The inverse non-stationary problem of identification of defects in an elastic rod

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Abstract: *Non-stationary inverse problems of deformed solid mechanics are among the most underexplored due to, inter alia, increasing dimension of non-stationary problems per unit as compared with stationary and static problems, as well as necessity to consider the initial conditions. In the context of the continuing progress of the aviation and aerospace industries, the question arises about technical condition monitoring of aircraft for the purposes of their safe operation. A large proportion of an aircraft structure consists of beam and rod elements exposed to various man-made and natural effects which cause defects inaccessible for visual inspection and required to be identified well in advance. It is well known that defects (such as cracks, cavities, rigid and elastic inclusions) are concentrators of stresses and largely cause processes, which lead to the destruction of elastic bodies. Therefore, the problem of identification of such defects and their parameters, i.e. the problem of identification, represents a great practical interest. Mathematically, the problem of identification represents a non-linear inverse problem. The development of methods of solving such problems is currently a live fundamental research issue.*

Key Words: *Inverse problem, elastic rod, influence function, Fourier series, integral transformations, integral equations, quadrature formulas*

1. INTRODUCTION

This work presents an analytical solution of a direct non-stationary problem for a three-step elastic rod. There has been developed and implemented a numerical and analytical method of solving an inverse non-stationary problem of identification of defects in an elastic rod. The basics of the solutions of inverse problems were set forth in the fundamental works by J. Hadamard [1], A. N. Tikhonov [2] and [3], A. O. Vatulian [4], [5], [6] et al. Various issues related to solving non-stationary problems for bodies and structures (such as creating mathematical models of non-stationary interactions, theoretical and numerical methods of investigation of non-stationary problems of dynamics) were addressed in the works of A.G. Gorshkov, D. V. Tarlakovsky et al. [7], [8], [9], [10]. Solutions of inverse non-stationary problems of identification of nonstationary loads upon Timoshenko beam were addressed in the works of Y. A. Vahterova, G. V. Fedotenkov, D. V. Tarlakovsky [11] and Y. A. Vahterova, G. V. Fedotenkov [12]. There are reasonably large number of works addressing inverse problems of rod mechanics, for instance, works [13], [14], [15], [16], [17]. However, in all existing publications on this subject the inverse problems for a rod are considered as either static or stationary ones. This article probable for the first time presents the solution of a non-stationary inverse problem of the identification of a defect in an elastic rod. The problem of identification of defects in an elastic rod is a key problem arising during non-destructive control of materials and elements of structures [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33]. The practical importance of this article is the development of a new method of identification, which allows for finding defects based on data of motion at the end of an elastic rod.

2. MATERIALS AND METHODS

There is an elastic rod of a finite length with one end rigidly fixed while an axial force with a predetermined time law is applied to another one (Figure 1). The rod has variable geometrical characteristics, which change over the length depending on the existing defect. The geometrical characteristics are cross section areas, as well as coordinates of changes of these areas. These geometrical characteristics will be referred to as defect parameter of the rod. Let us note that even in this way of problem statement finding a precise analytical solution of the stated direct problem seems impossible.

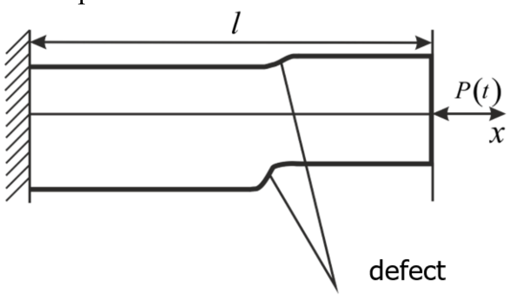

Fig. 1 - Elastic rod of a finite length with one end rigidly fixed

For the purpose of an analytical solution of the problem, the real defect will be replaced with a model representing an abrupt change of cross section area at some a-priori unknown part of the rod. Therefore, the problem is reduced to the representation of the rod with a real defect in form of a three-step rod. One of the steps is the defect area whose geometrical characteristics (cross section area, coordinates of the beginning and the end) are to be found from the solution of the inverse problem [34], [35], [36].

The inverse non-stationary geometrical problem is to identify one, several or all unknown parameters of defect with predetermined other parameters, zero initial conditions and boundary conditions of fixation on one end. The opposite end of the rod is affected by some predetermined time-dependent axial force. It is assumed that the dependency of the displacement of the end face of the rod exposed to this force is known from the displacement sensor which represents an additional condition necessary to solve the inverse problem (Figure 2).

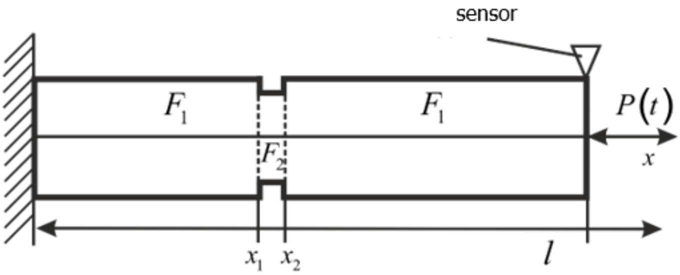

Fig. 2 – Displacement sensor which represents an additional condition necessary

For the purposes of finding a precise analytical solution of the direct problem which serves a basis for the solution of the inverse problem, the rod will be divided into three segments with three different cross section areas (Figure 3).

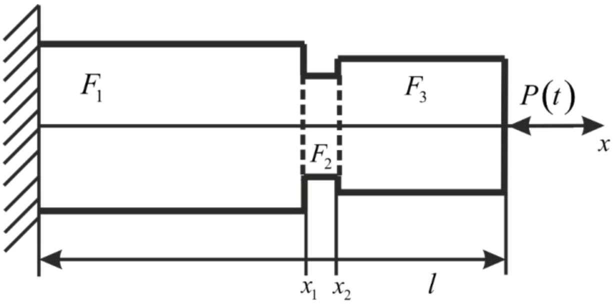

Fig. 3 – Rod which divided into three segments with three different cross section areas

3. RESULTS AND DISCUSSIONS

Before starting to solve inverse problems it necessary to develop the technique of solving a direct non-stationary problem for a three-step rod. The statement of a direct problem involves finding the displacements of the elastic rod.

The mathematical statement of the direct problem includes the equation of motion of a rod of variable cross sections, conditions of conjugation at the points of probable defect, boundary conditions and zero initial conditions [7]:

$$\begin{aligned}p\frac{\partial^2 u_n}{\partial t^2} = E\frac{\partial^2 u_n}{\partial x^2}, n = \overline{1,3}, u_n(x,t)|_{x=x_n} = u_{n+1}(x,t)\ |_{x=x_n}, n \\ = \overline{1,2}, EF_n u_n'(x,t)|_{x=x_n} = EF_{n+1}, u_{n+1}'(x,t)|_{x=x_n} n \\ = \overline{1,2}, u_1(x,t)|_{x=x_0} = 0, E\ F_3 \left.\frac{\partial u_3}{\partial x}\right|_{x=l} = -P(t), \\ u_n(x,t)|_{t=0} = 0 \left.\frac{\partial u_n(x,t)}{\partial t}\right|_{t=0} = 0, n = \overline{1,3}.\end{aligned} \tag{1}$$

where n is a number of segment; u_n is longitudinal displacements over the segment n; E, ρ are Young's modulus and density of the rod; F_n is the area of the r od at its segment, x_1 and x_2 are coordinates of the ends of the first and second segments.

Let us introduce non-dimensional values (dimensional parameters are primed):

$$x = \frac{\acute{x}}{l}, u = \frac{\acute{u}}{l}, F = \frac{\dot{F}}{F_0}, \tau = \frac{\mathrm{ct}}{\mathrm{l}},\ c^2 = \frac{E}{\rho}, P = \frac{\acute{P}}{\mathrm{E}F_0} \tag{2}$$

where τ is non-dimensional time; c is velocity of propagation of longitudinal waves in the rod; F_0 is some reference area. Then, in a non-dimensional form the problem (1) takes the form:

$$\begin{aligned} \ddot{u_n} = U_n'', n = \overline{1,3}, u_1|_{x=0} = 0, F_3\acute{u}_3|_{x=1} = -P\ (\tau), u_n|_{x=x_n} = u_{n+1}|_{x=x_n}, n \\ = \overline{1,2},\ F_n\acute{u}_n\ \mid x = x_n = \ F_{n+1}\acute{u}_{n+1}\ |_{x=x_n}, n = \ \overline{1,2}\ , F_n\acute{u}_n\ |_{x=x_n}, n \\ = \overline{1,2} \end{aligned} \tag{3}$$

$$u_n\ |_{\tau=0} = 0, \dot{u_n}\ |_{\tau=0} = 0, n = \overline{1,3} \tag{4}$$

From now on the dot over a value will denote its derivative with respect to time τ, while a prime, with respect to coordinate x. To find the real load, a problem of influence function for a three-step rod is to be solved. It is displacement of the rod in response to the load $\mathrm{P}(\tau) = \delta(\tau)$, where $\delta(\tau)$ is Dirac delta function. This function is convenient because the solution of the problem with random load will take form:

$$\mathrm{u} = \ G * P \tag{5}$$

where * is the convolution of the influence function with the real load by time τ.

Let us set the load with the Dirac delta function $\mathrm{P}(\tau) = \delta(\tau)$. By applying the Laplace integral transformation with respect to time to the problem (3), with the influence function and property of the delta function $\delta^L\ (\tau) = 1$ taken into consideration, we will come up with the following problem in the following transforms:

$$\begin{aligned} s^2\ddot{G}_n^L, n = \overline{1,3}, G_1^L\ |_{x=0} = 0, F_3\acute{G}_3^L\ |_{x=1} = \ -1, G_n^L\ |_{x=x_n} = G_{n+1}^L\ |_{x=x_n}, n \\ = \overline{1,2}, F_n\acute{G}_n^L\ |_{x=x_n} = F_{n+1}\acute{G}_{n+1}^L\ |_{x=x_n}, n = \overline{1,2}, \end{aligned} \tag{6}$$

where the superscript L of the function denotes its Laplace transform; s is a parameter of the Laplace transformation.

By solving the differential equation from (6) we will obtain:

$$G_n^L = A_n e^{sx} + \ B_n e^{-sx}, n = \overline{1,3}. \tag{7}$$

To satisfy the boundary conditions, let us find the first derivative with respect to x of the expression (7):

$$\acute{G}_n^L = A_n s e^{sx} - \ B_n s e^{-sx}, n = \overline{1,3}. \tag{8}$$

By replacing (7) and (8) to the boundary conditions of the problem (6) we arrive at a system of equations in unknown coefficients $A_1, A_2, A_3, B_1, B_2, B_3$:

$$\begin{aligned} A_1 + B_1 = 0, F_3(A_3 s e^s - B_3 s e^{-s}) = \ -1, A_1 e^{sx_1} - B_1 e^{-sx_1} = A_2 e^{sx_2} + B_2 e^{-sx_1} \\ = A_3 e^{sx_2} + B_3 e^{-sx_1}, F_1(A_1 e^{sx_1} - B_1 e^{-sx_1}) \\ = F_2(A_2 e^{sx_2} + B_2 e^{-sx_1}), F_2\ (A_2 e^{sx_2} + B_2 e^{-sx_2}) \\ = F_3(A_3 e^{sx_2} + B_3 e^{-sx_2}) \end{aligned} \tag{9}$$

By solving the system of linear algebraic equations in unknown coefficients, we obtain:

$$A_1 = \ -B_1 = \frac{-4e^{-s}F_{13}}{s}\mathrm{f}^L(s), \tag{10}$$

$$A_2 = -\frac{2\left(e^{-s(2x_1+1)}F_{12} + e^{-s}\right)F_{321}}{s}f^L(s) \tag{11}$$

$$B_2 = \frac{2(e^{-s(1-2x_1)}F_{12} + e^{-s}F_{321}}{s}f^L(s) \tag{12}$$

$$A_3 = -\frac{e^{-s(2x_1+1)}F_{123} + e^{-s(2x_2+1)}F_{213} + e^{-s(1-2x_1+2x_2)}F_{312} + e^{-s}}{sF_3}f^L(s) \tag{13}$$

$$B_3 = \frac{e^{-s(1-2x_1)}F_{123} + e^{-s(1-2x_2)}F_{213} + e^{-s(2x_1-2x_2+1)}F_{312} + e^{-s}}{30}f^L(s), \tag{14}$$

where:

$$F_{123} = \frac{F_1F_2 + F_1F_3 - F_2F_3 - F_2^2}{F_1F_2 + F_1F_3 + F_2F_3 + F_2^2}; \tag{15}$$

$$F_{213} = \frac{F_1F_2 - F_1F_3 - F_2F_3 + F_2^2}{F_1F_2 + F_1F_3 + F_2F_3 + F_2^2} \tag{16}$$

$$F_{321} = \frac{F_1 + F_2}{F_1F_2 + F_1F_3 + F_3F_2 + F_2^2} \tag{17}$$

$$F_{312} = \frac{F_1F_2 - F_1F_3 + F_2F_3 - F_2^2}{F_1F_2 + F_1F_3 + F_2F_3 + F_2^2}, F_{12} = \frac{F_1 - F_2}{F_1 + F_2}, \tag{18}$$

$$F_{13} = \frac{F_2}{F_1F_2 + F_1F_3 + F_3F_2 + F_2^2}, f^L(s) = \frac{1}{G1} \tag{19}$$

$$\begin{aligned} G1 = 1 + e^{-2s} + F_{123}\left(e^{-2sx_1} + e^{-2s(1-x_1)}\right) + F_{213}\left(e^{-2sx_2} + e^{-2s(1-x_2)}\right) \\ + F_{312}\left(e^{-2s(x_1-x_2+1)} + e^{-2s(x_2-x_1)}\right) \end{aligned} \tag{20}$$

Let us expand the function $f^L(s)$ exponentially in series:

$$f^L(\mathrm{s}) = \sum_{n=0}^{\infty}(-1)^n \sum_{|\alpha|=n} Z_{\alpha,n}\, F_\alpha e^{-2s\varphi(\alpha,x_1,x_2)}, \tag{21}$$

$$\boldsymbol{z}_{\boldsymbol{\alpha},\boldsymbol{n}} = \frac{\mathrm{n}!}{a_1!\,a_2!\,a_3!\,a_4!\,a_5!\,a_6!\,a_7!}, \tag{22}$$

$$F_\alpha = F_{123}^{\alpha_2+a_3}F_{213}^{\alpha_4+a_5}F_{312}^{\alpha_6+\alpha_7} \tag{23}$$

$$\alpha\,| = \sum_{i=1}^{7}\alpha_i\,, \tag{24}$$

$$\begin{aligned} \varphi(\boldsymbol{\alpha}, x_1, x_2) = \alpha_1 + \alpha_2x_1 + \alpha_3(1 - x_1) + \alpha_4x_2 + \alpha_5(1 - x_2) + \alpha_6(x_1 - x_2 + 1) \\ + \alpha_7(x_2 - x_1) \end{aligned} \tag{25}$$

where $\boldsymbol{z}_{\alpha,n}$ is a multinomial coefficient, $\varphi(\boldsymbol{\alpha}, x_1, x_2) > 0$.

Then, with the exponential expansion in series taken into consideration, let us substitute the found expressions for coefficients into (7):

$$G^L = G_1^L H(x_1 - x) + G_2^L H(x - x_1)H(x_2 - x) + G_3^L H(x - x_2), \tag{26}$$

$$G_1^L = \frac{-4}{s} F_{13} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{j=1}^{2} (-1)^n F_\alpha \sum_{j=1}^{2} (-1)^{j+1} e^{-sy_j} \tag{27}$$

$$G_2^L = \frac{-2F_{321}}{s} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{m=1}^{4} a_m e^{-sy_m}, \tag{28}$$

$$G_3^L = \frac{-1}{sF_3} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{k=1}^{8} b_k e^{-sy_k}, \tag{29}$$

$$\begin{gathered} a_1 = -a_4 = 1, a_2 = -a_3 = F_{12}, b_2 = -b_1 = 1, b_3 = -b_4 = F_{123}, b_5 = -b_6 \\ = F_{213}, b_7 = -b_8 = F_{312}. \end{gathered} \tag{30}$$

$$\begin{gathered} y_1 = 1 + 2\varphi(\alpha, x_1, x_2) - \mathrm{x};\ y_2 = 1 + 2\varphi(\alpha, x_1, x_2) + \mathrm{x}; \\ y_3 = 1 + 2\varphi(\alpha, x_1, x_2) - \mathrm{x} - 2x_1; y_4 = 1 + 2\varphi(\alpha, x_1, x_2) + \mathrm{x} - 2x_1 \\ y_5 = 1 + 2\varphi(\alpha, x_1, x_2) + \mathrm{x} + 2x_1; y_6 = 1 + 2\varphi(\alpha, x_1, x_2) - \mathrm{x} - 2x_2 \\ y_6 = 1 + 2\varphi(\alpha, x_1, x_2) + \mathrm{x} - 2x_1 + 2x_2; y_8 = 1 + 2\varphi(\alpha, x_1, x_2) - \mathrm{x} + 2x_1 - 2x_2 \end{gathered} \tag{31}$$

The construction of the original is not challenging, however, for solving a problem with random load based only on the property of Laplace transformation, knowing the transformation of the influence function is enough, because its structure allows for solving the problem with random load without evaluating the integral of convolution type (5). Let us note, that with predetermined value of τ the expression for u_n will contain only a finite number of non-zero summands [37], [38], [39].

Indeed, let us denote $\int_0^\tau P(t)dt = Q(\tau)$, then, according to the lag theorem [11], the originals adapted to the real load $P(\tau)$, formulas (5) and properties of transformation of convolution operation by time, and the original of the function of displacement of the rod will take form:

$$\mathrm{u}(\mathrm{x}, \tau) = u_1(\mathrm{x}, \tau) H(x_1 - x) + u_2(\mathrm{x}, \tau) H(x - x_1) H(x_2 - x) + u_3(\mathrm{x}, \tau) H(x - x_2), \tag{32}$$

$$u_1 = \frac{-4}{s} F_{13} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{j=1}^{2} (-1)^{j+1} \mathrm{P}(\tau - y_j) \tag{33}$$

$$u_2 = \frac{-2}{s} F_{321} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{m=1}^{4} a_m \mathrm{P}(\tau - y_m) \mathrm{H}(\tau - y_m), \tag{34}$$

$$u_3 = \frac{1}{F_3} \sum_{n=0}^{\infty} \sum_{|\alpha|=n} z_{\alpha,n} (-1)^n F_\alpha \sum_{k=1}^{8} b_k \mathrm{P}(\tau - y_k) \mathrm{H}(\tau - y_k). \tag{35}$$

Now let $F_1 = F_2 = F_3 = F, x_1 = x_2 = l$, which corresponds to a rod of constant cross section. Then, the formulas (11)–(13) imply:

$$u_1(\mathrm{x},\tau)=-\frac{1}{F}\sum_{n=0}^{\infty}(-1)^n\,(P(\tau-(1+2n)+x)H[\tau-(1+2n)+x]\\ -P(\tau-(1+2n)-x)H[\tau-(1+2n)-x]) \tag{36}$$

which coincides with the solution for a rod of constant cross section.

For solving the inverse non-stationary problem of identification of a defect, the rod is broken down into three segments (Figure 3) in the way that the first and third segments have equal areas of cross section while the third (middle) segment differs from those two. Here F_1 is the area of cross section of the rod at the defect-free segment; F_2 is the area of cross section at the defected segment; x_1 and x_2 are coordinates of the localization of defect. As it was noted above, the middle segment will describe the defect, if any, while the displacements of the end of the third segment should coincide with the displacements registered by the sensor (Fig.3). The required parameters are well within the found solution of the problem (10). Therefore, we arrive at the following equation:

$$\mathrm{u}(\mathrm{l},\tau)=\mathrm{u}(\mathrm{l},\tau,x_1,x_2,F_2)=U_d(\tau), \tag{37}$$

where $U_d(\tau)$ are the values of displacements registered by the sensor.

Equation (1) at the predetermined moment of time τ represents a non-linear algebraic equation with three unknowns serving as parameters of the defect. Therefore, for creating a system of a closed equation system with respect to three unknown, it is sufficient to fix three moments of time $\tau_k, k=1,2,3$ and, correspondingly, obtain three equations from which the required unknown parameters can be found:

$$U_k(x_1,x_2,F_2)=0, k=1,2,3, \tag{38}$$

where: $U_k(x_1,x_2,F_2)=\mathrm{u}(\mathrm{l},\tau,x_1,x_2,F_2)-U_{dk}\,, U_{dk}=U_d(\tau_k)$.

Therefore, the geometrical inverse problem will be reduced to solving a system of non-linear equations with respect to unknown parameters of the defect of the rod. For solving the system of non-linear equations the Newton method is applied.

$U_k(x_1,x_2,F_2)$ are non-linear functions defined and continuously differentiable in some area $G\subset\mathbb{R}^3$; τ_n is a fixed moment of time. Let us write it in a vector form:

$$\mathrm{x}=(x_1,x_2,F_2)^T, U(x)=(U_1(x),U_2(x),U_3(x))^T, U(x)=0 \tag{39}$$

It is necessary to find a vector $\boldsymbol{x}^*=(x_1^*,x_2^*,F_2^*)^T$, which, when placed to the initial system, will turn each equation into a true numerical equality.

With such approach, the formula for finding the solution is a natural generalization of the formula of one-dimensional integral method [8], [13]:

$$\mathrm{x_{n+1}}=\mathrm{x_n}-\mathrm{W}^{-1}(\mathrm{x_n})\mathrm{U}(x_n), n=\overline{0,N} \tag{40}$$

where:

$$\mathrm{W}=\begin{matrix}\frac{\partial U_1(x)}{dx_1} & \frac{\partial U_1(x)}{dx_2} & \frac{\partial U_1(x)}{dF_2}\\ \frac{\partial U_2(x)}{dx_1} & \frac{\partial U_2(x)}{dx_2} & \frac{\partial U_2(x)}{dF_2}\\ \frac{\partial U_3(x)}{dF_2} & \frac{\partial U_3(x)}{dx_2} & \frac{\partial U_3(x)}{dF_2}\end{matrix}\ \text{is the matrix of Jacobi.}$$

With an additional assumption $U_k(x_1, x_2, F_2)\epsilon C^2$, there is a quadratic convergence of the method.

Usually, the condition $\|\boldsymbol{x}_{n+1} - \boldsymbol{x}_n\| < \mu$, is taken as a criterion of the end of iteration process, where μ is the required precision of the solution.

For solving the direct and inverse problem let us set the following parameters for the three-step rod: $N = 10, x_1 = 0.49, x_2 = 0.51, F_1 = F_3 = 0.07, F_2 = 0.01, P(\tau) = 10^{-3}\cos(\tau), \varepsilon = 10^{-3}$ is noise; $\mu = 0.01$ is the prescribed precision of the solution; $u_3(\tau)$ are displacements known from the solution of the direct problem (10).

Let us solve a direct problem with the parameters specified above. The Figure 4 illustrates the displacements for a three-step rod at moments of time $\tau = 1$ (full line), $\tau = 2$ (dashed line), $\tau = 1$ (dash-and-dot line):

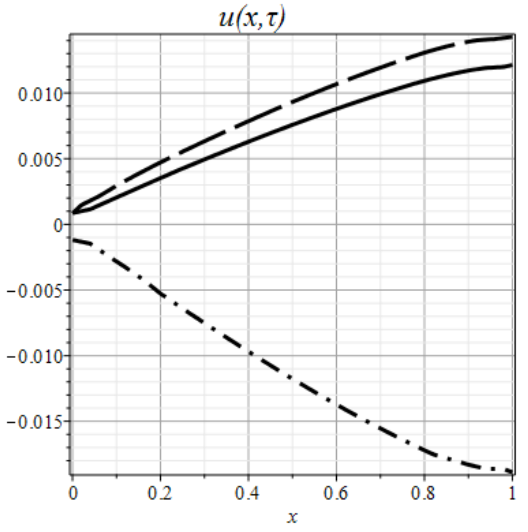

Fig. 4 – Displacements for a three-step rod at moments of time

To identify a defect in an elastic rod it is necessary to find F_2, x_1, and x_2 with three predetermined moments of time $\tau_1 = 1, \tau_2 = 2$ and $\tau_3 = 3$, while the area F_1 is already known. Based on the geometrical sense of the problems, the sought-for parameters are additionally limited with: $0 < F_2 < 0.1, 0 < x_1 < x_2 < 1,$. Here $x_1 = x_2 = F_2 = 0$ are initial values of the sought-for parameters corresponding to the zero iteration of the Newton method (17). Let us substitute the known values into the formula (13) and solve the system of non-linear equations with respect to F_2, x_1, and x_2. We will obtain $F_2 = 0.01, x_1 = 0.49$ and $x_2 = 0.51$ which corresponds to the predetermined parameters for the given rod. Let us solve the same problem with the noise taken into account. We will obtain$F_2 = 0.01001, x_1 = 0.4899$ and $x_2 = 0.5099$

4. CONCLUSIONS

As can be seen from this article, a non-stationary inverse problem of identification of a defect in an elastic rod can be solved correctly without regularization methods. The reason is the equations solving the inverse problem immediately result from the solution of the direct problem which explicitly contains the required values as parameters. Non-stationary nature of the problem allowed for fixing three moments of time and creating a system of non-linear equations with respect to the sought-for parameters of the defect. A solution has been investigated for an inverse problem with noised measurements. The solution has demonstrated the minimal influence of the noise to the solution of an inverse problem, which proves the stability of the proposed method. The algorithm of solving non-stationary direct and inverse problems for a three-step elastic rod was converted into code and successfully tested.

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