Implementation of a correction factor for the Pohlhausen laminar boundary layer applied on the CEVA curved wall jet model

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Abstract: *Curved wall jets have many technical applications, ranging from aeronautical circulation controlled wings to micro-fluidics and cryogenics. This paper addresses the issue of correctly estimating the boundary layer separation for laminar curved wall jets. For this, the Pohlhausen model was used in conjunction with the CEVA wall jet model with a semi-empirical modification which increases the accuracy for very thin jets. The method is therefore a mix of analytical equations with curve fitted experimental data in order to produce a simple yet effective way of estimating the boundary layer velocity profile along the curved wall. In order to cross-check the results, Newman's empirical equation – which only provides a separation location but no information regarding the velocity profile - for boundary layer separation was used with good results. The hereby model could be used as a pre-design tool for rapid assessment of aeronautical high-lift applications such as Upper Surface Blown (USB) or entrainment wings.*

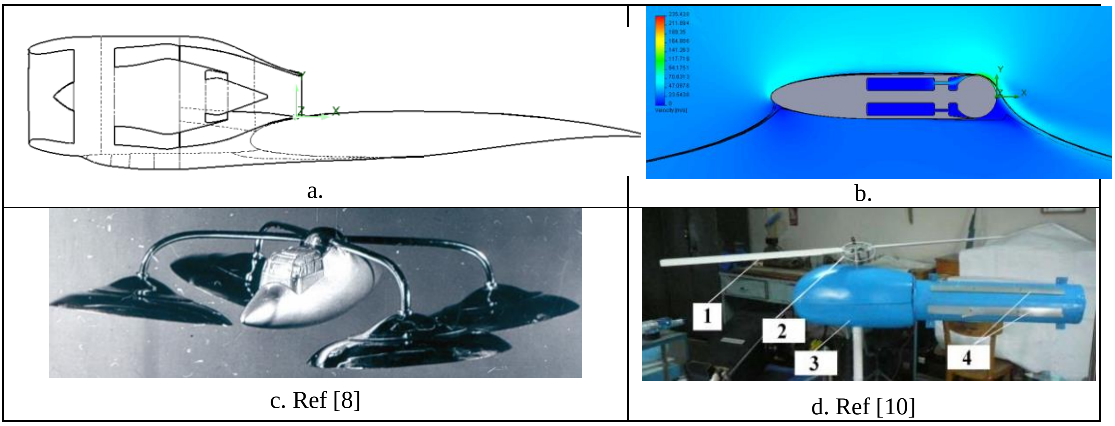
Key Words: *Coandă effect, boundary layer, Pohlhausen method, wall jet, experimental equation*

NOMENCLATURE

h – initial jet thickness
R – ramp radius
u_j – initial jet velocity
u_m – maximal local jet velocity
$y_{1/2}$ – wall distance at which the local velocity equals half of local maximum velocity
θ_{sep} – separation angle [degrees]
Λ – Pohlhausen coefficient

1. INTRODUCTION

Applications of curved wall jets – subject to the Coandă effect – have long been implemented in the aeronautical industry in obtaining higher lift – such as the USB Refs [1-3] Fig.1 a. or entrainment Refs [4-7] Fig.1 b. wings, or replacing the wing altogether Refs [8,9] Fig.1 c; other applications refer to replacing stability control devices such as the helicopter tail rotor with a Coandă-effect curved wall jet on the rotorcraft tail boom Ref [10-11] Fig. 1 d.



a.

b.

c. Ref [8]

d. Ref [10]

Fig. 1 - Applications for the Coandă effect lift a. USB wing, b. Entrainment wing,
c. the original Lenticulair Aerodine Ref [8], d. Coandă effect helicopter tail rotor Ref [10].

Although analytical models were developed for estimating the flow fields over curved surfaces Ref [12], or dedicated turbulence models which compensate for wall curvature Ref [13] or [14], the semi-empirical models such as the ones described by Lewinsky and Yeh [15] or Seed [16] still offer the advantage of being experimentally validated in their design range.

Reference [17] presents, amongst other calculations, a method for boundary layer detachment prediction in the case of curved wall jets subject to the Coandă effect. The paper approaches the matter by introducing the model proposed by Rodman, Wood and Roberts (RWR) Ref [18] into the Pohlhausen boundary layer equation [19]. Fundamentally there are two limitations to the method regarding the estimation of the detachment point. Firstly, the RWR model is validated only for very thin wall jets with a height to radius (h/R) ratio lower than 3-4%. An extended (h/R<10%), semi-empirical, model (CEVA) is described in Ref [20]. The second limitation is the Pohlhausen boundary layer equation itself, which has some difficulties in estimating the laminar boundary layer separation, improved models being presented in Refs [21] and [22].

It is the purpose of this paper to integrate the CEVA model into the Pohlhausen boundary layer equation with an added correction which is based on the reinterpretation of the experimental data from Wygnanski Ref [23].

The detachment point is then calculated numerically and cross-checked with the empirical equation of Newman Refs [24], [25]. Newman’s equation is regarded to be accurate up to a 10% height to Radius ratio and therefore it is suited for the validation of the current method. However, the formula does not provide a way to calculate the radial velocity profile along the ramp.

2. INITIAL CALCULATIONS AND COMPARISONS

For a simple curved wall jet – over a circular cross-section ramp Fig. 2 – in a quiescent ambient, Newman provides an empirical equation for estimating the detachment point of the flow as a function of the height to radius ratio Eq (1).

$$\theta_{sep} = 245 - 391\frac{h/R}{1+1{,}125h/R} \tag{1}$$

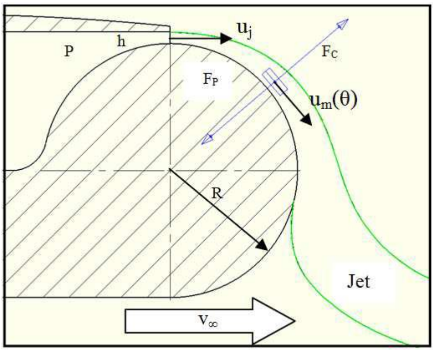


Fig. 2 – Example of curved wall jet over a circular ramp

$$\frac{u}{u_m}=\left(2+\frac{\Lambda}{6}\right)\frac{y}{y_m}+\frac{\Lambda}{2}\left(\frac{y}{y_m}\right)^2+\left(\frac{\Lambda}{6}-2\right)\left(\frac{y}{y_m}\right)^3+\left(1-\frac{\Lambda}{6}\right)\left(\frac{y}{y_m}\right)^4 \tag{2}$$

$$\Lambda=\frac{y_m^2}{\nu}\frac{du_m}{dC} \tag{3}$$

After the numerical implementation (the derivative dum/dC is solved numerically) of the CEVA model into the Pohlhausen boundary layer equation, the separation point was calculated for various h/R ratios using the criteria of Λ=12.

The results are plotted against the h/R ratio for comparison with the empirical Eq. (1) in the chart presented in Fig. 3.

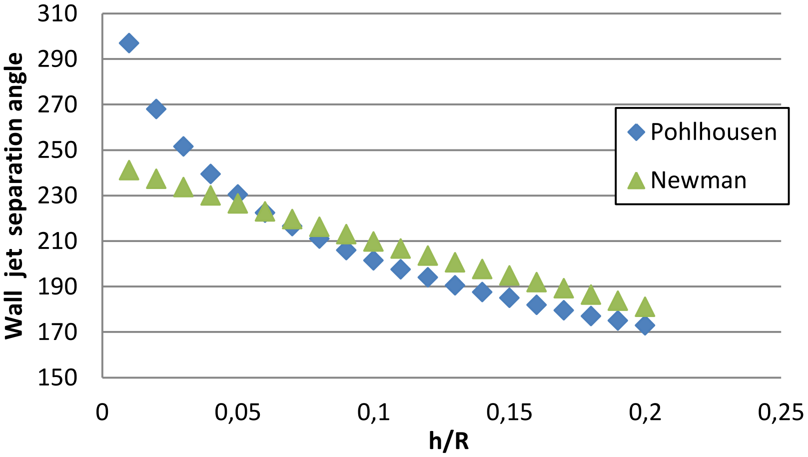


Fig. 3 – The comparison between the separation points calculated with the CEVA-Pohlhausen model versus Newman’s empirical equation

It is apparent that for h/R ratios higher than 5%, the two methods provide similar results, within a margin of 3° to 4°. On the other hand, for thinner jets - with an h/R ratio lower than 5% - the CEVA-Pohlhausen method clearly overestimates the separation angle.

As stated before, the literature offers more precise boundary layer models however, the nature of the CEVA model equations may allow for a different approach. By introducing additional experimental data regarding the boundary layer of curved wall jets, it may be possible to correct the overestimation in the <5% ratios.

3. THE CORRECTED CEVA-POHLHAUSEN MODEL

In this approach we maintain the original Pohlhausen laminar boundary layer equation (2), (3) while modifying the original formulations of the CEVA model with a new equation based on Wygnanski’s experimental data provided in Ref [23].

As shown there, the thickness ratio between the boundary layer, y_m, and the reference wall distance, $y_{1/2}$ (the distance for which the local velocity is half of the local maxima), has a steep increase for angular positions above 180°.

This can be explained by the transition of the jet from laminar to turbulent. Therefore, it must be pointed out that the self-similarity assumption becomes obsolete for non-laminar flows but, for the purpose of calculating an estimate for the separation point, the method might still be useful.

By using iterative non-linear regression methods Refs [26, 27], analytical equations have been deduced to fit the experimental curve.

Because, in this case, the extrapolation of the experimental data is important (i.e. for thinner jets, which have separation points at more than 180°) two equations were proposed.

Both of the two equations (4) and (5) provide good fitting for the existing experimental data but have almost diametrically opposed behaviour when extrapolated.

As shown in Fig. 4, Eq. (4) tends to continue its increase beyond the 180° threshold whereas Eq. (5) is more conservative, levelling off just above 180°.

$$y_m = y_{\frac{1}{2}}\left[0{,}1371 - 0{,}00119\frac{C}{h} + 4{,}35\cdot 10^{-5}\left(\frac{C}{h}\right)^2 - 5{,}1\cdot 10^{-7}\left(\frac{C}{h}\right)^3 + 1{,}85\cdot 10^{-9}\left(\frac{C}{h}\right)^4\right] \tag{4}$$

$$y_m = y_{\frac{1}{2}}\left[0{,}1314 + \frac{0{,}33}{1 + 10^{4{,}9\cdot 10^{-2}\cdot\left(177{,}3 - \frac{C}{h}\right)}}\right] \tag{5}$$

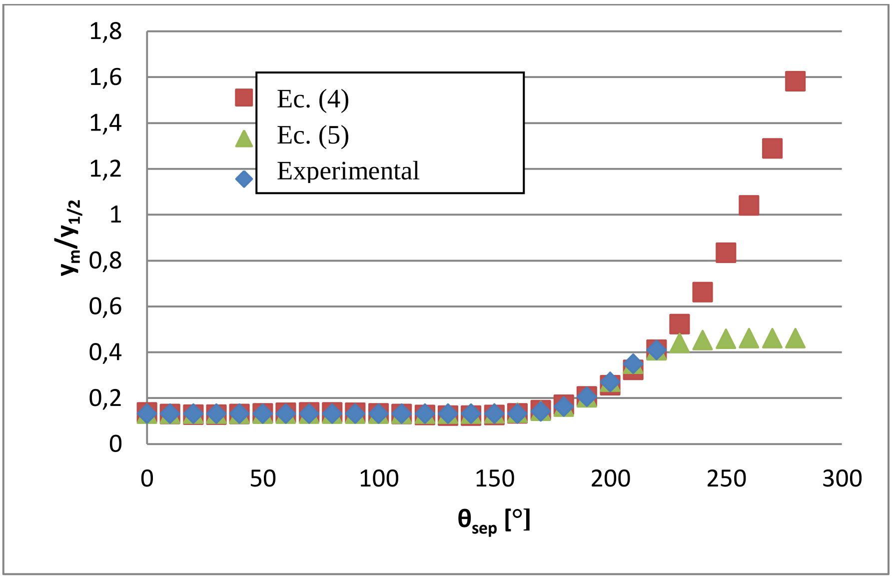


Fig. 4 – The correlation of the experimental data with the proposed analytical equations

Equation 5 has been used for a new estimation of the separation point with the CEVA-Pohlhausen model.

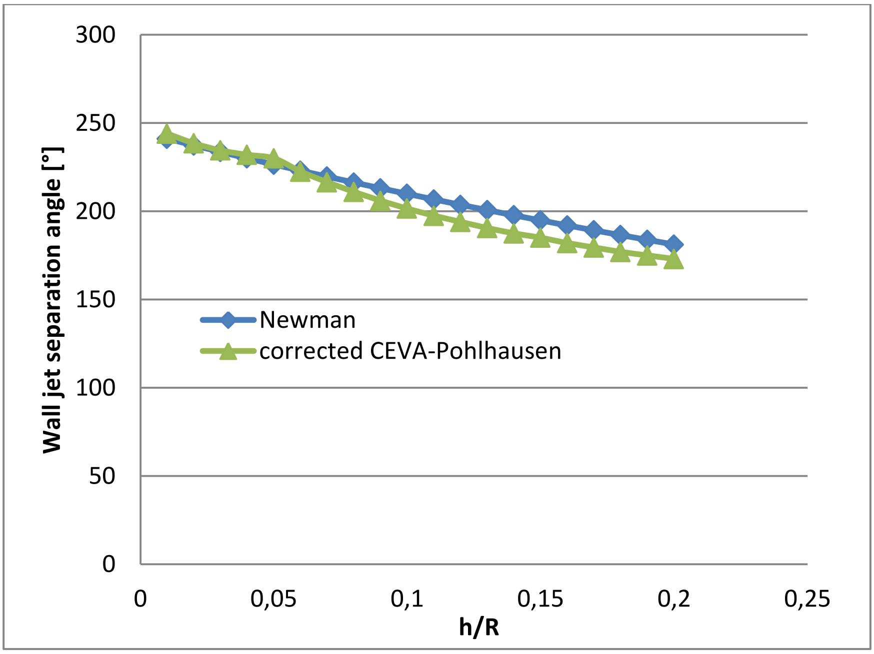


Fig. 5 – The separation points calculated trough the corrected CEVA-Pohlhausen method vs Newman's equation

4. CONCLUSION

The current study investigates the possibility of integrating a corrected semi-empirical curved wall jet model with the Pohlhausen boundary layer method with the intent of obtaining a more accurate estimation of flow separation.

The paper is dwelling on the previous work provided in Ref [20] which proposes a semi-empirical Coandă effect model and that presented in Ref [Pohl] in which the Pohlhausen boundary layer is applied to the Rodman et al. formalism. An initial test is made with the baseline CEVA model which was implemented numerically in Eq.(2) and (3), the results were then compared with the empirical equation for the separation point fitted by Newman Refs [24, 25].

The results displayed a good correlation for jets thicker than 5%, however the baseline CEVA-Pohlhausen model overestimated the detachment point for thinner jets – which tend to stay attached longer than thicker jets. This is the effect of flow transition from the laminar region, near the ejector slot, to the turbulent region located at very high angular locations (above 180°).

Although it has been pointed out that the self-similar velocity profile cannot be rigorously considered, the formulas can be adjusted so that they will correlate better with the reference considered i.e. Newman's equation.

Therefore, based on the experimental research provided by Wignansky in Ref [23], two correction factors were developed and then combined in order to re-iterate the comparison. The proposed corrected model achieved improved correlation with the Newman equation on the entire h/R range tested.

Further improvements on the proposed model may be achieved by implementing the more elaborate boundary layer polynomial model described in Refs [21, 22].

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