

# Tuning of a Wavelet Filter for Miniature Accelerometers Denoising based Joint Symbolic Dynamics (JSD) Method

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**Abstract:** *The paper exposes a wavelet filtering mechanism related to the noise suppression in the acceleration sensors, with direct application in the strap-down inertial navigation systems. The presented procedure is related to the actual trend in the inertial navigation field to use miniaturized inertial measurement units, which includes MEMS or NEMS sensors. Beside the already wavelet filtering used method, based on different thresholding mechanisms, the here proposed work refers to the use of an alternative tuning mechanism for the wavelet filters, based on the Joint Symbolic Dynamics (JSD) method. The main idea of the proposed method is to process and analyze signals received from the sensors in the inertial measurement unit of the navigator by using the Wavelet transform until optimal levels of decomposition are established and the useful signals are achieved.*

**Key Words:** *miniaturized inertial measurement unit, wavelet filtering, tuning method, optimal level of decomposition, joint symbolic dynamics.*

## 1. INTRODUCTION

The continuous change at the level of the inertial sensors fabrication technologies in the last two decades, targeting the size and cost reductions, enlarged a lot their application field, but, in the same time, conducted to a multitude of low-cost and low-sizes architectures with a reduced performance level. On the other way, due to the market intense requests related to the new unmanned small vehicles positioning, navigation and monitoring, the use of such miniature sensors is irreplaceable ([1-4]). Therefore, the scientific community develops a lot of researches to find numerical algorithms helping at their performance improvement. Having in mind that the strap-down inertial navigation system (SDINS) can be used as a stand-alone system or in an integrated positioning structure (near the GPS or other navigation system) on a monitored vehicle, the ideas of the algorithms allowing the sensors precision improvement varies significantly ([5-9]).

According to the literature, one of the most important components of the inertial sensors errors, near the bias, is the sensors noise. The problem with this noise resides in its filtering

which is impossible to be done by using traditional methods due to its frequency components superposed over the frequencies characterizing the dynamics of the monitored vehicle. Multiple research studies proposed a statistical filtering of this noise, based on “second opinion” data provided by another positioning system, integrated with the SDINS ([4], [6-10]). Other studies proposed its statistical filtering based on data fusion in redundant inertial sensors networks, with various configurations ([5], [10]). Another already used method is the wavelet transform method, based on different thresholding mechanisms. This method was also used for single-ended SDINS or for integrated positioning structures ([11-15]).

The here proposed work refers to the use of an alternative tuning mechanism for the wavelet filters, based on the Joint Symbolic Dynamics (JSD) method. The main idea of the proposed method is to process and analyze the signals received from the sensors in the inertial measurement unit of the navigator by using the Wavelet transform until optimal levels of decomposition are established and the useful signals are achieved.

## 2. PROPOSED TUNING METHOD

To capture the strong nonlinear nature of interactions that occur within dynamical systems, classical linear approaches proved insufficient and approaches using nonlinear time series seem more appropriate. In capturing these complex nonlinear interactions, nonlinear approaches are partially based on the notion of Granger causality (GC) ([16]).

The term causality refers to a relationship of cause and effect, an event causing another event. In our investigations we are not interested in the causality of signals, but in the dependence of signals, by modelling noisy signals for removing “the noise” superimposed on signals of interest. To observe the noise signal behavior in relation to the useful signal, causality analysis methods, used in dynamic systems correlation investigations, may be addressed.

Causality can be defined also in terms of predictability based on the chronological ordering of causal time series, which are incorporating causal influences from a signal to another signal (i.e. system response in chronological order). This definition can be applied for bivariate analysis (of two time series) and multivariate analysis (more than two time series). The definition of causality also generalizes the concept of causality related to the interactions between the two time series ([17]).

Using multivariate time series, several methods have emerged as a natural extension of univariate approach, allowing the analysis of the relationships between different types of signals. Granger causality can be evaluated both by means of linear and nonlinear methods. Methods based on local linear models have been proposed by Chen et al. ([18]). They introduced an extended conditional model for assessing GC, a joint dynamics spatial reconstruction developed based on an exogenous auto-regressive classic (ARX) model. The main advantages of the ARX model approach are: the possibility of investigating/ evaluating short data recordings, the necessity of defining a smaller number of parameters and the possibility of a more realistic interpretation of the investigated system.

Symbolic methods allow a detailed quantitative assessment of the dynamics of short time series. The direct analysis of successive amplitudes of signals is based on discrete states (symbols) analysis. The here proposed to be used method, Joint Symbolic Dynamics (JSD) method, introduced by Baumert ([19]) is based on the bivariate dynamic processes analysis by means of symbols. By using *JSD* method, short term oscillating signal changes can be captured and an overall assessment of complex system dynamic correlations (between two processes/signals) is possible on short-term. A bivariate vector  $X$  which characterizes two

synchronized time series  $(x, y)$  is transformed by using the *JSD* method in an *S* type bivariate symbol vector as follows:

$$X = \{[x_n, y_n]^T\}_{n=0,1,\dots} \quad x \in R \xrightarrow{\text{Transformation}} S = \{[s(x_n), s(y_n)]^T\}_{n=0,1,\dots} \quad s \in \{0, 1\} \quad (1)$$

For example, a bivariate vector that contains the characteristic data of two signals (a clean signal, an ideal signal, and a noisy signal, superimposed on the ideal signal) can be translated into an *S* type bivariate symbol vector containing binary encoded information about both time series investigated.

For conversion into symbols, an alphabet containing the '0' and '1' symbols has been used; symbol '1' was used for increasing the amplitude values, while '0' symbol, for decreasing or unchanged/constant amplitude values. Subsequently, models of short sequence of symbols were created, each consisting of three successive symbols, called suggestive words and noted with  $w$ . In order to measure the probability occurrence of each type of word  $w$ , dependent on  $x$  and  $y$  time series, the occurrence probability  $p(w_{x,y})$  of all reunited types of words was estimated by using a  $W \times W$  density distribution matrix, with values from  $(000, 000)^T$  to  $(111, 111)^T$  (the sum of all types of words occurrence probabilities  $\sum p(w_{x,y}) = 1$ ) ([19]). In this paper we used an improved version of the method *JSD*. The occurrence probability of each type of words in  $W$  matrix was calculated, as a measure of the global complexity. Also each type of words probability of occurrence on the main diagonals of matrix was calculated and investigated. Most types of words occurrence on the main diagonal suggested the correlation of the investigated signals, while most types of words located on the secondary diagonal suggested a lack of correlation between the investigated signals.

This approach has the advantage of being insensitive to non-stationary time series and is able to capture nonlinear bivariate correlations.

The final target of the proposed algorithm is the noise filtering of signals received from inertial measurement units (IMU), as suggested in Fig. 1. According to the figure, the signals received from the sensors in the inertial measurement unit (IMU) of the navigator are processed and analyzed by using the Wavelet transform until optimal levels of decomposition are established and the useful signals are achieved. In this structure, the reference signals are provided by a GPS, while the disrupted input signals in *JSD* are the outputs of the inertial navigation system (INS).

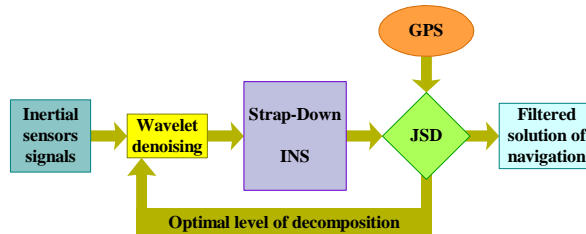


Fig. 1 - Architecture of the tuning method for the inertial sensors denoising

### 3. TESTED ARCHITECTURE AND SOFTWARE MODELS

The here presented work describes the first phase of the method testing, i.e. the denoising analyze for some acceleration signals without implying the strap-down INS mechanism. In this way, we use a simplified testing scheme, which implies some acceleration reference and perturbed signals (Fig. 2). Therefore, the here tested mechanism refers to a monodimensional filtering situation. In order to perform the method validation, an accurate reference signal is

needed. In the same time, the JSD method should work on a perturbed acceleration signal similar to a real one. As a consequence, an accelerometer software model developed by the authors is used to generate these necessary signals (Fig. 3 [5], [10], [20]).

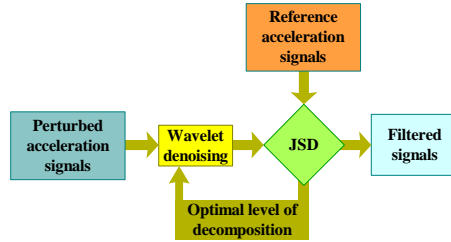


Fig. 2 – Architecture of the monodimensional filtering case

The used model was conceived based on the IEEE inertial sensors standards and on the parameters given by producers in a series of accelerometers data sheets. It covers the main errors of the acceleration sensors: bias, scale factor error, sensitivity axis misalignment, cross axis sensitivity and noise. The model can be successfully used in the numerical simulation of any strap-down inertial navigation system, to create similar conditions with the real ones from the point of view of the perturbations that affects the useful acceleration signal  $a_i$ , when passing through any accelerometric detection device desired to be implemented in the sensing block of the navigator ([5], [10], [20]).

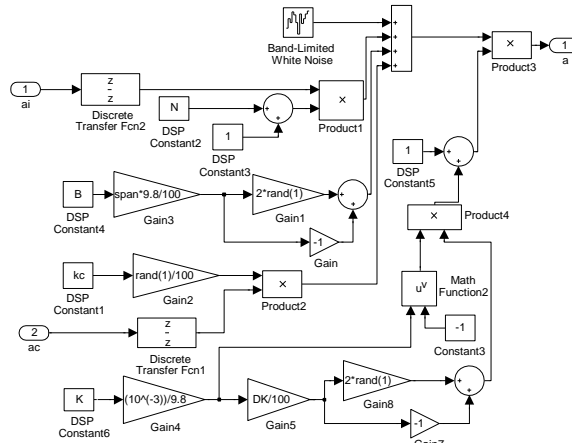


Fig. 3 - Matlab/Simulink accelerometers model

The model in Fig. 4 is used to generate our needed acceleration signals. The “Accelerometer model” block in Fig. 4, obtained by grouping Matlab/Simulink schema in Fig. 3, has the acceleration  $a_i$  applied along of the sensitivity axis and the cross-axis acceleration  $a_c$ , as inputs and the perturbed acceleration  $a$ , as output.

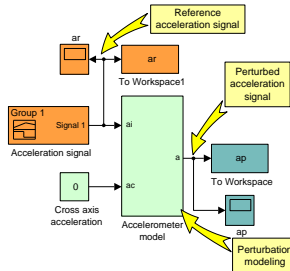


Fig. 4 - Matlab/Simulink model used to generate the simulation signals

The signals used in our study are the input and output signals of this model, as follows: the signals entering the model ( $a_i$ ) are considered clean and used as reference signals, while the model's output signals ( $a$ ) are the noisy signals. During the numerical simulations, the noisy signals are correlated with the reference signals with the JSD method for various levels of decomposition of the wavelet filter in order to prove the viability of the proposed method. Finally, based on a quality parameter (named "Coupling Level" (CL-a) in the next sentences) provided by the JSD method (on the maximum value of this parameter), the optimal level of decomposition of the wavelet filter is chosen. It should leads to the achievement of more accurate data from the sensor.

#### 4. NUMERICAL SIMULATION RESULTS

Based on the model in Fig. 4, various reference/perturbed signals were generated and used to test the proposed method. The generated signals had various noise levels, with different durations (15 seconds or 100 seconds), and different dynamics (constant, chirp signal with a constant amplitude, complex combined sinusoidal signals, or successive ramp signals) as in Fig. 5. Table 1 shows the values of the JSD estimated "Coupling Level" (CL-a) for these signals, for various values of the wavelet filter level of decomposition ( $D$ -Level) between 1 and 10.

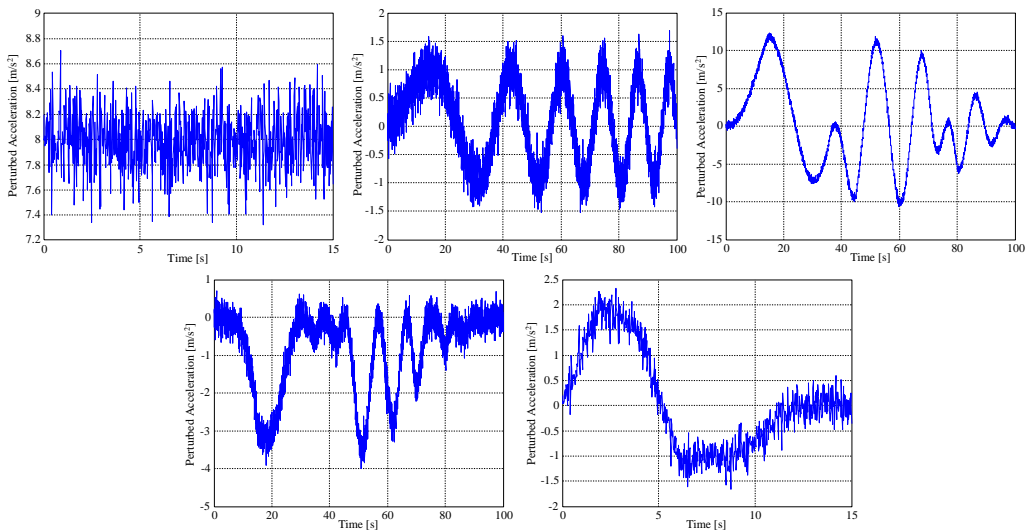


Fig. 5 – Various perturbed signals used in numerical simulation

For the first case, when a constant acceleration of  $8 \text{ m/s}^2$  was considered as reference signal, the wavelet filter optimal decomposition level was found with the value  $D\text{-Level}_{opt}=9$ , being associated with the maximum value of the coupling level  $CL\text{-}a_{max}=1$ .

The graphical results for this case, depicting the coupling level evolution as a function of the wavelet filter decomposition level, the comparison between the perturbed and filtered signals, and the comparison between the reference and filtered signals, are presented in Fig. 6. In the second case, a chirp signal with a constant amplitude equaling the unit was used as reference.

The numerical simulations conducted at the value  $D\text{-Level}_{opt}=8$ , associated with a coupling level  $CL\text{-}a_{max}=0.9744$ . The graphical results corresponding to this case are shown in Fig. 7.

Table 1 - JSD estimated  $CL-a$  for various values of the wavelet filter level of decomposition ( $D$ -Level)

$D$ -Level	$CL-a$ (Case 1)	$CL-a$ (Case 2)	$CL-a$ (Case 3)	$CL-a$ (Case 4)	$CL-a$ (Case 5)
1	0.1551	0.1711	0.2511	0.1609	0.1631
2	0.3075	0.3535	0.5724	0.3569	0.3583
3	0.4225	0.5080	0.8147	0.5322	0.5882
4	0.4559	0.7071	0.9426	0.7039	0.6725
5	0.5267	0.8677	0.9674	0.8407	<b>0.7901</b>
6	0.5374	0.9632	<b>0.9888</b>	0.9274	0.7527
7	0.6377	0.9720	0.9780	0.9188	0.7674
8	0.6444	<b>0.9744</b>	0.9760	0.9224	0.7634
9	<b>1.0000</b>	0.9736	0.9756	<b>0.9290</b>	0.7741
10	0.9910	0.9738	0.9758	0.9234	0.7727

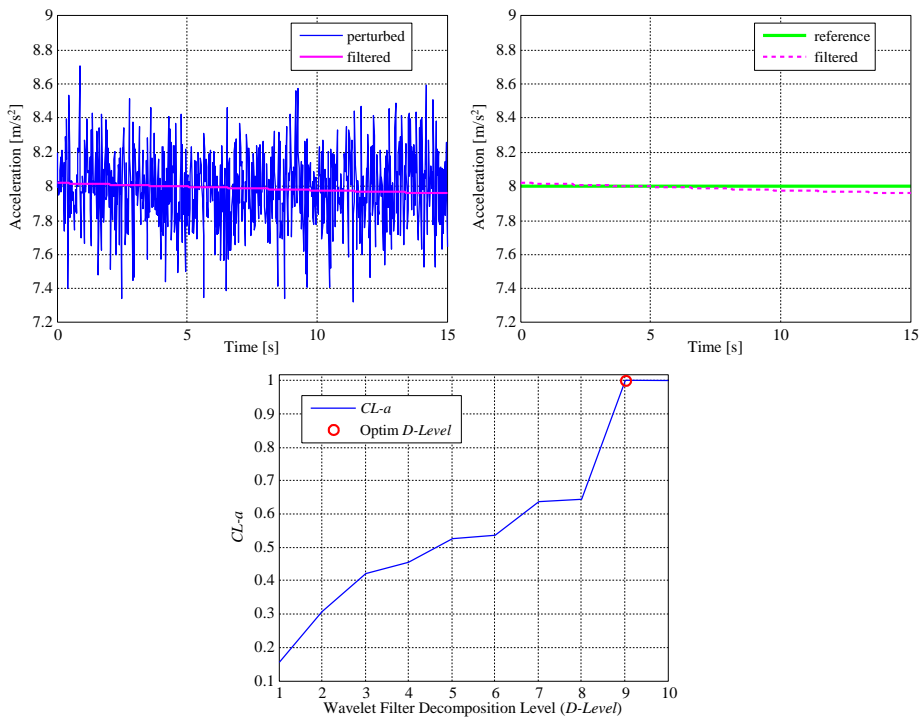


Fig. 6 – The graphical results for the 1<sup>st</sup> testing case

Signals in cases 3 and 4 revealed  $D$ -Level<sub>opt</sub>=6 and  $D$ -Level<sub>opt</sub>=9 values for the wavelet filter optimal decomposition levels, but associated with  $CL-a$ <sub>max</sub>=0.9888 in the 3<sup>rd</sup> case and with  $CL-a$ <sub>max</sub>=0.9290 in the 4<sup>th</sup> case. Fig. 8 and Fig. 9 expose the graphical results for the 3<sup>rd</sup> testing case, and for the 4<sup>th</sup> testing case, respectively.

The 5<sup>th</sup> numerical testing case included as reference signal a repeated ramp signal, offering at the output  $D$ -Level<sub>opt</sub>=5 corresponding to  $CL-a$ <sub>max</sub>=0.7901. The corresponding graphical results are shown in Fig. 10.

From all figures depicting the numerical simulation results can be easily observed that the acceleration filtered signals follow the acceleration reference signals. Moreover, Fig. 11 exposing the time evolutions of the reference signal, of the filtered signal with  $D$ -Level<sub>opt</sub>=5, of the filtered signal with  $D$ -Level<sub>opt</sub>=4 and of the filtered signal with  $D$ -

$Level_{opt}+1=6$  for the 5<sup>th</sup> case, proves that the best reproduction of the reference signal following wavelet filtering is obtained for the optimal decomposition level  $D-Level_{opt}=5$ .

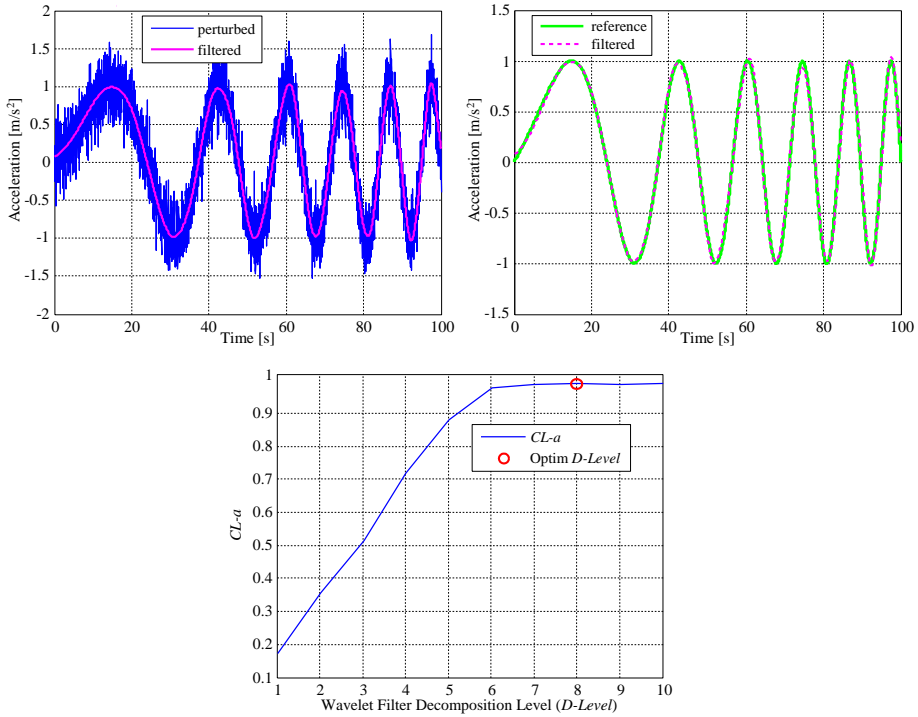


Fig. 7 – The graphical results for the 2<sup>nd</sup> testing case

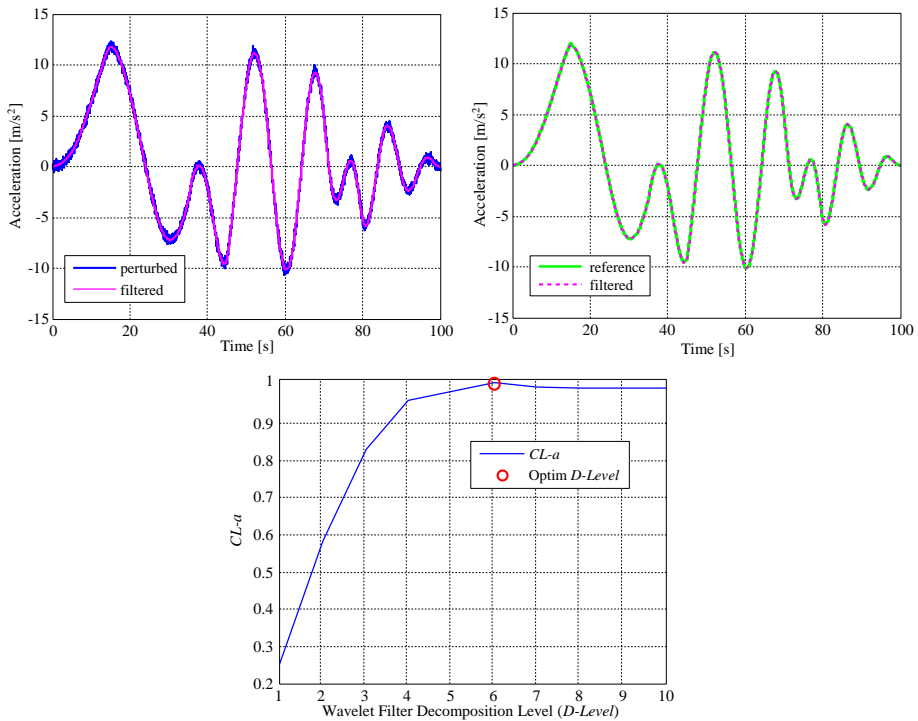


Fig. 8 – The graphical results for the 3<sup>rd</sup> testing case

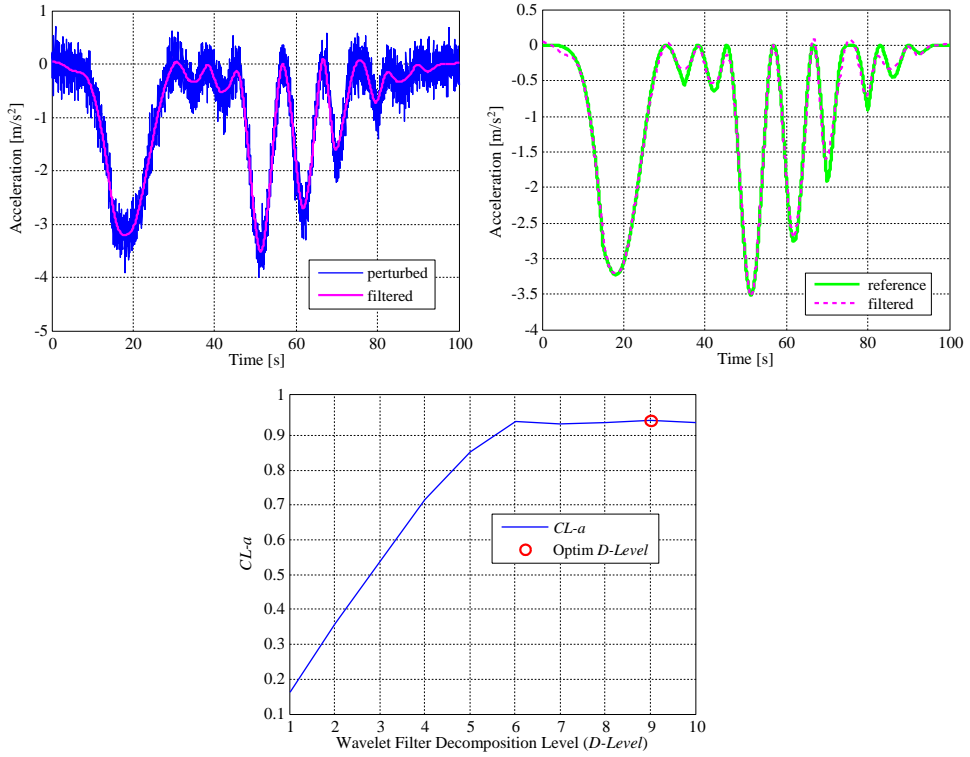


Fig. 9 – The graphical results for the 4<sup>th</sup> testing case

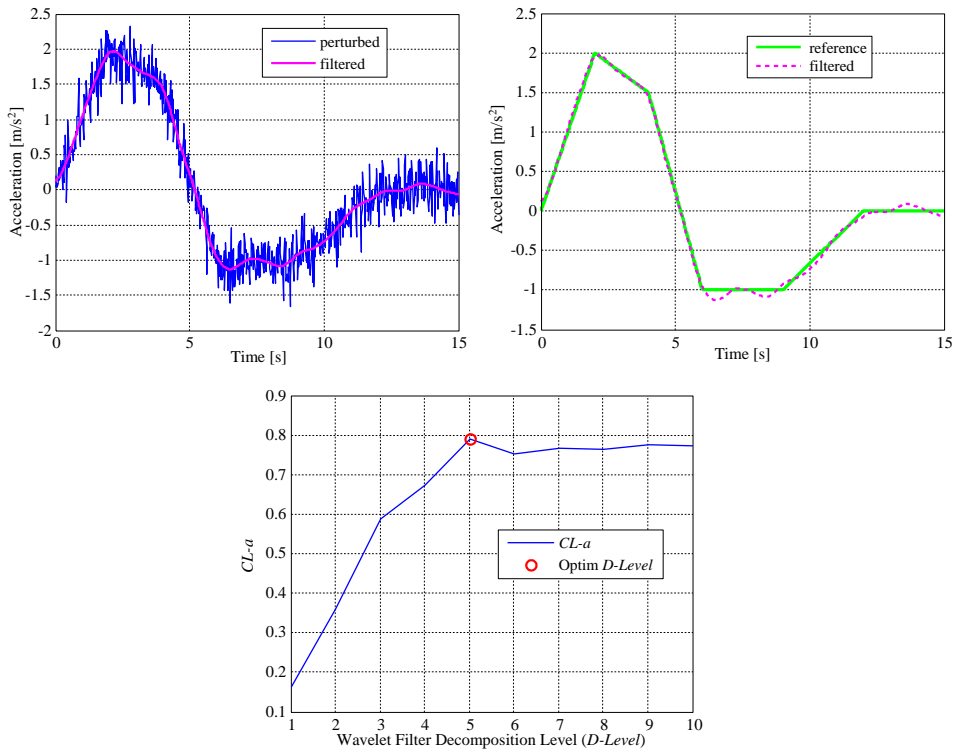


Fig. 10 – The graphical results for the 5<sup>th</sup> testing case



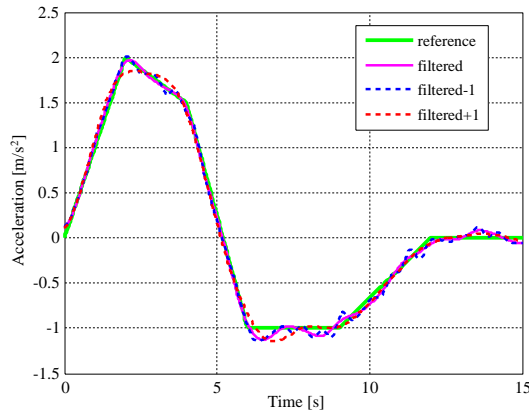


Fig. 11 – The time evolutions of the reference and filtered signals in the 5<sup>th</sup> testing case

## 5. CONCLUSIONS

A new tuning method for the wavelet filters optimal decomposition level was here proposed. The aim of the proposed wavelet filtering is to reduce the noise affecting the miniature inertial sensors used in strap-down inertial navigators. The exposed tuning mechanism is based on the Joint Symbolic Dynamics (JSD) method, which evaluates the coupling levels between some perturbed and reference signals. In the generalized case, the proposed method is intended to be used for the off-line determination of the optimal decomposition level of wavelet filters associated with each sensor in the navigator inertial measurement unit. As reference signals the GPS solution of navigation components are to be used, while as perturbed signals the inertial navigator outputs are considered.

The here presented work describes the first phase of the method testing, i.e. the denoising analyze for some acceleration signals without using the SDINS mechanism. As a consequence, the here tested mechanism referred to a monodimensional filtering situation. In this way, we used a simplified testing scheme, which implies some acceleration reference and perturbed signals. For numerical testing, various reference/perturbed signals were generated and used to test the proposed method. All numerical simulation results showed that the acceleration filtered signals with the tuned wavelet filters follow the acceleration reference signals, creating in this way the premises for a further test of the method on the SDINS/GPS integrated mechanism, based on the use of the GPS signals as reference signals and of the SDINS signals as perturbed signals.

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