), and to obtain finaly:

$$tg\delta_F = \frac{1-\xi}{\gamma_1\cdot M_1^2 + 1 - \xi}\cdot\sqrt{\frac{\frac{2\cdot\gamma_1\cdot M_1^2}{\gamma_2+1}\cdot\left[1+\frac{\gamma_1\cdot(\gamma_2-1)\cdot\frac{Q}{a_1^2} - \frac{\gamma_1-\gamma_2}{\gamma_1-1}}{1-\xi}\right]}{\frac{\gamma_2-1}{\gamma_2+1}+\xi} - 1} \tag{9}$$

This relation is the equation of the flame deflagration polar, as the relation:

$$tg\delta = \frac{\xi-1}{\gamma\cdot M_1^2 + 1 - \xi}\cdot\sqrt{\frac{\frac{2\cdot\gamma\cdot M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} - \xi}{\xi + \frac{\gamma-1}{\gamma+1}}} \tag{10}$$

represents the shock polar. Otherwise, if in Eq. (9) we make Q=0, $\gamma_1=\gamma_2=\gamma$ and we change the sign, we obtain Eq. (10).

In the above formulas M_1 is the Mach number of the flow of the unburnt gas from zone.

If it is possible to consider $\gamma_1=\gamma_2=\gamma$ (the case of the poor mixtures), we can simplify the Eq. (9). We denote:

$$\left.\begin{aligned} & b=\gamma M_1^2+1,\quad L=\frac{\gamma-1}{\gamma+1}, \\ & s=\frac{2\gamma M_1^2}{\gamma+1}\left[1+\gamma(\gamma+1)\frac{Q}{a_1^2}\right]-L. \end{aligned}\right\} \tag{11}$$

With this notations, Eq. (8) became:

$$\mathrm{tg}\delta_F=\frac{1-\xi}{b-\xi}\cdot\sqrt{\frac{s-\xi}{L+\xi}}. \tag{12}$$

The reaction heat can be computed, after the methods which was developed in [6].

The interferences between the shock and combusion waves may produce complex flow configurations, whith may contain, apart from shock waves, expansion fans of Prandtl-Meyer type.

For the Prandtl-Meyer type expansion, we have [4]:

$$\begin{aligned} \delta= & -\sqrt{\frac{\gamma_j+1}{\gamma_j-1}}\mathrm{arctg}\sqrt{\frac{(\gamma_j-1)M_1^2-(\gamma_j+1)\xi^{\frac{\gamma_1-1}{\gamma_1}}+2}{(\gamma_j+1)\xi^{\frac{\gamma_1-1}{\gamma_1}}}}+ \\ & +\sqrt{\frac{\gamma_j+1}{\gamma_j-1}}\mathrm{arctg}\sqrt{\frac{\gamma_j-1}{\gamma_j+1}\left(M_1^2-1\right)}+ \\ & +\mathrm{arctg}\sqrt{\frac{(\gamma_j-1)M_1^2-(\gamma_j+1)\xi^{\frac{\gamma_1-1}{\gamma_1}}+2}{(\gamma_j-1)\xi^{\frac{\gamma_1-1}{\gamma_1}}}}-\mathrm{arctg}\sqrt{M_1^2-1}, \end{aligned} \tag{13}$$

$$\frac{T_2}{T_1}=\xi^{\frac{\gamma_j-1}{\gamma_j}} \tag{14}$$

(where M_1 denote the Mach number before the Prandtl-Meyer expansion and γ_1, the specific heat ratio)

$$M_2^2=\frac{(\gamma_j-1)\cdot M_1^2-2\cdot\left(\xi^{\frac{\gamma_j-1}{\gamma_j}}-1\right)}{(\gamma_j-1)\cdot\xi^{\frac{\gamma_j-1}{\gamma_j}}}. \tag{15}$$

The Eq. (9) and (12) represent two branchs of a continuous curve $\delta=\delta(\xi)$ [4]. The common tangent in the combination point ($\xi=1$) is:

$$\left[\frac{d\delta}{d\xi}\right]=\frac{\sqrt{M_1^2-1}}{\gamma_1\cdot M_1^2}. \tag{16}$$

According to the before considerations, the construction of the shock-expansion-combustion polar can be realized with the methods stated in [4] and [6].

4. The Mach number of the flame propagation

Because the purpose of this paper is to announce the theoretical possibility of the existence of such configurations, we can suppose that the Mach number of the flame propagation may be obtaind with the Passauer formula:

$$v_{1n}=AT_1^2, \tag{17}$$

where v_{1n} is the propagation velocity of the flame, A, an experimental coeficient, but which can be approximated with some data from literature [6] and [10]. T_1 is the unburnt gas temperature. If we note with m_1 and m_2 the Mach numbers in two arias with the temperatures T_1 and T_2 repectively, we can write:

$$m_1=\frac{v_{1n}}{a_1}=\frac{A}{\sqrt{\gamma R}}T_1^{3/2} \tag{18}$$

and:

$$m_1/m_2=(T_1/T_2)^{3/2} \tag{19}$$

5. The simple penetration

In this paper we will examine a possible configuration of the flow which was named „*simple penetration*". We will taken into consideration the flow of a gaseous combustible mixture whose Mach number is M_1 (Fig. 2).

The mixture is ignited and a fully supersonic flame front, F_1PF_2, will appear. Behind this flame front is the zone of the burnt gas.

A shock wave, S_1P, comes through the flame front, bringing forth the refracted shock wave, PS_2.

In the point P the flame front is changing his inclination because the temperatures in the regions 2 and 3 are different, and, according to (18), the flame Mach numbers are not the same.

We will take into consideration a poor combustible gas mixture which flows with a supersonic speed at $M_1 = 2$ and $T_1 = 300^oK$. Because the mixture is poor, we will consider that $\gamma_1 = \gamma_2 \cong 1.4$.

As previously stated, this paper tries to show only the possibility, at least theoretical, of the existence of such a flow and for this we will use, arbitrarily, the following caracteristic values (but which are near enough of the real gas mixture values):

$q_1 = 10$; $R = 287\ m^2/s^2 \times {}^oC$; $m_1 = 0.1$; $\xi_{13} = 1.2$; (20)

(the index $_{ij}$ corresponds to the regions noted by i and j, which are separeted by a flame front, a shock wave or an expansion fan).

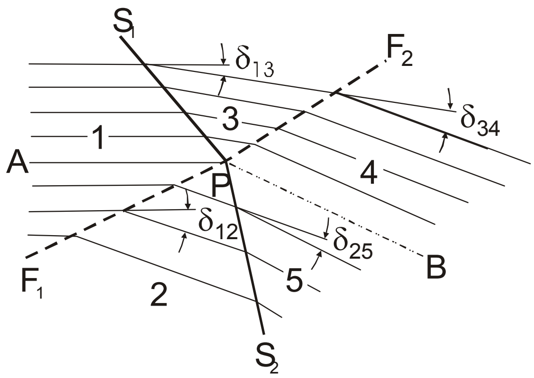

Fig. 2

For the tracing of the flame fronts PF_1 and PF_2, we have used the Eq. (1), (3), (5), (6), (7), (8) and (9). and some results and informations from [1], [3], [5], [6], [10].

Fig. 3 represent the shock and flame polars, obtaind with the aid of the equations mentioned before.
The principal numerical results are presented in the below table.

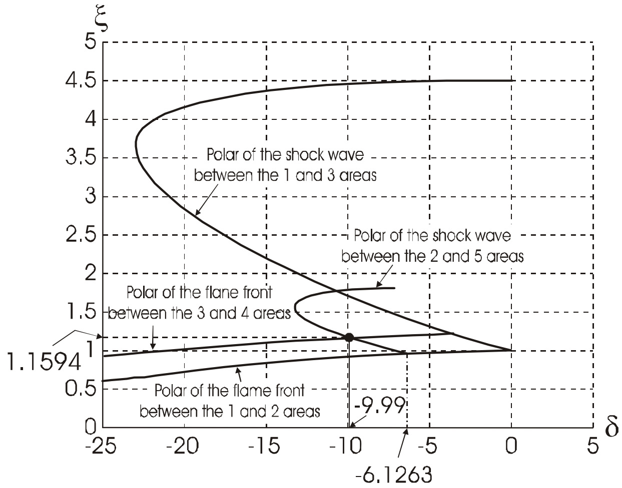

Fig. 3

Area 2	Area 3	Area 4	Area 5
M_2=1.3120	M_3=1.8819	M_4=1.2056	M_5=1.8819
T_2=919.6^{O}K	T_3=316.1^{O}K	T_4=934.9^{O}K	T_5=316.1^{O}K
δ_{12}=-6.1263^O	δ_{13}=--3.2939	δ_{34}=--9.76	δ_{25}=--3.2939
ξ_{12}=0.9697	ξ_{13}=1.2	ξ_{34}=0.9662	ξ_{15}=1.1591
λ_{12}=3.1610	λ_{13}=1.1391	λ_{34}=3.0610	λ_{25}=1.1187

Similar configurations can be obtaind taking into consideration another values than (20).

Also we could to use another law than the Passauer law (18) to compute the flame mach number. This law and the data (20) was used only to demonstrate the possibility of the (at least) theoretical existence of such a flow configuration.

6. The complex penetrations

In this paragraph we remark some theoretical flow configurations which can be produced when a shock wave penetrates in the space of the hot burnt gas mixture from behind a flame front.

We will consider a gas mixture which flows with $M_1 = 2$ and $T_1 = 300^oK$ and $\gamma_1 = \gamma_2 \cong 1.4$. The characteristic values in this new case will be:

$q_1 = 10$; $R = 287\ m^2/s^2 \times {}^oC$; $m_1 = 0.12$; $\xi_{13} = 1.4$;(21)

Proceeding as before, we obtain the shock and flame polars shown in Fig. 4.

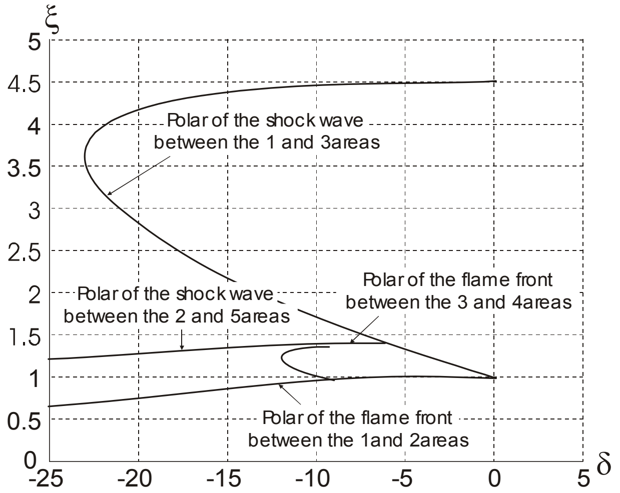

Fig. 4

We notice that the polar of the shock wave between the 2 and 5 areas can not intersect the polar of the flame front between the 3 and 4 areas. In this case it is not possible to achieve the equality between the deflection angles δ_{14} and δ_{15} and between the pressures p_4 and p_5. To obtain this equality a new shock and expansion wave system are necessary, a matter that will be examined in a future work.

7. Conclusions

In this paper we presented a plausible configuration of a supersonic two dimensional flow in which a shock wave penetrates through a flame front in the burnt gas zone. It was demonstrated that such a configuration is in a total concordance with the laws of conservation of mass, momentum and energy, which make possible (even probable) his apparition.

At the same time, the possible apparition of other, more complicated configurations was noted, an aspect that will be examined in a future paper.

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