The Structured Wall-Turbulence, a Galilean Relativistic Phenomenon

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Abstract: *The relationship between heavenly bodies and earthly behavior along with its importance took many centuries before the rigor scientific understanding enabled the true influences on Earth, such as its complicated motion and perceived other regularities in the behavior of earthly objects. One of these was the tendency for all things in one vicinity to move in the same downward direction according to the influence that is known as gravity property. Moreover, matter was observed to transform, sometimes, from one form into another, such as with melting of ice or vaporizing/cavitation of water, but the total quantity of that matter never seemed to change, which reflects the law at which we now refer to as the conservation/ integrity of mass, including its latent energy. Much latter it is noticed that planet Earth forms a self-regulating complex system, i.e. the Earth's surface is alive, that is known as the Gaia hypothesis, reflected in the Newton-Galilei dynamics through the law of equal action and reaction for stress vector and tensor, respectively. In addition, at was noticed that there are many material bodies with the important property that they retain their shapes, excepting the flowing fluids, whence the idea of rigid spatial motion arose, and it becomes possible to understood spatial relationships in terms a precise, well-defined geometry, the Euclidian three-dimensional geometry. Though the heavenly bodies are permanently moving in a self-built on universe like a timeless perpetuum mobile, the time remains an important property for the behaviors/motions of an Earth-bound object due to their relativity as against the diurnal rotation depending on the velocities of the impacted object. In contrast to the constant inertia condition where for small starting velocities and accelerations the Newton's determinist principle is applied, the onset of a motion of the Earth-bound material bodies, at higher velocities and accelerations (O(g)), involves changes of moving matter/inertia under influence of gravitational field via some intrinsic latent motions/processes. They achieve the kinetic-gravitational mutual energy transfer obeying the Galilei's law of inertia for self-equilibrating impact forces. The intrinsic motions, at the cellular scale (10^{-6} m), are responsible for the kinetic trinity of the momentum, kinetic energy and power, and they represent what it is called structured turbulence, i.e. a Galilean space-time structure according to the mathematical idea of a bundle (or fibre bundle) and its gauge connection. The bundle and gauge connection are a kind of Galilean transformation to a system moving with constant velocity carrying its relativistic non-inertial fraction as a blend of structure less turbulence and non-rigorously defined intermittency of a non-inertial motion.*

Key Words: *Galilean space-time structures, laminar-turbulent transition, contact hydrodynamic structures, structured turbulence*

1. INTRODUCTION

The starting impact/collision is a process of momentum exchange between two colliding bodies within a short time of contact. With respect to single impacted body or structure, the loading in such a process acts with high intensity during this short period of time. As a result, the initial momentum rapidly changed in the contacting area where a contact force or contact/twist pressure in fluids arouses, and concomitantly the Earth bound reaction engenders gravitational-shear waves that propagate with finite speeds through the whole body, in the form of kinetic trinity of momentum, kinetic energy and power (the change of rate in the kinetic energy). The starting process of fluid flows, triggered off at the onset of movement, is an intrinsic latent process of Gaian self-regulation type.

It is of importance to make from the outset a clear distinction between the common longitudinal compression process of elastic fluids, in both gaseous and liquid steady flows under the action of their dynamic head $q = 1/2\rho V^2$, and the lateral impact/contact compression at the onset of movement. At the starting the all wall-bounded flows are subjected to considerable compressibility effects due to twist/torsion of fluid at fluid-solid contact, even for small velocities less the incompressibility limit considered for gaseous flows ($M < 1/3$). These contact twisting/torsion stresses and their post-impact latent evolution was up to now taken into consideration only through a frictional shearing stress, known as Newton's law of skin friction [1]. However, the outcome of twisting/torsion compressibility is a complicated local deformation of the boundary vorticity where from its impact excitation spring up microscopic ordered structures of molecular thermal nature along with an active inertia self-adjusting to the wall-guided flow, i.e. the fluid becomes a thixotropic one [2]. The final macroscopic flow field does not explicitly depend on the intrinsic variable of microscopic states, herein molecular inertia.

The phenomenon due to the complicated deformation of vortices by mutual straining is of statistical nature and the macroscopic kinetic states are differentiated rather qualitatively through visual images as laminar, transitional, or less-understood disordered "turbulent" flows.

The idea of the intrinsic microscopic states is not new, and there have been several attempts to build up some statistical hydrodynamics, beginning with the pioneering work of Onsager [3]. The various attempts to approximate the continuous Euler system by methods of statistical mechanics to fluid dynamics such as finite point-vortex models [4], [5], Fourier decomposition of vorticity models [6], [7] and Robert-Sommeria's models of statistical equilibrium states [8], [9], have showed that the finite-dimensional approximations of Euler's equations can provide a good representation of the flow during a finite time. However, the information that the thermodynamics (long-time dynamics involving turbulence) of such systems gives on the behavior of the full system is highly questionable, and the mentioned models cannot yield reference formulae that permit quantitative comparison to check accuracy and correctness of experiments and numerical simulations.

To introduce our approach, it is important to state the mathematical justification of the present impact force model based on the concept of fibre, bundle with its gauge connection that can calculate contact stress and integrate the equations of intrinsic motion during the impact, so-called structured turbulence, governed by the wall-kinetic trinity of momentum, kinetic energy and power. The kinetic trinity replaces Prandtl's friction law for turbulent shear flows.

2. FIBRE BUNDLES AND GAUGE CONNECTIONS

The frictional shearing fluid flows at large Reynolds numbers involve the twist deformation of mutual straining vortices along with particle interactions, depending on high contact accelerations of the order O(*g*) produced at the beginning of a moving process.
The Earth-bound relativity describing such as contact physical forces along with the twist effects of geometry change in the presence of matter experienced by high accelerations, can be approached based on a rather complicated mathematical theory, on contact geometry [10]. Instead of regarding the smallest internal dimensions and their time-evolution, the particle interactions are more appropriate to think of them in bulk as what is called a fibre bundle (or simply a bundle) over a structure/entity space-time where each of structure space-time is another space, called a fibre, consisting of all the internal dimensions, according to the considered physical interaction.
Then the bundle concept must necessarily be tied to the physical interpretation of a particular interest. The gauge connection (or covariant derivative operator, or simply connection) refers to a parallel transport of certain physical quantities (vectors and tensors) localized at a point different of the common local tangent space at the point of a manifold.

The mathematical concept of a bundle. A *bundle* $\mathcal{B}$ is a manifold with some structure, which is defined in terms of two other manifolds $\mathcal{M}$ and $\mathcal{V}$, where $\mathcal{M}$ is called the base space (which for a physical application is space-time itself/intrinsic), and where $\mathcal{V}$ is called the fibre (the internal space of the physical application). The bundle $\mathcal{B}$ itself is thought, as being completely made up of a whole family of fibres $\mathcal{V}$, i.e. it is constituted as an $\mathcal{M}$'s worth of $\mathcal{V}_S$, Fig. 1.

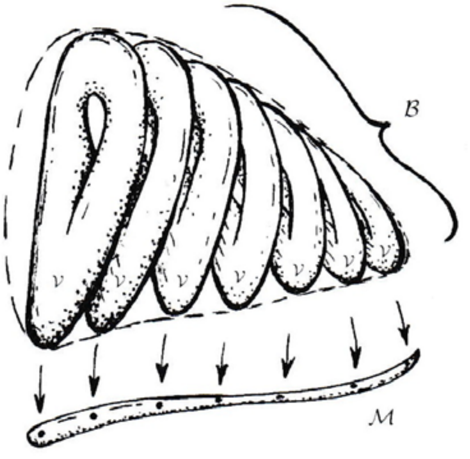


Figure 1. – A bundle $\mathcal{B}$ with base space $\mathcal{M}$ and fibre $\mathcal{V}$ as a '$\mathcal{M}$'s worth of '$\mathcal{V}$'s; the canonical projection from $\mathcal{B}$ down to $\mathcal{M}$ is viewed as the collapsing of each fibre $\mathcal{V}$ down to a single point

The simplest kind of bundle is what is called a *product space*. This can be a *trivial or untwisted bundle* of $\mathcal{V}$ over $\mathcal{M}$, $\mathcal{M}\times\mathcal{V}$ (pairs of elements (*a*, *b*) with $a\in\mathcal{M}$ and $b\in\mathcal{V}$, Fig. 2a) and the twisted bundles $\mathcal{B}$ over $\mathcal{M}$, with fibre $\mathcal{V}$, locally resembling $\mathcal{M}\times\mathcal{V}$, i.e. the part of $\mathcal{B}$ over any sufficiently small open region of $\mathcal{M}$ is identical in structure with that part of $\mathcal{M}\times\mathcal{V}$, lying over of same open region of $\mathcal{M}$, Fig 2b. The space $\mathcal{V}$ also has some symmetries that give freedom for the twisting and making the bundle concept appropriate for turbulence approaching.

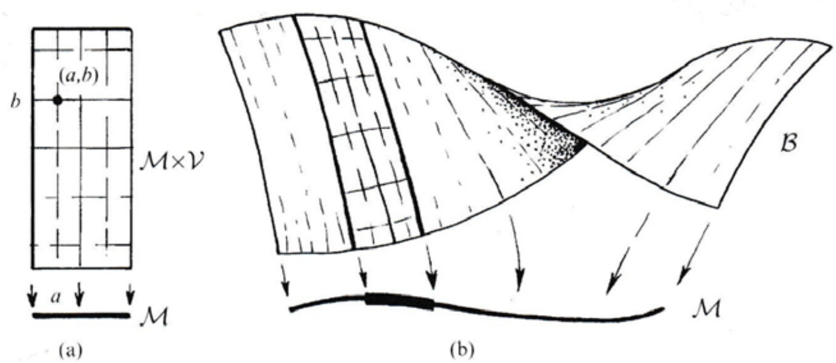


Figure 2. – a) The particular case of a “trivial” bundle, as the product space $\mathcal{M}\times\mathcal{V}$ of $\mathcal{M}$ with $\mathcal{V}$,

b) The general “twisted” bundle $\mathcal{B}$ over $\mathcal{M}$, with fibre $\mathcal{V}$, locally resembling $\mathcal{M}\times\mathcal{V}$

Covariant derivative operator (or connection) simply is the reply of a twisted bundle produced by a parallel transport definite mathematically by the notion of differentiation of vector or tensor fields. Physically, this is a kind of reaction to the parallel-transport conception. The essential idea is to compare the way in which a vector or tensor field actually behaves in some direction away from a point *p* with the parallel-transport of the same vector taken in that same direction *p*, subtracting the latter from the former i.e. relative to the former. The covariant derivative (operator $\boldsymbol{\nabla}$) connected to the concept of parallel transport of a ξ along a path γ represents the variation of a vector field ξ on $\mathcal{M}$ ($\rightarrow$) measured by its departure from that standard provided by parallel transport ($\longrightarrow$), depending only upon the tangent vector **w** of γ at *p* ($\Rightarrow$), Fig. 3.

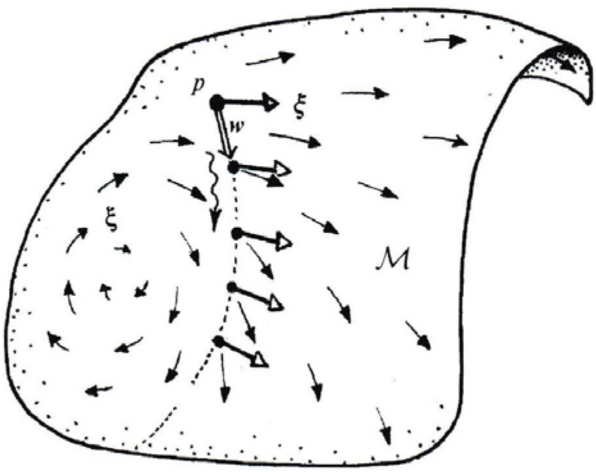


Figure 3. – The notion of covariant derivative understood in relation to parallel transport

A fundamental requirement of such an operator is its linear dependence on the vector **w**, defined by the displacement (direction) of **w** by two displacement vectors **w** and **u**,

$$\underset{\mathbf{w}+\mathbf{u}}{\nabla} = \underset{\mathbf{w}}{\nabla} + \underset{\mathbf{u}}{\nabla}, \tag{1}$$

and a scalar multiplier λ:

$$\underset{\lambda\mathbf{w}}{\nabla} = \lambda\underset{\mathbf{w}}{\nabla} \text{ or } (\lambda\mathbf{w}^a)\underset{a}{\nabla} = \lambda\left(\mathbf{w}^a\underset{a}{\nabla}\right), \quad (2)$$

with Einstein summation convention,
The action on vector fields, $\boldsymbol{\nabla}$ satisfies the kind of rules that the differential d (exterior derivative) satisfies

$$\nabla(\xi + \eta) = \nabla\xi + \nabla\eta, \quad (3)$$

and the Leibniz law

$$\nabla(\lambda\xi) = \lambda\nabla\xi + \xi\nabla\lambda, \quad (4)$$

where ξ and η are vector fields and λ is a scalar field. A particular connection is the action of $\boldsymbol{\nabla}$ on a scalar Φ that is identical with the action of the gradient d on that scalar

$$\nabla\Phi = \mathrm{d}\Phi, \quad (5)$$

The extension of $\boldsymbol{\nabla}$ to a general tensor field is uniquely determined by the following two natural requirements: the Leibniz like additively (for equivalent tensors **T** and **U**)

$$\nabla(\mathbf{T} + \mathbf{U}) = \nabla\mathbf{T} + \nabla\mathbf{U}, \quad (6)$$

and a little modified form of Leibniz law (for non-equivalent tensors **T** and **U**)

$$\nabla(\mathbf{T} \cdot \mathbf{U}) = (\nabla\mathbf{T}) \cdot \mathbf{U} + \mathbf{T} \cdot (\nabla\mathbf{U}), \quad (7)$$

where dot indicates some form of contracted product of the set of upper and lower indices of **T** and **U** (for its vacuous indices with no contraction at all). All these properties can be formulated as the coordinate derivative operator $\boldsymbol{\partial/\partial x^a}$ in place of $\boldsymbol{\nabla_a}$, allowing defining in any one coordinate small open region of $\mathcal{M}$ (different from zero), a particular connection in that region, called *the coordinate connection*. The coordinate connection is a special kind of connection that defines the concept of parallel transport independent of the path, in the form of the switch of coordinate derivative operators (symmetry in its indices *a*, *b*)

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \text{ or } \frac{\partial^2}{\partial x^a \partial x^b} = \frac{\partial^2}{\partial x^b \partial x^a},$$

For a general connection $\boldsymbol{\nabla}$, the symmetry property does not hold for $\boldsymbol{\nabla_a \nabla_b}$, its anti-symmetric part $\boldsymbol{\nabla}_{[\boldsymbol{a}}\boldsymbol{\nabla}_{\boldsymbol{b}]}$ given rise to two special tensors, one of valence $\begin{bmatrix}\mathbf{1}\\\mathbf{2}\end{bmatrix}$ called the *twist/torsion* tensor $\boldsymbol{\tau}$ and the other of valence $\begin{bmatrix}\mathbf{1}\\\mathbf{3}\end{bmatrix}$ called the *curvature* tensor **R**. Torsion is present when the action of $\boldsymbol{\nabla}_{[\boldsymbol{a}}\boldsymbol{\nabla}_{\boldsymbol{b}]}$ on a scalar quantity fails to vanish.
In most physical theories, $\boldsymbol{\nabla}$ is taken to be *torsion-free*, i.e. $\boldsymbol{\tau} = 0$**,** and this certainly makes their approach easier, but less accurate. The long-lasting and continuing paradigmatic crisis in the basic research of turbulence is the result of the non-understanding of the starting process when the torsion engendering gravitational-shear waves is present in all wall-bounded flows. Curvature is the essential quantity that expresses the path dependence on the connection at the local scale. In the case of a parallel-transport defined by $\boldsymbol{\nabla}$, **R,** the curvature measures how much that vector has changed when it returns to the starting point.
Gauge connection is a generalization of the coordinate connection in the case of the parallel-transport of the specific quantities for a particular concrete physical interest. The gauge connections give the near local twist effect, for certain physical fields under the action of

gravity like forces along with their realm of a straight propagation lines for inertial particle motions. More on the contact geometry and fibre bundles you can find in [11].

3. SPACE-TIME AND DIMENSIONAL SPREADING

The space-time relativity refers to the dynamical framework of the physical motions observed from an Earth-bound locations and its perception depending on the inertia, mass (*m*) or density (ρ) and starting accelerations (a_s) of material bodies/particles. For both solid and fluid bodies, the starting of any motion involves relativistic aspects connected principally to matter/inertia changes and return/reactive force, which have opposite tendencies producing an intrinsic pre-compressing torsion stress of matter, that long-time was ignored in the successive development of motion after start. The pre-tensed state of matter is the outcome of the less or more intense impact/contact of colliding bodies, herein a fluid-solid surface, and the matter/inertia changes from the Newtonian inertial state (common sized up bodies) to a non-inertial one for molecular and cellular motions including apparent structure less turbulence. Both non-inertial contact motions of molecular and/or cellular process type, at alive cell scale ($<10^{-6}$ m) are caused by the ability of atoms and/or molecules in a microstructure to occupy two different energy states, known as Schottky effect [12]; i.e. at any time the fluid dynamics have to include both kinetic and gravitational/potential effects. At such as dimensional scales the gravitational field activated by incoming kinetic energy engenders an *active gravitational mass*, different from the Newtonian passive mass, transported by means of gravitational waves under the form of a trinity of momentum, kinetic energy and power.

The dual behavior of molecular and cellular processes reflects a non-known Gaian property, of gravitational field that for given size and excitation conditions it is be able to distinguish between the Newtonian inertial mass and an active/heavy non-inertial mass.

Space-time relativity. In the sequel, the non-inertial/inertial mass proportion is analyzed in according to the twisted bundle concept for three Earth-bound dynamical schemes developed in a long course of time [12]

(*A*) Aristotelian space-time relativity (temporal simultaneity) is simply the product

$$A = E^1 \times E^3, \tag{8}$$

of the pairs (t, $\mathbf{x}$), where t ranges over Euclidian 1-space E^1 and X range over Euclidian 3-space E^3 (uncertain location of an event), Fig. 4. This instantaneous time portal describes the thermodynamic states for quantum values of $\boldsymbol{a_s/g - 1 < 1/3}$, and size < 10^{-7} m.

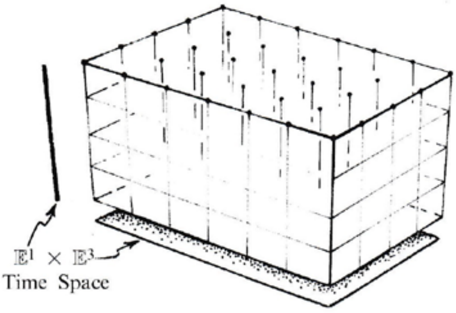


Figure 4. – Aristotelian space-time $A = E^1 \text{ X } E^3$ as the space of pairs (t, $\mathbf{x}$), where t (“time”) ranges over a Euclidian 1-space E^1 and $\mathbf{x}$ (“point in space”) ranges over a Euclidian 3-space E^3

(*G*) Galilean space-time relativity is a fibre bundle with base space E^1 and fibre E^3 with the canonical projection from *G* to *E* (absolute time), Fig. 5. Its main features are:

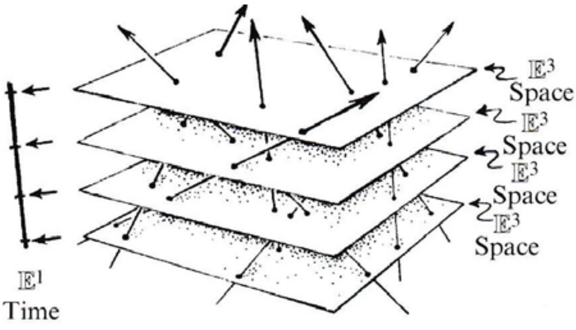


Figure 5. – Galilean space-time *G* as a fibre bundle with base space E^1 and fibre E^3, without no point-wise identification between different E^3 fibres (no absolute space), each space-time event being assigned a time via the canonical projection (absolute time)

a) in a fibre bundle there is no point wise identification between one fibre and the next (the fibre forgets its origin);

b) the fibres fit together to form a connected whole, i.e. each space-time event is naturally assigned a time, as a particular element of one specific "clock space" E^1 without a spatial location in one specific location space E^3, and the natural assignment of a time is achieved by the canonical projection from *G* to E^1;

c) the particle histories are cross-sections of the bundle $\mathcal{B}$ (intermittency of a non-inertial motion) and the inertial particle motions is depicted as *G*'s specific structures (dynamic equilibrium lines for Galilei's law of inertia).
The absolute/ Gaian time portal describes the motions of cellular structures for quantum values of $\boldsymbol{a_s/g-1>2/3}$ and size > 10^{-7} m.

(*N*) Newtonian space- time relativity (absolute space) is a fibre bundle with base space E^1 and fibre E^3, just as was in the case for the previous Galilean space-time *G*, but containing some structures on *N* different from that of *G*, representing the family of "straight" equilibrium lines of different inertial motions, Fig. 6 (a, b, c):

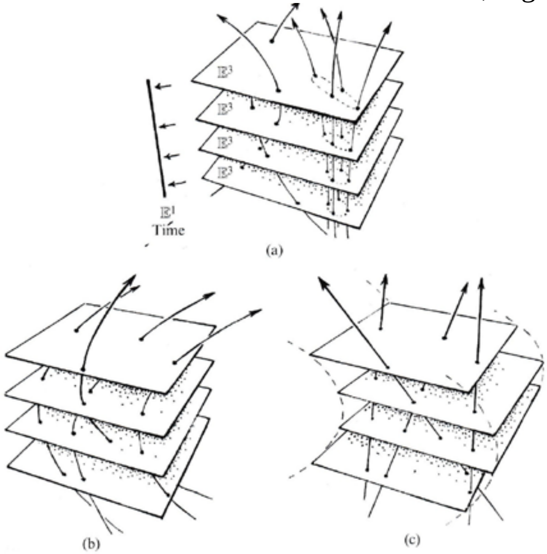


Figure 6. – (a) Newton-Cartan space-time *NC* like the particular Galilean case *G*, as a bundle with base-space E^1 and fibre E^3; (b) The special case of a Newtonian gravitational field *N* (constant over all space); (c) the structure Newton-Galilei *NG* completely equivalent to that of *G*, as seen by "sliding the E^3 fibres horizontally until the world lines of free fall are all straight

a) Special Newton-Cartan case represents rolling friction contact motions, including the wall-bounded turbulent flows and jet-like flows, for $\mathbf{1/3} \leq (\boldsymbol{a_s/g} - \mathbf{1}) \leq \mathbf{2/3}$ and size > 10^{-7} m;
b) Classical Newton case represents shearing friction contact motions, including laminar flows for $(\boldsymbol{a_s/g} - \mathbf{1}) \leq \mathbf{1/2}$ and size > 10^{-6} m;
c) The Newton-Galilei case represents the special contact motions of full/random turbulent flow along with structured all turbulence in pipe-like flows, for $(\boldsymbol{a_s/g} - \mathbf{1}) > \mathbf{2/3}$ and size < 10^{-7} m.

From the above dynamical representations of contact/twisted motions, where their starting is determinant for the post-impact evolution, it can define an intermittency factor measured as the heavy to inert mass ratio, $\boldsymbol{I_r} = \frac{\boldsymbol{m_h}}{\boldsymbol{m}} = \mathbf{2} - \frac{\boldsymbol{a_s}}{\boldsymbol{g}}$, of gravitational nature.

Dimensional spreading. The wall-bounded/contact local fluid flows during the starting impact, are subjected to the strain action of both elastic longitudinal (*E*) and twist lateral (*G*) compression, which obeys the generalized Hooke's law, defined in terms of the Lame constants by Poisson ratio

$$\nu_P \equiv \frac{E}{2(E+G)} = \frac{1}{2\left(1+\frac{G}{E}\right)}, \tag{9}$$

For $\left(\frac{\boldsymbol{G}}{\boldsymbol{E}} = \frac{\mathbf{2}}{\mathbf{3}}\right), \boldsymbol{\nu_P} = \mathbf{0.3}$ is an isotropic state of gaseous flows, relaxing after start, Fig. 6c.

In contrast to the external dimensions of twist-free motions, corresponding to the situation for rectangular Euclidian geometry holding the triangle inequality, the twisted contact motions of wavy nature are in the corresponding situation for the "floppy" Lorentzian geometry "smoothing" the corners of triangle. The continuously dimensional change from 1d-3d dimensions suitable for a wavy motion, is achieved by means of dimensional continued fractions and eigen-solutions of some linear second order differential equations along straight equilibrium lines.

4. CONTACT RELATIVISTIC HYDRODYNAMICS AND ITS GAUGE CONNECTIONS

The all relative motions as against the diurnal rotation are the result a more or less intense collision or shock called starting impact, of its intrinsic contact effect on the motion of fluids was ignored or less understood. Our derivation of equations referring to contact hydrodynamics is based on four basic principles, which we shall treat in turn:

1. Mass is neither created nor destroyed;
2. The rate of change of momentum of a portion of the fluid equals the force applied to it, Newton's second law $(\boldsymbol{d}\mathbf{p}/\boldsymbol{dt} = \mathbf{F})$;
3. Energy is neither created nor destroyed;
4. Impact work/power is asymptotically conserved through momentum, kinetic energy and reactive power carried on by gravitational waves, Gaia self-regulating hypothesis.

The Earth-bound matter, including fluids, due to its molecular thermal structure has a dual behavior at any change of matter state (Schottky effect [13]), so that the starting impact produces a change of molecular inertia like the inertia of a moving wheal. The change of molecular structures generates a geometric orientability and energy mutation under the mutual action of kinetic and gravitational fields, phenomena of a statistical nature.

Redistributing mass/inertia process after an impact. The key of microscopic ordered structures of a contorted fluid when it is impacted at a wall, is the relationship between the starting acceleration (measured in quanta of $\mathbf{g}$) and the twist state of impacted fluid (measured by $\left.\frac{dU}{dy}\right|_{y=0}$) as against the wave number ($\boldsymbol{\lambda} = \boldsymbol{n}\frac{\pi}{2}$, n – integer) of a beta-β decay process whereby a concentrated mass disintegrates spontaneously with the emission of β - heavy/gravitational mass (β*gm*) and inert mass (*m*). The β-decay process describes the mutual interaction between the instantaneously twisted kinetic field and the gravitational field. The route of inertia from the starting impact to its end (the isotropic residual inertia – “ashes” of the continuum) is a statistic process modelled by Wallis’ integrals (the relationship between Beta and Gamma functions), Fig. 7.

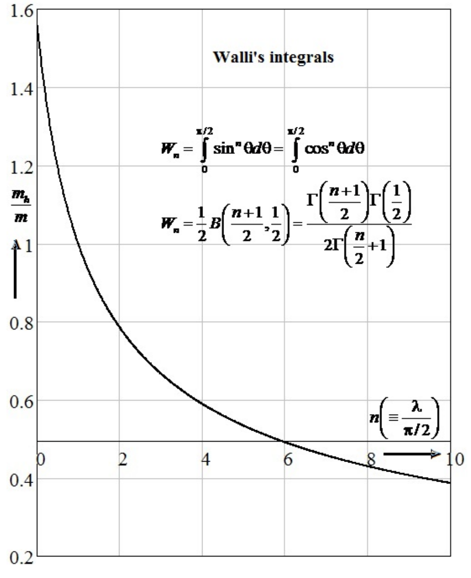


Figure 7. – The β-decay process of gravitational/heavy mass after a starting shock (independent of the total mass)

The behaviour of matter at a shock is a twist/torsion one, connected to the complicated contact geometry given by the Lie-Poisson bracket [14, 15]; briefly it is a matter of the Poisson ratio/coefficient of matter reflecting its dual behaviour when it is strained as an elastic ($\boldsymbol{\nu_P} = \frac{1}{2}$) one and torsion/twist ($\boldsymbol{\nu_P} = \frac{1}{3}$) another; the matter energy of a molecular thermal nature is a genuinely inertial kinetic energy embodying the gravitational contact wave energy.

The gravitation (g), the angular velocity (ω_E =7.27 10^{-5} rad/s), as well as the Poisson ratio (ν_P) connecting intrinsic symmetric-antisymmetric motions, are timeless geophysical phenomena caused by the Eigen rotation of the Earth.

Intrinsic contact equations. The underlying philosophy is that energy and momentum are nothing else than functions of mass/inertia and velocity, that under suitable conditions, happen to be conserved. Our starting point is the definition of kinetic energy for a particle, as a scalar quantity whose change equals the intrinsic work done on the particle after starting. Mathematically, one first defines the impact power (work per unit time)

$$W_S \equiv \mathbf{F}_S \cdot \mathbf{u}, \tag{10}$$

where $\mathbf{F_S} \equiv \boldsymbol{m\beta}\mathbf{g} \equiv \boldsymbol{m_h}\mathbf{g}$, is the total impact/starting force acting on the particle, $\boldsymbol{m_h} = \boldsymbol{m\beta}$ is an active or gravitational mass, different of constant Newtonian inertia, and $\mathbf{u}$ is the impact velocity.

Then, using Newton’s second law which holds both in the non-relativistic and the relativistic regimes, one gets also

$$W = \frac{d\mathbf{p}}{dt} \cdot \mathbf{u}. \tag{11}$$

Finally, one defines the kinetic energy as a function $E_k(m_h, \mathbf{u})$ such that

$$\frac{dE_k}{dt} \equiv W, \tag{12}$$

thus from (11), (12) obtaining

$$dE_k = \mathbf{u} \cdot d\mathbf{p}, \tag{13}$$

$$\mathbf{u}(\mathbf{u} \cdot d\mathbf{p} - dE_k) = 0. \tag{14}$$

Equations (13, 14) constitute a contact fundamental formulation (for an interacting particle system) between reactive power, kinetic energy and momentum, that stands up on its own, independently of Eqs. (11, 12) and a reference frame. It expresses the simultaneous conservation of energy and momentum for different flow classes/groups, depending on the starting power, W_s. Since $W_s \approx M$ (Eq. 10), then for $M < \ln 2$ (half-life of sonic impact W_s (M = 1)) only the Eq. 13 is requested to kinetic energy – momentum conservation, while above $M \geq 0.7$ both Eqs. (13, 14) are requested for the balance of momentum, kinetic energy and power, so called the kinetic trinity of a relative motion carried by the gravitational – shear/contact waves.

Mathematically, the flow classes correspond to Lie group structure [15], each having own the theory of differential equations: vortex models/technology, (the Kelvin circulation theorem), momentum models (the Navier-Stokes equations, NSE) and impact force models (the self-sustained structured turbulence by reactive power).

Balance of impact power (twist effect). The energy-momentum tensor in empty space is zero, so the gravitational wave energy has to be measured in some other way that is not locally attributable to an energy density; gravitational energy is a genuinely non-local quantity. This does not imply that there is no mathematical description of gravitational energy, however. The relative motions of matter, herein fluids, as against the timeless rotation of the Earth, produce perturbations of gravitational energy gendering local gravitational-shear/contact waves through which the matter can be transported in a form preserving the kinetic trinity (momentum, kinetic energy and power) in whole matter, without its disintegration. In the case of fluids, the impact power is the product ($p_c V_c$), between the stagnation/contact/wall pressure (p_w) and a twist-free velocity ($\mathbf{V}_\infty$), where $\mathbf{V}_\infty \ln 2$ is the propagation velocity of material/inertial waves with a wave drag. The wall pressure differs from the twist-free pressure ($\boldsymbol{p}_\infty$) or static pressure, but commonly they are assumed equal, $\boldsymbol{p_w} \approx \boldsymbol{p}_\infty$ (Prandtl's approximation) and the stagnation or total pressure is measured using a total pressure probe (Pitot-Prandtl) to investigate a velocity field from $\boldsymbol{\Delta p} = \boldsymbol{p_w} - \boldsymbol{p_0} = \boldsymbol{K_P} \frac{\boldsymbol{\rho_0 V_0^2}}{\mathbf{2}}$ (the Bernoulli equation) where Δp is the pressure difference between holes of the pressure probe, K_P is the correction or gauge coefficient depending on the local velocity V_0 ($\boldsymbol{K_P} \approx \mathbf{1}$ for $V \leq 100$ m/s) and ρ_0 is given.

The balance of impact power refers to changes of geometric orientation and energy mutation under the mutual action of kinetic and gravitational fields of a molecular thermal nature, to restore the equilibrium thermodynamic of fluid after impact.

The timeless gravitational energy perturbed through the instantaneous starting/shock of a relatively moving matter, herein fluids, is absorbed into the moving whole matter by changes of a kinetic trinity (i.e. a coupling of momentum, kinetic energy and power) carried by gravitational-shear waves depending on the impact velocity. The visual perception of the inertial wave like transitional-turbulent flows from some experimental coherent hydrodynamic contact structures: soliton-like coherent structures [16, 17] and coherent packets of hairpin vortices [18-20], was poor and/or wrong interpreted and explained. The decomposition of impact energy into inertial/material waves corresponding to the gravitational wave energy is the only explanation for the turbulence phenomena, that under suitable conditions can be produced. The inertial waves redistribute the inertial Newtonian/passive mass through a partition process at a molecular scale, independent of own mass (matter quantity), so called the kinetic trinity (momentum, kinetic energy and power). However, this dynamic partition is influenced by the mode of propagation (transmission) of matter that is a form of propagation of guided waves characterized by a particular field pattern (**g**) in a plane transverse to the direction of propagation ($\mathbf{V}_{\infty}$). In the case of the direction of propagation along $\mathbf{V}_{\infty}$, the mode of propagation is a longitudinal acoustic mode independent of frequency, while for the direction of **g** the transverse gravitational mode depends on both frequency and amplitude. The modulation of inertial waves caused by a starting impact, under suitable scales shows linear/straight evolutions, so called gauge connections.

A generic gauge is a term used by Herman Weyl [11] in the restricted sense of gauge of a railway track, herein the inertial waves, and refers to a scale factor. The evolution of kinetic trinity is described by means of three relativistic parameters: Richardson like number, $\mathbf{Ri} = \boldsymbol{g\beta/(dU/dy)^2}$, a starting parameter representing the ratio of the potential/gravitational energy to the energy recovered from a shearing contact fluid, Mach number, $\boldsymbol{M = \frac{V}{a}; a = \sqrt{\gamma\frac{p}{\rho}} = \sqrt{\gamma RT}, \gamma = 1.4}$ (for air), and Reynolds number, $\boldsymbol{Re = \frac{Vl}{\nu_E}}$, $\boldsymbol{\nu_E = 1m^2 \cdot \omega_E/2\pi = 1.158 \cdot 10^{-5} m^2/s}$, the moving fluid to Earth circulation ratio. For Ri = 1 and different ballistic trajectories of contact particles: $\boldsymbol{\frac{dU}{dy} > 1}$: $\boldsymbol{\sqrt{3}}$ (60 deg), 2 (63 deg) and 2.6 (69 deg), one obtains:

$$\beta \equiv M \approx 1/3, 1/2, 2/3 \text{ and } 1 + \beta = 1.3, 1.5, 1.66, \tag{15}$$

the range of Mach numbers M and polytropic constants λ, respectively, of adiabatic thermodynamic processes in which changes of pressure p and density ρ are related according to

$$\boldsymbol{p\rho^{-\lambda} = p_0\rho_0^{-\lambda}},$$

where subscript zero denotes initial values of variables. Commonly, the compressibility effects was referred only at the isentropic elastic process (γ = 1.4) for gaseous flows at M > 1/3, while the twist effect $\boldsymbol{(1/3 \le \lambda \le 2/3)}$ was ignored. The gaseous flows dominated by kinetic energy-momentum invariance (Eq. 13) are called pre compressible flows, including the turbulent flows.

The energy-momentum invariance mathematically is described by a tensor with 2-dimension curvature [14]. The self-regulation process of Gaian type for starting shocks in the range of M = 1/3 – 2/3 is called Poisson bracket, describing the self-sustaining mechanism of turbulence. If one notes $\nu_P \equiv 1/3$ (Poisson ratio) and $\mathbf{ln\,2} \approx \boldsymbol{2/3}$ (half-life of twist), then the relationship $\boldsymbol{1 \equiv (1/3)^2 - 2(2/3)}$ is held and expresses the existence condition of Eq. 13, i.e. the kinetic energy-momentum balance condition.

Using the Mach partition of subsonic flow field ($\boldsymbol{M_\infty < 1/3}$ - incompressible flow, $1/3 \le M < 2/3$ - adiabatic pre-compressible flow and $\boldsymbol{M_\infty \ge 2/3}$ non-adiabatic compressible flow), and the bundle flow concept where the relevant linear part (parallel field) is subtract from the non-linear transport one gets the decomposition of starting/impact energy as momentum, kinetic energy, power inertial waves $(\boldsymbol{\delta_m}, \boldsymbol{\gamma_k}, \boldsymbol{\beta_w})$, Fig. 8.

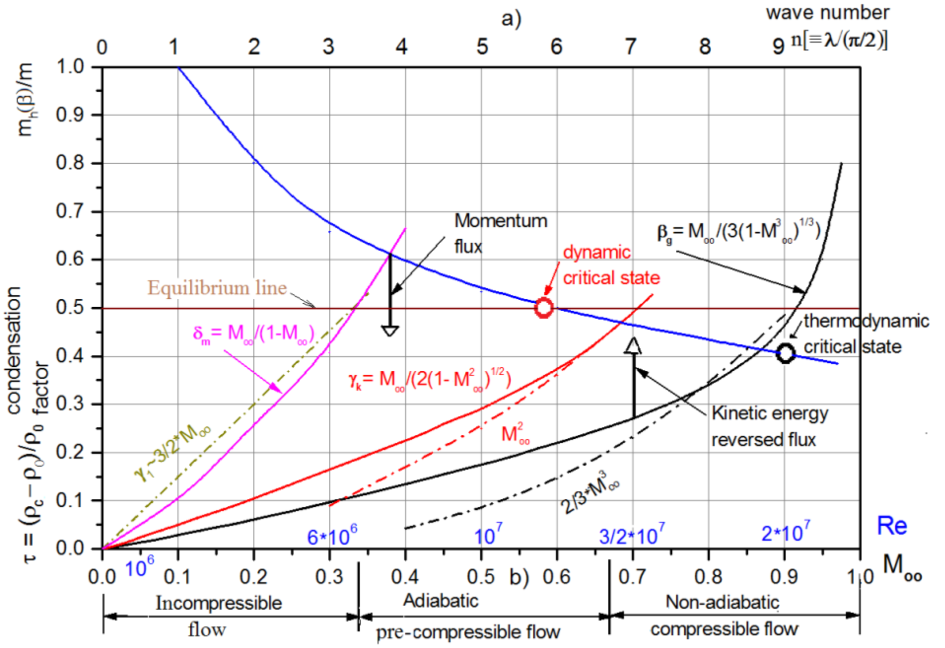


Figure 8. – Comparison between: a) the heavy/active mass decay versus b) the kinetic trinity (inertial self-equilibrating waves: momentum (δ_m) kinetic energy (γ_k) and power (β_g).

Figure shows the range of momentum-kinetic energy mutation, Eq. 13, for $\mathbf{1/3} \leq \boldsymbol{M} < \mathbf{2/3}$ (including self-sustained turbulence, as seen by traveling "soliton-like coherent structures") and the range of momentum-kinetic energy-reactive power mutation for $\boldsymbol{M}_\infty \geq \mathbf{2/3}$, Eqs. (13, 14), (including turbulence decay by thermal diffusion, viewed as wall standing waves, so called "hairpin vortex coherent structures" and herein identified as structured wall-turbulence). The subsonic flow range following Mach number is the Mach gauge (15) depending on the starting conditions, that in a wavy terminology represents the modulation amplitude of subsonic flows in their transition from incompressible $\left(\frac{\mathbf{p}}{\boldsymbol{\rho}} = \text{const.}\right)$ to compressible $\left(\frac{\mathbf{p}}{\boldsymbol{\rho}^{\gamma}} = \text{const.}\right)$ fluid.

A more precise gauge for hydrodynamic "traffic" is the Reynolds number scale, where the critical Reynolds number, $\mathbf{Re}_c \equiv \frac{\mathbf{1}}{\boldsymbol{\nu}_E}, \left(\boldsymbol{\nu}_E[\boldsymbol{m}^2/\boldsymbol{s}], \text{kinematic viscosity}\right)$ is a universal constant [21] used to measure the circulation of fluids at a molecular scale, as the viscosity (μ) to density (ρ) ratio of physical properties of fluids, depending on the velocity via shear flow and temperature.

The kinetic trinity reveals physical causes of the motion, like $\boldsymbol{F} = \boldsymbol{ma}$ for Newtonian relativity, as a whole: kinematics and dynamics, depending on the external (*Vl*) to internal (ν) circulation ratio, $\mathbf{Re}_c = \frac{\boldsymbol{Vl}}{\boldsymbol{\nu}}$.

The Reynolds number is a relativistic local parameter that depends on the starting impact (β or *M*), such that

$$\log \mathrm{Re}^{g\beta} \equiv \text{Reynolds gauge,} \tag{16}$$

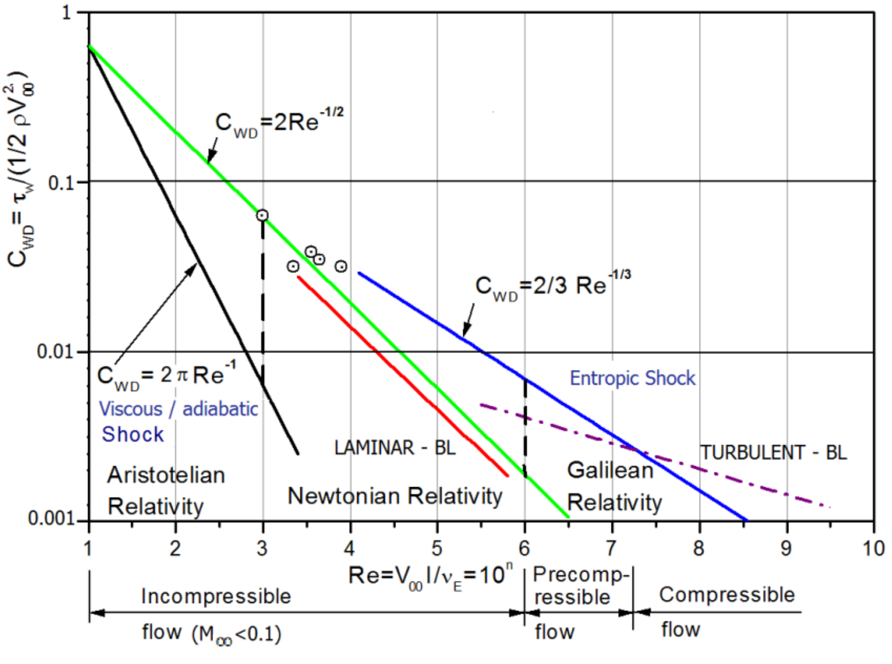
where: $\mathbf{Re}_c \equiv \mathbf{10}(\boldsymbol{M} = \mathbf{0.01}, \boldsymbol{V_0} \approx \mathbf{3}\boldsymbol{m/s})$ is the unity of Reynolds scale;
$\mathbf{Re} \equiv \mathbf{10^3}$ (the onset of a viscous molecular-thermal process);
$\mathbf{Re_{cr}} \equiv \mathbf{10^5}$ (the universal critical Reynolds number) is the half scale (5 unities);
$\mathbf{Re_{cd}} \equiv \mathbf{10^6}$ (the critical dynamic point/state) is the reversible Rayleigh process described by Eq. 13;
$\mathbf{Re}_{\boldsymbol{cth}} \equiv \mathbf{10^7}$ (the critical thermodynamic point/state) is the irreversible Lorentz process described by the Poisson bracket, Eqs. (13, 14).
The starting impact-relaxation relationship in hydrodynamics is the response to an abrupt change of the fluid power (p_0V_0) to an equilibrium state that can be dynamically and/or thermodynamically reached.
For a velocity given, the abrupt change of pressure, $\frac{\boldsymbol{\Delta p}}{\boldsymbol{p_0}} = \frac{\boldsymbol{p_w - p_0}}{\boldsymbol{p_0}}$, usually does not produce instantaneous, corresponding change in flowing fluid, but the new equilibrium frequently is approached exponentially.
If one defines a function $\boldsymbol{C_{WD}}(\mathbf{Re})$ such that

$$\overline{\Delta p} = \frac{p_w - p_0}{\rho_0/2V_0^2} \equiv C_{WD}, \tag{17}$$

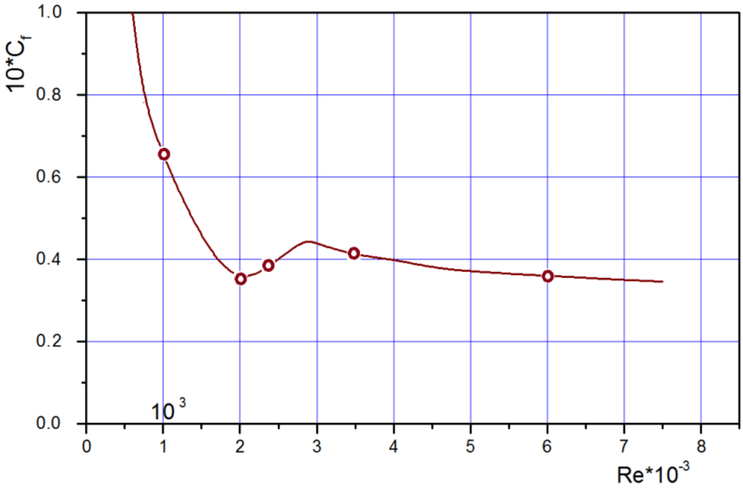
where $\overline{\boldsymbol{\Delta p}}$ is the starting pressure jump for a given velocity V_0 of flowing fluid, and C_{WD} is a friction/wave drag coefficient, then

$$\boldsymbol{C_{WD}} = \mathbf{Re}^{-\mathbf{1}/(\boldsymbol{dV/dy})_{\boldsymbol{w}}}, \left(\frac{\boldsymbol{dU}}{\boldsymbol{dy}}\right)_{\boldsymbol{w}} = \mathbf{1, 2, 3}$$

is a fundamental relationship of contact hydrodynamics, including the transition behaviours ($\mathbf{Re} \equiv \mathbf{10^6}$, reversible Rayleigh process and $\mathbf{Re} \equiv \mathbf{10^7}$, irreversible Lorentz process).



a)



b)

Figure 9. – The self-equilibrating lines of external forces: a) wave drag (skin friction) coefficients,
b) turbulent flow at Re ; 2700 (water in a circular pipe).

Figure 9 shows the dynamic equilibrium lines (logarithmic scales) that gives clear reference formulae yielding quantitative comparison to experiments and numerical simulations, as well as a non-empiric Pitot correction.

The Reynolds gauge represents the modulation amplitude of wall-bounded flows for three characteristic/fundamental frequencies: $\mathbf{Re = 10^3}$ (the vortex-like models based on Aristotelian relativity of temporal simultaneity), $\mathbf{Re = 10^6}$ (the NSE – like models based on Newtonian relativity) and $\mathbf{Re = 10^7}$ (the structured turbulence – like models based on Galilean relativity), the Reynolds number playing the role of a reduced frequency.

5. CONCLUSIONS

Let us move on the only some results we believe are new, without to recall the standard ingredients of classical hydrodynamics.

In §4 we introduce the fundamental impact/shock force characterizing the kernel of such a relative motion. This result is used in an essential way to show the equivalence of the base and bundle concepts in the effective case of turbulence. The impact forces generalize the classical Newtonian motion forms, including the turbulence of fluid dynamics at Reynolds numbers exceeding $Re > 10^6$.

One important ingredient of this approach is the property of contact fluids to self-adjust the thermodynamic state and geometric orientability of a molecular thermal nature, so called thixotropy. Another outcoming is the Ω – shape model for energy-momentum mutation of a gravitational nature.

The paper also clears up the dilemma of the hydrodynamics as how large Re and smallest relevant scales reflected by the NSE, showing that the NSE along with the Stokes's approximation $(3\lambda + 2\mu = 0)$ for $Re > 10^6$ is ill defined and its solution requests local Poisson bracket-like corrections in the contact region.

At the same time the paper yields and its solution request theoretical reference formulae that permit quantitative comparison to check accuracy and correctness of experiments and numerical simulations. A last observation but not less important refers to the Galilean relativity based on the proportionality of the gravitational and inertial mass, that one observe in turbulent flows without any reference whatever to the velocity of light, and differs from the Newtonian relativity based only on passive masses. While the Aristotelian and Newtonian relativities refers to molecular processes of geometric orientation change (i.e. viscous diffusion effects, center of mass model), the Galilean relativity refers to both geometric orientation and energy mutation processes at cellular scales (i.e. inertial dispersion effects, barotropic model).

REFERENCES

[1] H. Schlichting, *Boundary-Layer Theory*, 1968.

[2] H. Dumitrescu, V. Cardos, R. Bogateanu, Random turbulence versus structured turbulence, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol. **10**, Issue 4, pp. 45-60, https://doi.org/10.13111/2066-8201.2018.10.4.5, 2018.

[3] L. Onsager, Statistical hydrodynamics, *Nuovo Cim. Suppl.*, **6**, 279, 1949.

[4] E. A. Novicov, Dynamics and statics of a system or vortices, *Sov. Phys. JETP*, **41**, pp. 937-943, 1976.

[5] P. G. Saffman, G. R. Baker, Vortex interactions, *Ann. Rev. Fluid Mech.*, **11**, pp. 95-122, 1979.

[6] T. D. Lee, On some statistical properties of hydrodynamic and magneto-hydro dynamical fields, *Q. Appl. Maths.* **10**, pp. 69-74, 1952.

[7] R. H. Kraichnan, Statistical dynamics of two-dimensional flow, *J. Fluid Mech.*, **67**, pp. 155-175, 1975.

[8] R. Robert, J. Someria, Statistical equilibrium states for two-dimensional flows, *J. Fluid Mech.*, vol. **229**, pp. 291-310, 1991.

[9] J. Someria, C. Staquet, R. Robert, Final equilibrium state of a two-dimensional shear layer, *J. Fluid Mech.*, vol. **233**, pp. 661-689, 1991.

[10] H. Geiges, A brief history of contact geometry and topology, *Expositiones Mathematicae*, **19**, pp. 25-53, 2001.

[11] R. W. Sharpe, *Differential Geometry; Cartan's generalization of Klein's Erlangen Program*, Springer 2010.

[12] R. Penrose, *The Road to Reality, Jonathan Cape Random House*, London, 2004.

[13] D. Van Nostrand Company Inc. ed., *The international dictionary of applied mathematics*, New York, 1960.

[14] J. Jost, *Geometry and Physics*, Springer-Verlag, Berlin Heidelberg, 2009.

[15] I. V. Arnold, A. B. Khesin, *Topological methods in hydrodynamics*, Springer-Verlag New York Berlin Heidelberg, 1998.

[16] C. B. Lee, J. Z. Wu, Transition in wall-bounded flows, *Applied Mechanics Reviews*, vol. **6**, 030802: 1-21, 2008

[17] X. Y. Jiang, C. Lee, Numerical of K-, N- and O- regime boundary-layer transition at early nonlinear stage, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol. **11**, Issue 3, pp. 77-86, https://doi.org/10.13111/2066-8201.2019.11.3.7, 2019.

[18] J. Zhou, R. J. Adrian, S. Balachandar, T. M. Kendall, Mechanism for generating coherent packets of hairpin vortices in channel flow, *J. Fluid Mech.*, vol. **387**, pp. 353-396, 1999.

[19] W. Schoppa, F. Hussain, Coherent structure generation in near-wall turbulence, *J. Fluid Mech.*, vol. **453**, pp. 57-108, 2002.

[20] R. J. Adrian, Hairpin vortex organization in wall turbulence, *Physsics of Fluids*, **19**, 041301: 1-16, 2007.

[21] H. Dumitrescu, V. Cardos, R. Bogateanu, Al. Dumitrache, Relativistic contact-wall effects at start-up, *INCAS Bulletin*, (online) ISSN 2247–4528, (print) ISSN 2066–8201, ISSN–L 2066–8201, vol. **11**, Issue 2, pp. 85-96, https://doi.org/10.13111/2066-8201.2019.11.2.7, 2019