PENETRATION OF A SHOCK WAVE IN A FULLY SUPERSONIC FLAME FRONT WITH THE FORMATION OF AN EXPANSION FAN

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Abstract: *In a previous paper [3] was treated the „simple penetration" of an incident shock wave through a fully supersonic flame front in the space of the hot burnt gases, situated in a supersonic two-dimensional flow of an ideal homogeneous /combustible gas was treated in a previous paper [3]. In the present paper takes into consideration, a configuration, in which an expansion fan is produced, is take into consideration the shock polar and expansion polar are used for the analyze of the interference phenomena.*

Key Words: shock waves, flame front, expansion fan

Notation

$_i$ or $_j$	–	subscripts indicating the i or j areas of the flow,
$_{i,j}$	–	subscript for function or variable at the boundary among the two jointed areas,
M_i	–	Mach number in the i area,
T_i	–	absolute temperature in the i area,
γ_i	–	specific heat ratio in the i area,
Q	–	heat liberated by chemical reactions per unit mass of the gas mixture,
a_i	–	sound velocity in the i area,
p_i	–	pressure in the i area,
ρ_i	–	density in the i area,
m_i	–	flame Mach number in the i area,
R	–	gas constant,
q_i	–	the ratio $2Q(\gamma_i^2-1)/a_i^2$ in the i area,
δ_{ij}	–	deflection angle of the flow behind the flame front (or shock wave) between the i and j areas,
$\lambda_{ij} = \dfrac{\rho_i}{\rho_j}$		density ratio between the i and j areas,
$\xi_{ji} = \dfrac{p_j}{p_i}$		pressure ratio between the j and i areas,

1. INTRODUCTION

In [3] we have signalized a possible flow configuration which can appear when a shock wave penetrates in the space of the hot burnt gas mixture, which flows behind a fully supersonic flame front. The characteristic values of this configuration was:

$M_1 = 2$; $T1 = 300^{O}C$; $q_1 = 10$; $R = 287\ m^2/s^2\times{}^{o}C$; $m_1 = 0.1$; $\xi_{13} = 1.2$;

With these characteristics it was obtained the possible configuration named *simple penetration* and having these characteristics, which are similar to those reproduced from [3] in fig. 1.

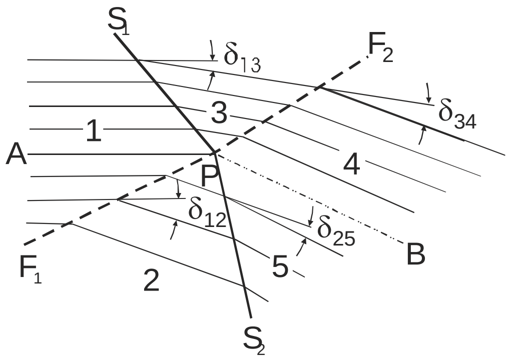

Fig. 1

The polars of this configuration are represented in fig. 2 (from [3]):

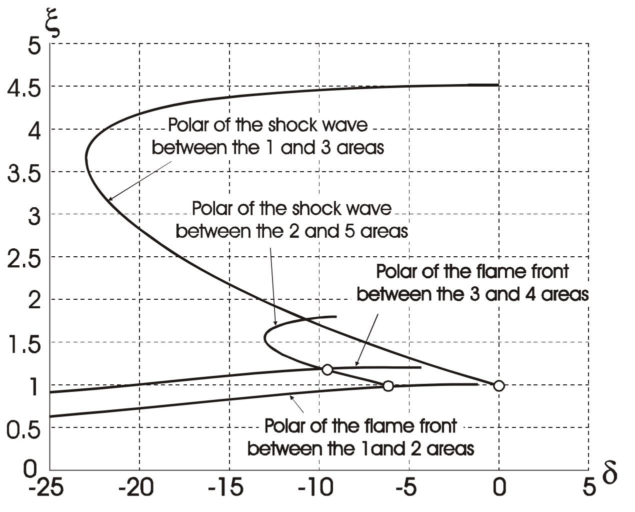

Fig. 2

2. PENETRATION WITH THE FORMATION OF AN EXPANSION FAN

In this paper, we will analyze another case, which was signalized in [3], in which the simple penetration of the shock wave is not possible.

As in [3], we will suppose that $\gamma_1 \cong \gamma_2 \cong 1.4$ and, because of this, the specific heat ratio will be writed without index.

The characteristics values of the gas mixture chosen for the considered case are:

M_1=2; q_1=12; γ=1.4; R_1=287; T_1=300; m_1=0.12.

Owing to these characteristics, the polar of the shock wave between the 2 and 5 areas can not intersect the polar of the flame front between the 3 and 4 areas (see Fig.1 - reproduced from [3]).

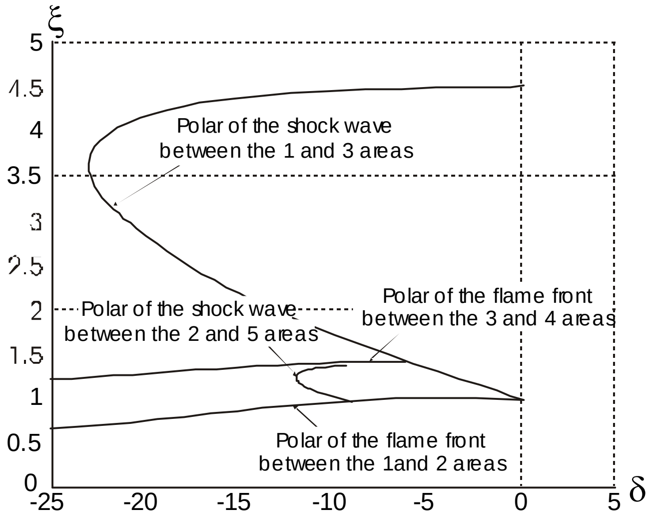

Fig. 3

In this case it is not possible to realize the equality between the deflection angles δ_{14} and δ_{15} and between the pressures p_4 and p_5, as in the case showed in fig. 1 and fig. 2.

We will try to realize these equalities by introducing a Prandtl-Meyer expansion fan after the shock wave which is occurring between the 1 and 3 areas (fig. 5).

If we will take into consideration the polars from Fig. 3, and if instead of the flame front polar between the 3 and 4 areas we will introduce the Pramdtl-Meyer expansion polar, the configuration showed in the Fig. 4. will be obtained.

The configuration of the flow is shown in the Fig. 5

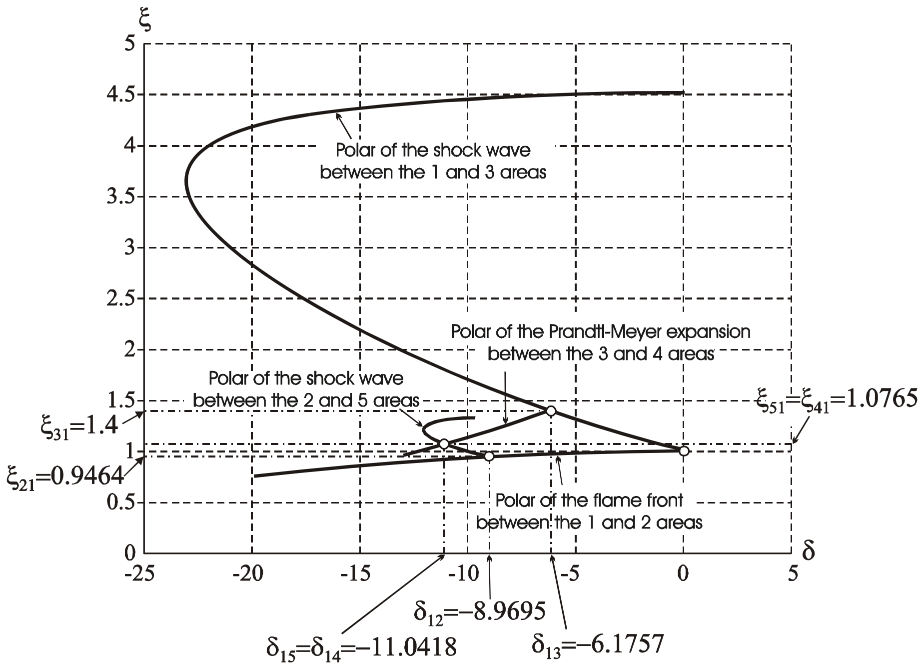

Fig. 4

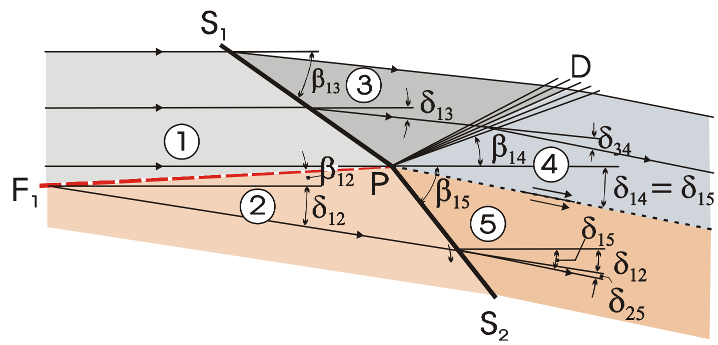

Fig. 5

The principal numerical results are presented in the table below.

Area 2	Area 3	Area 4	Area 5
M_2=1.1657	M_3=1.7793	M_4=1.9500	M_5=1.0624
T_2=1039.3 ^{O}K	T_3=330.6 ^{O}K	T_4=306.7250 ^{O}K	T_5=1078.3 ^{O}K
δ_{12}=-8.9695 ^{O}	δ_{13}=-6.1757 ^{O}	δ_{14}= -11.0419 ^{O}	δ_{15}= -11.0414 ^{O}
β_{12}=3.4398 ^{O}	β_{13}=-35.4090 ^{O}	β_{14}=-28.0191 ^{O}	β_{15}=-54.0521
ξ_{21}=0.9464	ξ_{31}=1.4	ξ_{41}=1.0765	ξ_{51}=1.0765
λ_{12} =3.6606	λ_{13}=0.7872	λ_{14}=0.9498	λ_{15}=3.3390

3. THE COMPUTATION RELATIONS

The above results was got with using the following relations:

3.1. *For the shock waves*

$$\mathrm{tg}\delta_{ij} = \frac{\xi_{ji} - 1}{\gamma\square M_i^2 + 1 - \xi_{ji}} \cdot \sqrt{\frac{\frac{2\cdot\gamma\cdot M_i^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} - \xi_{ji}}{\xi_{ji} + \frac{\gamma-1}{\gamma+1}}} \tag{1}$$

$$M_j^2 = \frac{\left[(\gamma+1)\xi_{ji} + (\gamma-1)\right]M_i^2 - 2\left(\xi_{ji}^2 - 1\right)}{\xi_{ji}\left[(\gamma-1)\xi_{ji} + (\gamma+1)\right]}, \tag{2}$$

$$\sin(\beta_{ij}) = \sqrt{\frac{\xi_{ji}(\gamma+1) + (\gamma-1)}{2\gamma M_i^2}}, \tag{3}$$

$$T_j = T_i \frac{(\gamma-1)\xi_{ji} + (\gamma+1)}{(\gamma+1)\xi_{ji} + (\gamma-1)}, \tag{4}$$

$$\lambda_{ij} = \frac{(\gamma-1)\xi_{ji} + (\gamma+1)}{(\gamma+1)\xi_{ji} + (\gamma\_-1)}. \tag{5}$$

3.2. *For the expansion fan*

$$\delta_{ij} = -\sqrt{\frac{\gamma+1}{\gamma-1}}\left[\operatorname{a\,tan}\sqrt{\frac{(\gamma-1)M_i^2-(\gamma+1)\xi_{ji}^{\frac{\gamma-1}{\gamma}}+2}{(\gamma+1)\xi_{43}^{\frac{\gamma-1}{\gamma}}}}-\operatorname{a\,tan}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_i^2-1\right)}\right]+ \tag{6}$$

$$+\operatorname{a\,tan}\sqrt{\frac{(\gamma-1)M_i^2-(\gamma+1)\xi_{43}^{\frac{\gamma-1}{\gamma}}+2}{(\gamma_1-1)\xi_{ji}^{\frac{\gamma-1}{\gamma}}}}-\operatorname{a\,tan}\sqrt{\left(M_i^2-1\right)}\,,$$

$$M_j^2 = \frac{(\gamma-1)M_i^2-2\left(\xi_{ji}^{\frac{\gamma-1}{\gamma}}-1\right)}{(\gamma-1)\xi_{ji}^{\frac{\gamma-1}{\gamma}}}\,. \tag{7}$$

$$\beta_{ik} = \delta_{ij}+\beta_{jk} = \delta_{ij}+\operatorname{a\,sin}\left(\frac{1}{M_j}\right), \tag{8}$$

$$\frac{T_j}{T_i} = \xi_{ji}^{\frac{\gamma-1}{\gamma}}\,, \tag{9}$$

$$\lambda_{ik} = \lambda_{ij}\cdot\lambda_{jk} = \lambda_{ij}\cdot\left(\xi_{kj}^{-\frac{1}{\gamma}}\right). \tag{10}$$

3.3. *For the combustion wave*

$$\operatorname{tg}\delta_{ij} = \frac{1-\xi_{ji}}{\gamma M_i^2+1-\xi_{ji}}\cdot\sqrt{\frac{\frac{2\cdot\gamma\cdot M_i^2}{\gamma+1}\cdot\left[1+\frac{\gamma\cdot(\gamma-1)\frac{Q}{a_i^2}}{1-\xi_{ji}}\right]}{\frac{\gamma-1}{\gamma+1}+\xi_{ji}}-1}\,, \tag{11}$$

In [3] it was shown that we can admit the relation:

$$m_i/m_j = (T_i/T_j)^{3/2}\,, \tag{12}$$

$$M_j^2 = \frac{\gamma\cdot R\cdot T_i\cdot\left[M_i^2+\left(\lambda_{ij}^2-1\right)\cdot m_i^2\right]}{\gamma\cdot R\cdot T_j}, \tag{13}$$

$$\beta_{ij} = \operatorname{a\,sin}\left(\frac{m_i}{M_i}\right), \tag{14}$$

$$T_j = T_i \cdot \lambda_{ij} \cdot \xi_{ji} \,, \tag{15}$$

$$\lambda_{ij} = 1 + \frac{1}{(\gamma+1)\, m_i^2} \cdot \left[1 - m_i^2 - \sqrt{\left(1 - m_i^2\right)^2 - q_i m_i^2} \right]. \tag{16}$$

7. CONCLUSIONS

The configuration of the two dimensional supersonic flow presented in this paper, as the flow presented in [3], is a plausible one, because it is in a total concordance with the conservation laws of mass, momentum and energy, making possible (even probable) his apparition; but only the experimental confirmation can award the certainty of the physical existence of such a flow.

REFERENCES

[1] E. Carafoli, *Aerodinamica vitezelor mari (Fluide compresibile)*, Bucureşti, Edit, Academiei, 1957.
[2] H. W. Emmons, *Fundamentals of gas dynamics,* vol. **3**, London, Oxford University Press, 1958.
[3] D. Pantazopol, *Penetration of a shock wave in a flame front*, INCAS Bulletin No 1/ 2009.
[4] J. F. Clarke, *Some remarks on the treatment of fully supersonic oblique flames as gas dynamical discontinuity,* Co. A. Report Aero, 179, 1965.
[5] D. Pantazopol, F. Munteanu, *Polara şoc-combustie (detentă-combustie) cu conside-rarea variaţiei raportului căldurilor specifice al gazului la trecerea prin frontul de flacără*, St. cercet. mec. apl. 3, 1972.
[6] D. Mateescu, *Two-dimensional supersonic flow with flame sheet*, St. cercet. mec. apl. 3, 1972.
[7] F. Munteanu, *Calculul curgerii amestecurilor gazoase combustibile prin frontul de flacără cu considerarea variaţiei căldurilor specifice*, St. cercet. mec. apl. 3, 1974.
[8] D. Pantazopol, O. Manole, *Determinarea unor configuraţii de curgeri şi a spectrului curgerii în prezenţa undelor de şoc şi a fronturilor de flacără*, INCAS, 1998.