Non-stationary diffraction problem of a plane oblique pressure wave on the shell in the form of a hyperbolic cylinder taking into account the dissipation effect

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Abstract: *The plane non-stationary problem of the dynamics of a thin elastic shell in the form of a hyperbolic cylinder immersed in a liquid under the action of an oblique acoustic pressure wave is considered. To solve this problem, a system of equations is constructed in a related statement. In this case, hydroelasticity problems are reduced to equations of shell dynamics, the damping effect of the liquid (dissipation effect) is taken into account by introducing an integral operator of the convolution type in the time domain. The problem is solved approximately on the basis of the hypothesis of a thin layer taking into account the damping forces in the liquid. The integro-differential equations of shell motion are solved numerically based on the difference discretization of differential operators and the representation of the integral operator by the sum using the trapezoid rule. The kinematic and static parameters of the system are given.*

Key Words: *non-stationary dynamics, damping in a liquid, first-order theory, transitional surface functions, damping effect of a liquid*

1. INTRODUCTION

An important problem of modern mechanics is the study of the non-stationary interaction of shock waves propagating in continuous media with various deformable barriers. Research in this area is of considerable interest both from the point of view of developing mathematical methods for solving initial-boundary-value problems of mechanics, and for a number of technical applications, in particular, the calculation of thin-walled structural elements loaded by shock waves in a fluid.

Here we study the dynamic behavior of a thin-walled elastic isotropic shell in the form of a hyperbolic cylinder immersed in a liquid and exposed to acoustic shock waves. The main focus is on the construction of approximate models of the interaction of a deformable shell with a wave diffracting on it. The main mathematical apparatus developed in the work is the transition functions – the fundamental solutions of the non-stationary initial-boundary-value problem of diffraction of an acoustic medium on a smooth convex surface.

Application of the transition functions provides a transition from solving the associated non-stationary problem of joint motion of the acoustic medium and the deformable obstacle to solving the problem only for the obstacle, the mathematical model of which takes into account interaction with the external environment in the form of integral relations. Thus, the dimension of the problem is reduced. This makes it possible to significantly simplify the numerical solution on the basis of the finite-element or finite-difference approach, and in some important particular cases, to construct analytical solutions and estimate the error introduced by the accepted hypotheses. Therefore, the solution to the problem is based on the apparatus of the transition functions, which are fundamental solutions to the non-stationary initial-boundary-value problem of diffraction of an acoustic medium on a smooth convex surface.

The problem of diffraction of a non-stationary plane oblique pressure wave by a thin elastic shell in the form of a hyperbolic cylinder placed in an acoustic medium is considered. To determine the hydrodynamic pressure acting on the shell, a transition function constructed on the basis of the thin layer hypothesis is used [1], [2], [3], [4]. The integration of the equations of motion of a shell of the Tymoshenko type obtained using the Maple 9.0 software environment is carried out with the finite-difference method using Matlab 6.5 [5], [6], [7].

2. MATERIALS AND METHODS

The mathematical formulation of the problem has the following form:

– acoustic environment (Eqs. (1-2)) [1]:

$$\frac{\partial^2\varphi}{\partial\tau^2}+2\beta\frac{\partial\varphi}{\partial\tau}=\Delta\varphi,\mathrm{p}=\frac{\partial\varphi}{\partial\tau},\mathrm{v}=\mathrm{grad}\varphi \tag{1}$$

$$\varphi|_{\tau=0}=\frac{\partial\varphi}{\partial\tau}|_{\tau=0}=0 \tag{2}$$

– elastic isotropic thin shell (Eqs. (3-5)):

$$\frac{\partial^2u_i}{\partial\tau^2}\mathrm{L_{ij}}(\mathrm{u_j})+(\mathrm{p_*}+\mathrm{P})\delta_{\mathrm{i3}},(\mathrm{i},\mathrm{j}=1,2,3) \tag{3}$$

$$u_i|_{\tau=0}=\frac{\partial\mathrm{u_i}}{\partial\tau}|_{\tau=0}=0 \tag{4}$$

$$N^{(m)}(\mathrm{u_i})|_{\xi^1=\xi^1_k}=0,(\mathrm{k}=1,2) \tag{5}$$

Here φ is the velocity potential in an acoustic medium, p is the pressure in the reflected and radiated waves, v is the velocity vector of the acoustic medium, u_i are the generalized displacements of the middle surface of the shell, $\mathrm{L_{ij}}$ are the known differential operators determined by the geometry of the shell, δ_{ij} is the Kronecker delta, β is the parameter that determines dissipation in a liquid [8], [9], [10], [11]. Equations (5) determine, with the help of operators $N^{(m)}(\mathrm{u_i})$, the boundary conditions depending on the shape of the shell and its fastening in space.

Next, the problem is solved in a dimensionless form. Moreover, all linear dimensions are assigned to the focal distance a velocities to the speed of sound in an acoustic medium c_0, quantities having a pressure dimension to a complex $\rho_0 c_0^2$, time τ to tc_0/a. From the conditions of the joint motion of the shell and adjacent particles of the acoustic medium, the conditions of non-leakage follow (Eq. (6)):

$$\frac{\partial w}{\partial \tau} = \frac{\partial \varphi_*}{\partial n}|_\Gamma + \frac{\partial \varphi}{\partial \tau}|_\Gamma \tag{6}$$

Here φ_* is the potential velocity of the wave incident on the shell, $\partial/\partial n$ is the derivative along the external normal to the shell, w is the deflection of the shell. The pressures p_1 and p_2 in both the reflected and radiated waves can be found using the transition function $G(x^i, \tau)$ constructed in the framework of the thin layer hypothesis (an asterisk denotes the convolution operation in time τ) (Eqs. (7-9)).

$$p_1(\xi^1, \tau) = \frac{\partial \varphi_*(\xi^1, 0, \tau)}{\partial n} * G_p(\xi^1, \tau) \tag{7}$$

$$p_2(\xi^1, \tau) = \frac{\partial w}{\partial t}(\xi^1, \tau) * G_p(\xi^1, \tau) \tag{8}$$

$$p = p_1 + p_2, G_p(\xi^1, \tau) = \frac{\partial G(x^1, \tau)}{\partial \tau}|_\Gamma \tag{9}$$

Moreover, the influence function $G(x^1, \tau)$ satisfies the following initial-boundary-value problem (Eqs. (10-12)):

$$\frac{\partial^2 G}{\partial \tau^2} + 2\beta \frac{\partial G}{\partial \tau} = c_0^2 \Delta_\xi G \tag{10}$$

$$G|_{\tau=0} = \frac{\partial G}{\partial \tau}|_{\tau=0} = 0 \tag{11}$$

$$\frac{\partial G}{\partial n}|_\Gamma = \delta(\tau), G(r, \tau) = O(1) \text{as } r \to \infty \tag{12}$$

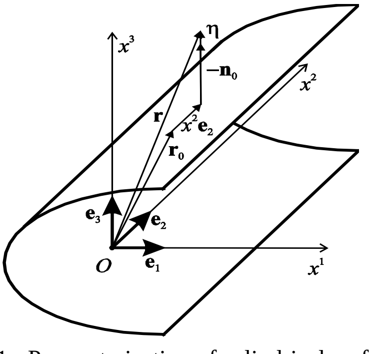

Fig. 1 - Parameterization of cylindrical surfaces

3. RESULTS AND DISCUSSIONS

In flat problems, we assume that the surface Π is a cylinder with a guide Γ and a generatrix parallel to the axis Ox^2 (Fig. 1) of a rectangular Cartesian coordinate system and take the form (Eq. (13)):

$$\Pi: r(\xi^1) = r_0(\xi^1) + x^2 e_2 \tag{13}$$

where (Eq. (14)):

$$\Gamma: r_0(\xi) = x^1 e_1 + x^3 e_3, \xi \in \omega \tag{14}$$

Moreover, the curvilinear coordinate system has $\xi^1 = \boldsymbol{\xi}, \xi^2 = x^1, \xi^3 = \eta$ (Eqs. (15-16)):

$$r = x^i e_i = r_0(\xi) + x^2 e_2 - \eta n_0(\xi) \tag{15}$$

$$r_0 = x^i e_i, n_0 = n_0^i e_i, n_0^2 = 1 \tag{16}$$

where $r_0(\boldsymbol{\xi})$ is radius vector of the curve Γ, and $n_0(\boldsymbol{\xi})$ is unit normal vector.

To determine the transition function $G(\xi^i, \tau)$ in the constructed coordinate system (ξ, η), we obtain the following initial-boundary-value problem (Eqs. (17-19)):

$$\frac{\partial^2 G}{\partial \tau^2} + 2\beta \frac{\partial \mathrm{G}}{\partial \tau} = \frac{1}{\mathrm{H}}\left[\frac{\partial}{\partial \eta}\left(\frac{1}{\mathrm{H}}\frac{\partial \mathrm{G}}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\mathrm{H}\frac{\partial \mathrm{G}}{\partial \eta}\right)\right] \tag{17}$$

$$G|_{\tau=0} = \frac{\partial \mathrm{G}}{\partial \tau}|_{\tau=0} = 0 \tag{18}$$

$$\frac{\partial \mathrm{G}}{\partial \mathrm{n}}|_{\eta=0} = \delta(\tau), \mathrm{G}(\mathrm{r}, \tau) = \mathrm{O}(1), \eta \to \infty \tag{19}$$

We introduce the curvilinear coordinate system (ξ, η) associated with the curve Γ. Let $r_0(\boldsymbol{\xi})$ be the radius vector of the curve Γ, and $n_0(\boldsymbol{\xi})$ be the vector of the unit normal to the shell surface in the form of a parabolic cylinder. Then the curvilinear coordinate system is defined as follows (differentiation is indicated by a subscript) (Eq. (20)):

$$r(\xi, \eta) = \mathrm{r}_0(\xi) - \eta \mathrm{n}_0(\xi) \tag{20}$$

The components of the metric tensor take the form (Eq. (21)):

$$g_{11} = \mathrm{H}_1^2 = \tau^2[1 + 2\eta \mathrm{k} + (\eta \mathrm{k})^2], \mathrm{g}_{12} = 0, \mathrm{g}_{22} = \mathrm{H}_2^2 = 1 \tag{21}$$

where (Eq. (22))

$$k = k(\xi) \tag{22}$$

is curvature of the Γ curve.

In a first approximation, we can assume that the main contribution to the hydrodynamic load comes from the medium moving along the normal to the surface [1], [2], [3], [4]. In this case, the motion of the medium along the Γ surface can be neglected. Therefore, the derivatives with respect to the coordinate ξ can be set identically equal to zero, and the Laplace operator can be calculated on the surface of the cylinder $\eta = 0$. So the initial-boundary-value problem (Eqs. (17-19)) will have the form (Eqs. (23-25)):

$$\frac{\partial^2 G}{\partial \tau^2} + 2\beta \frac{\partial \mathrm{G}}{\partial \tau} = \frac{\mathrm{c}_0^2}{\mathrm{H}_1}\left[\frac{\partial}{\partial \eta}\left(\mathrm{H}_1 \frac{\partial \mathrm{G}}{\partial \eta}\right)\right]|_{\eta-0} \tag{23}$$

$$G|_{\tau=0} = \frac{\partial \mathrm{G}}{\partial \tau}|_{\tau=0} = 0 \tag{24}$$

$$\frac{\partial \mathrm{G}}{\partial \mathrm{n}}|_{\eta=0} = \delta(\tau), \mathrm{G}(\mathrm{r}, \mathrm{t}) = \mathrm{O}(1), \eta \to \infty \tag{25}$$

The transition function of the effect $G_0(\xi,\eta)$ on the obstacle Γ surface is found by the operational method and has the form (Eq. (26)) [2]:

$$G_0(\xi,\eta) = \mathrm{k}(\xi)\Phi_1(\tau) - k(\xi)^2 \int_0^{\tau} \Phi_1(\tau - t)\,\Phi_2(\tau)dt \tag{26}$$

where (Eqs. (27-28)) is:

$$\Phi_1(\tau) = \frac{1 - \mathrm{e}^{-2\beta\tau}}{2\beta} \tag{27}$$

$$\Phi_2(\tau) = \mathrm{e}^{-2\beta\tau}\mathrm{J}_0(\mathrm{k}(\xi)^2 - \beta^2) \tag{28}$$

In this case, the expressions for the pressure in the reflected and radiated waves taking into account Eqs. (7-8) are represented as (Eqs. (29-31)):

$$p_1(\xi,\eta) = -\int_0^{\tau} \frac{\partial\varphi_*(\xi,0,\tau - \mathrm{t})}{\partial\eta} \mathrm{G_p}(\xi^1,\mathrm{t})\mathrm{dt} \tag{29}$$

$$p_2(\xi^1,\tau) = -\int_0^{\tau} \frac{\partial \mathrm{u}_1(\xi^1,\tau - \mathrm{t})}{\partial \mathrm{t}} \mathrm{G_p}(\xi^1,\mathrm{t})\mathrm{dt} \tag{30}$$

$$G_p(\xi,\tau) = \frac{\partial \mathrm{G}_0(\xi,\tau)}{\partial\tau} \tag{31}$$

The pressure behind the wave front in the coordinate system $Ox^i(i = 1,2)$ is given by the relation (Eqs. (32-33)) [2]:

$$p_*(x^1,\tau) = p_0 H(\tau - f(x^i,\vartheta)) \tag{32}$$

$$f(x^i,\vartheta) = \mathrm{x}^1\cos\vartheta + \mathrm{x}^2\sin\vartheta + \mathrm{C} \tag{33}$$

Here the constant C determines the position of the wave front at the initial moment of time $\tau = 0$; p_0 is amplitude pressure. To determine the constant C and coordinates of the point of tangency, we obtain the following system of equations (Eqs. (34-35)) [3]:

$$x^1(\xi_0^1)\cos\vartheta + \mathrm{x}^2(\xi_0^1)\sin\vartheta + \mathrm{C} = 0 \tag{34}$$

$$\frac{d^1(\xi_0^1)}{d\xi^1}\cos\vartheta + \frac{dx^1(\xi_0^1)}{d\xi^1}\sin\vartheta = 0 \tag{35}$$

where ξ_0^1 is the touch point parameter A.

The potential velocity of the incident wave $\varphi_*(x^i,\tau)$ (Eq. (36)) corresponds to the pressure (Eq. (32)):

$$\varphi_*(x^i,\tau) = -p_0(\tau - f(x^i,\vartheta))_+ \tag{36}$$

For the derivative normal to the surface of the incident wave potential from Eq. (37) we obtain (Eqs. (37-38)):

$$\begin{aligned}\frac{\partial\varphi_*(\xi^i,\tau)}{\partial\eta}\Big|_{\eta=0} &= p_0 \frac{\partial f(x^j,\vartheta)}{\partial x^k}\frac{\partial x^k}{\partial\eta} H\left(\tau - f(x^j,\vartheta)\right)\Big|_{\eta=0} \\ &= p_0(n_0^1 cos\vartheta + n_0^2 sin\vartheta)H\left(\tau - f_0(\xi^j,\vartheta)\right)\end{aligned} \tag{37}$$

$$f_0(\xi^i,\vartheta) = f(x^i(\xi^j),\vartheta)|_{\eta=0} \tag{38}$$

Subject to Eqs. (37-38) the pressure in the reflected wave is determined by the equality (Eq. (39)):

$$\begin{aligned} p_1(\xi,\tau) &= -p_0(n_0^1 cos\vartheta + n_0^2 sin\vartheta)\int_0^{\tau-f_{0(\xi,\vartheta)}} G_p(\xi,t)dt \\ &= -p_0(n_0^1 cos\vartheta + n_0^2 sin\vartheta)\, G_0(\xi,\tau - f_0(\xi,\vartheta)) \end{aligned} \tag{39}$$

where the function $G_p(\xi^1,0,\tau)$ is understood instead of $G_p(\xi,\tau)$ with the average surface curvature $k(\xi)/2$. Equation (39) allows approximately, within the framework of the thin layer hypothesis, to determine the reflected pressure in diffraction problems.

Let us consider an example of solving the problem of diffraction of a plane oblique pressure wave by various obstacles. At the initial moment of time $\tau = 0$, the shell and the medium are in an unperturbed state, which corresponds to homogeneous initial conditions (Eqs. (2) and (4)).

Let us consider the problem of diffraction of a plane step pressure wave by an elastic rigid stationary curvilinear obstacle [12], [13], [14], [15]. An oblique plane acoustic wave with a front making an angle v with the axis Ox^1 touches at the point A (Fig. 2) the surface of the cylinder with the guide Γ at the initial moment of time.

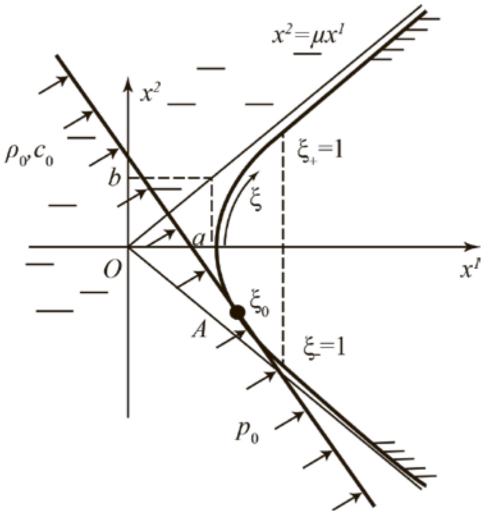

Fig. 2 - Plane oblique pressure wave on the shell in the form of a hyperbolic cylinder

This surface in a Cartesian rectangular coordinate system Ox^i with the asymptote (Eq. (40)):

$$x^2 = \mu x^1 \tag{40}$$

has the form (Eq. (41)):

$$\Gamma: x^1 = \frac{1}{\mu}\sqrt{\mu^2 + \xi^2}, x^2 = \xi, \xi \in R \tag{41}$$

where (Eq. (42)):

$$\mu = \tan(\varphi/2) \tag{42}$$

(φ is the angle between the asymptotes of the hyperbola).

The expression for the curvature and the components of the normal vector are determined by the expressions for the case of a planar problem (Eqs. (43-44)):

$$k(\xi) = \frac{\mu^2}{[\xi^2(1+\mu^2)+\mu^4]} \tag{43}$$

$$n_0^1 = \frac{\mu\sqrt{\mu^2+\xi^2}}{\sqrt{\mu^2(1+\mu^2)+\mu^4}}, n_0^2 = \frac{\mu}{\sqrt{\mu^2(1+\mu^2)+\mu^4}} \tag{44}$$

The coordinate of the touch point ξ_0 and the constant C are determined from Eqs. (34-35) and have the following form (Eq. 45):

$$\xi_0 = \frac{\mu^2 \sin\vartheta}{\sqrt{\cos^2\vartheta - \beta^2\sin^2\vartheta}}, C = -\sqrt{\cos^2\vartheta - \mu^2\sin^2\vartheta} \tag{45}$$

Figure 3 shows the spatio-temporal distribution of pressure $p(\xi,\tau)$ under the action of a plane direct pressure wave ($\vartheta = 0, p_0 = 1$) on a hyperbolic obstacle in an acoustic medium under the action of a single pressure jump ($\xi_0 = 0,\ C = -1$).

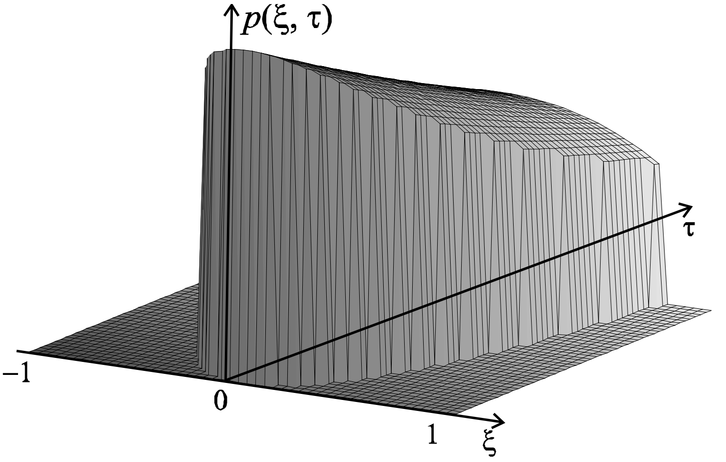

Fig. 3 - Space-time pressure $p(\xi,\tau)$ distribution

Shown in Figs. 4 and 5 are sections of this graph for various values ξ and τ, respectively.

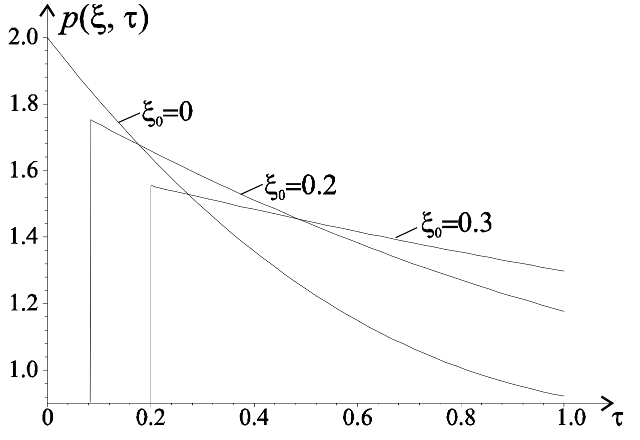

Fig. 4 - Temporal pressure $p(\xi,\tau)$ distribution

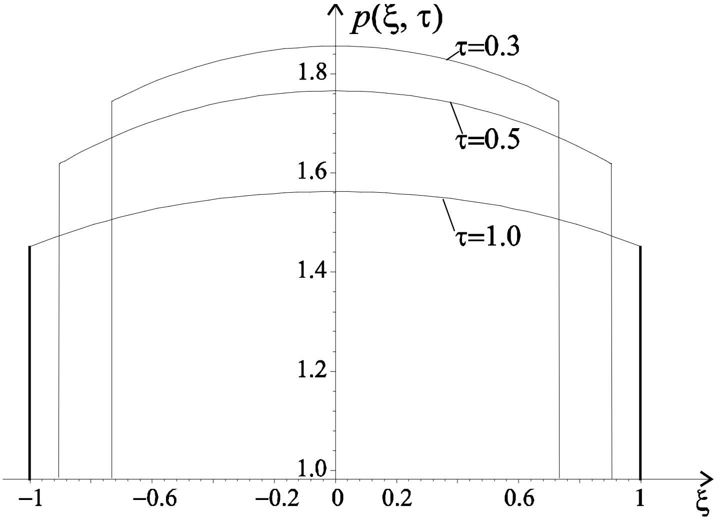

Fig. 5 - Spatial distribution of pressure $p(\xi, \tau)$ at various points in time

The resolving equations for the shell can be written in the operator form (Eqs. (46-47)):

$$\frac{\partial^2}{\partial \tau^2} = \mathrm{Lu} + \mathrm{p} \tag{46}$$

$$L = C\frac{d}{d\xi^2} + \mathrm{B}\frac{\mathrm{d}}{\mathrm{d}\xi} + \mathrm{A} \tag{47}$$

suitable for numerical solution of a discrete analogue of a problem (L is a linear operator of a problem, p is a vector function of the right-hand sides) [5].

In the general case, the construction of resolving Eqs. (46-47) in curvilinear coordinates associated with a surface of arbitrary shape, is very difficult.

At the same time, the use of computer algebra systems that support the basic operations of tensor analysis allows us to automate the transition from the general formulation of the problem to its operator record in a specific coordinate system.

In this case, the Maple 9.0 computer algebra system with the Tensor extension package was used.

The results of the solution are presented in Figs. 2-5 for steel thin shell in the form of a hyperbolic cylinder (density $= 7200 kg/m^3$, modulus of elasticity $E = 2 \cdot 10^6 MPa$, Poisson coefficient $v = 0.3$, shell thickness $h = 0.01m$, ratio between semiaxes $b/a = 0.5$), placed in water (density $\rho_0 = 1000 kg/m^3$, sound speed $c_0 = 330 m/s$, $\beta = 0.1$). The pressure intensity at the front of the incident wave at the initial time (Eq. (48)) is:

$$\rho_0 = 10^4 \mathrm{Pa} \tag{48}$$

Shown in Figs. 6-7 are the dependences of the deflection and normal velocity of the shell on the dimensionless time at the frontal point and the point of contact.

Figure 8 shows the time dependence of the total pressure on the shell at various points of the shell.

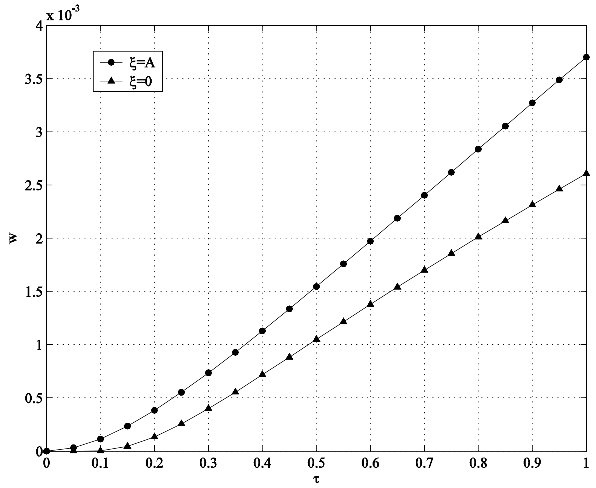

Fig. 6 - Shell deflection $w(\tau, \xi)$

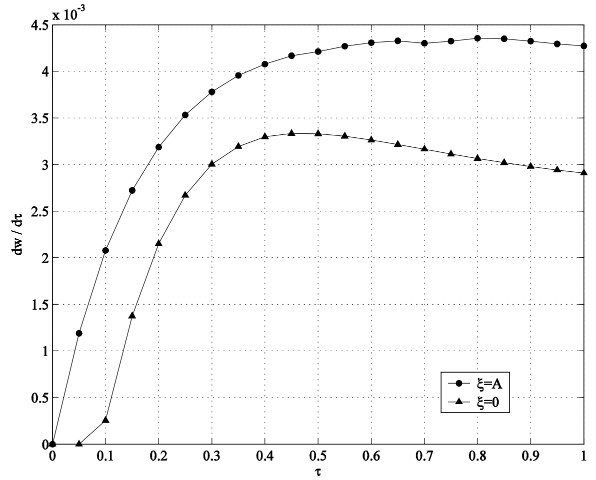

Fig. 7 - Normal shell speed $\frac{w(\tau,\xi)}{d\tau}$

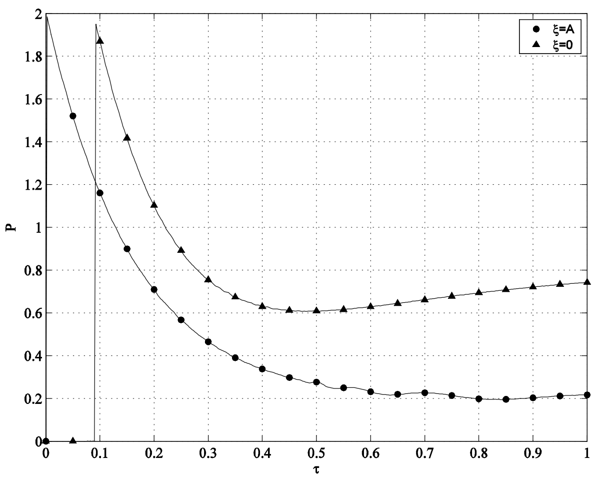

Fig. 8 - Total pressure $p(\tau, \xi)$

4. CONCLUSIONS

The following items were obtained in the present research:

– an approximate model of diffraction of an acoustic pressure wave on an elastic and rigid obstacle in the form of a hyperbolic cylinder was built;

– a fundamental solution to the problem of diffraction of an acoustic pressure wave on a canonical surface of the second order in the form of a hyperbolic cylinder was obtained in special functions;

– a numerical method for solving the obtained integro-differential equations of motion of an elastic shell interacting with an acoustic medium taking into account damping in a liquid has been developed;

– the non-stationary deformed state of thin shells of variable curvature in the form of a hyperbolic cylinder interacting with weak shock waves, based on the developed method, was studied;

– the integro-differential equations of motion of the elastic shell are obtained taking into account the interaction with the ideal fluid.

The obtained solutions (Figs. 6-8) show that taking damping in a liquid into account reduces the static and kinematic parameters of the system (marked with the symbol - Δ).

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