H_∞ control synthesis with antisaturation compensator

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***Abstract**: In the paper a technique for dealing with the saturation of an H_∞ control is proposed. This is a two-step method which extends the approach considered by the author in some recent works and consists in: 1) the synthesis of the H_∞ controller for the pair of systems composed by the physical plant and an internal model oriented to the tracking error decreasing in the presence of step input signals, and 2) the insertion of an antisaturating compensator based on the idea to make the state of the saturated system arbitrarily close to the state of the unsaturated one. A proof on the structure of the anti-windup compensator is performed.*

Keywords: *robust servomechanism problem solution, H_∞ control, stabilizing compensator, servocompensator, internal model, input saturation, anti-windup compensator.*

1. INTRODUCTION

All real world control systems are subject to either or both input and state constraints. These can represent hard constraints (physical constraints) or soft constraints (limitations imposed on relevant variables in order to meet certain safety regulations) [1]. Whatever constraint converts whatever hypothetical linear model into a nonlinear one. If one ignores the constraints when designing a control law, then significant degradation in the resulting closed loop performance may occur.

Different methods were adopted to deal with the control systems constraints. A possible, cautious approach involves a reduction of the demand on the control performance until the constraints are avoided in all operational regimes; this strategy may lead to conservative designs, possibly compromising the control efficiency. The fuzzy logic control enters in principle in this category; see [2]. Another approach consists in designing, first, a linear controller without considering the constraints, and then modifying the controller implementation in some way in order to compensate for the constraints on the closed loop performance [3], [4], [5]; this solution is easy to implement. The third strategy involves the inclusion of the constraints in the control design from the beginning [6], [7]; this potentially leads to improved performance but at the expense of an increased complexity.

The input typical constraint is the actuator saturation. This is a topic of recent active research in the control theory, since ultimately all physical actuators are constrained and can potentially saturate. When the saturations are ignored, the phenomenon referred to as reset windup [8] can produce the worst undesirable transients. So, in many applications of aerospace engineering, the actuator saturation is often the principal impediment to achieving significant closed loop performance.

The present paper continues earlier works of the author [4], [5]. The outline of the paper is as follows. The statement and solution of the problem are shown in Section 2. The approach consists in designing, first, a linear H_∞ controller without considering the constraints, and then modifying the controller implementation in some way in order to compensate for the constraints on the closed loop performance. A proof of a Proposition giving the structure of the antiwindup compensation matrix is performed. Some concluding remarks end the paper.

2. H_∞ CONTROL SYNTHESIS WITH ANTISATURATION COMPENSATOR

Consider the plant to be controlled described by a system of first order ordinary differential equations – the state equations

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \tilde{\boldsymbol{B}}_1\boldsymbol{w} + \boldsymbol{B}_2\boldsymbol{u} \tag{1}$$

$\boldsymbol{x} \in R^n$ is the state of the system, $\boldsymbol{u} \in R^p$ is the control, $\boldsymbol{w} \in R^q$ is the disturbance, $\boldsymbol{A}$, $\tilde{\boldsymbol{B}}_1, \boldsymbol{B}_2$ are matrices of appropriate dimensions.

The system output equations are grouped in *regulated* variables, which characterize the objectives to be attained through control and *measured* variables which represent directly the sensor output. The measurement equation is

$$\boldsymbol{y} = \boldsymbol{C}_2\boldsymbol{x} + \tilde{\boldsymbol{D}}_{21}\boldsymbol{\mu} + \boldsymbol{D}_{22}\boldsymbol{u} \tag{2}$$

where $\boldsymbol{y} \in R^m$ is the measured sensor output vector, $\boldsymbol{\mu} \in R^r$ is the vector of measurement (sensor) noise, and the specific form of the measurement matrices $\boldsymbol{C}_2$, $\tilde{\boldsymbol{D}}_{21}$ and $\boldsymbol{D}_{22}$ of appropriate dimensions depends on the type of measurement considered. When measuring quantities such as displacements or strains, the exogenous perturbation $\boldsymbol{w}$ and the control inputs $\boldsymbol{u}$ have no direct effect on the measured outputs. Then

$$\boldsymbol{D}_{22} = \boldsymbol{0} \tag{2$'$}$$

In its most general form, the output equation for the regulated variables $\boldsymbol{z} \in R^s$ can be hereby written

$$\boldsymbol{z} = \boldsymbol{C}_1\boldsymbol{x} + \tilde{\boldsymbol{D}}_{11}\boldsymbol{w} + \boldsymbol{D}_{12}\boldsymbol{u} \tag{3}$$

thus relating the regulated output to the system state as well as exogenous perturbations and control inputs; $\boldsymbol{C}_1$, $\tilde{\boldsymbol{D}}_{11}$ and $\boldsymbol{D}_{12}$ are matrices of appropriate dimensions. Usually, the output reflection of the actuator inputs is achieved through the matrix $\boldsymbol{D}_{12}$ whose first rows are null since the control input $\boldsymbol{u}$ has no direct effect on the modal coordinates and velocities. As well, the perturbations $\boldsymbol{w}$ have no direct effect on the regulated variables – therefore the matrix $\tilde{\boldsymbol{D}}_{11}$ will simply be a null matrix. Thus

$$\tilde{\boldsymbol{D}}_{11} = [\boldsymbol{0}],\ \boldsymbol{D}_{12} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \tag{3$'$}$$

Summarizing, we have the following basic equations of the controlled system, processed from the equations (1), (2), (3)

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}_1 + \boldsymbol{B}_2\boldsymbol{u}_2, \boldsymbol{y} = \boldsymbol{C}_2\boldsymbol{x} + \boldsymbol{D}_{21}\boldsymbol{u}_1 + \boldsymbol{D}_{22}\boldsymbol{u}_2, \boldsymbol{z} = \boldsymbol{C}_1\boldsymbol{x} + \boldsymbol{D}_{11}\boldsymbol{u}_1 + \boldsymbol{D}_{12}\boldsymbol{u}_2$$

$$\boldsymbol{u}_1 := \begin{bmatrix} \boldsymbol{w} \\ \mu \end{bmatrix}, \boldsymbol{u}_2 := \boldsymbol{u}, \boldsymbol{B}_1 = \begin{bmatrix} \tilde{\boldsymbol{B}}_1 & \boldsymbol{0} \end{bmatrix}, \boldsymbol{D}_{21} = \begin{bmatrix} 0 & \tilde{\boldsymbol{D}}_{21} \end{bmatrix}, \boldsymbol{D}_{11} = \boldsymbol{0} \tag{4}$$

For the sake of simplicity, let us consider these equations characterizing a SISO (Single-Input-Single-Output) system (thus, the input and output are scalars y, u_2, for which an H_∞ optimal control problem is posed: find a controller $\boldsymbol{K}(\boldsymbol{s})$ that will minimize the peak value of the frequency response of $\boldsymbol{T}_{\boldsymbol{z}\boldsymbol{u}_1}(s)$, the matrix-valued closed-loop transfer function from the system input to its output. In other words, the question is *to find a controller* $\boldsymbol{K}(\boldsymbol{s})$

$$\boldsymbol{K}(\boldsymbol{s}) = \left[\begin{array}{c|c} \boldsymbol{A}_c & \boldsymbol{B}_c \\ \hline \boldsymbol{C}_c (:= \boldsymbol{K}_\infty) & \boldsymbol{D}_c \end{array}\right], \quad \begin{array}{l} \dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c y \\ u_2 = \boldsymbol{C}_c\boldsymbol{x}_c + D_c y \end{array} \tag{5}$$

if one exists, that internally stabilizes the closed-loop system and that, given $\gamma > 0$*, satisfies the condition* (see details in [10], [11])

$$\left\|\boldsymbol{T}_{zu_1}\right\|_\infty := \sup_{\omega \in R} \bar{\sigma}\left[\boldsymbol{T}_{\boldsymbol{z}\boldsymbol{u}_1}(j\omega)\right] < \gamma \tag{6}$$

A justification for the optimal H_∞ control resides in the *minimax* nature of the problem, with the argument that minimizing the “peak” of the transfer $\boldsymbol{u}_1 \to \boldsymbol{z}$ necessarily renders the magnitude of $\boldsymbol{T}_{\boldsymbol{z}\boldsymbol{u}_1}$ small at all frequencies. In other words, minimizing the H_∞-norm of a transfer function is equivalent to minimizing the energy in the output signal due to the inputs with the worst possible frequency distribution. This improvement of the “worst-case scenario” has a direct correspondent in the active vibration control problem and seems particularly attractive for light structures with embedded piezoelectric actuators [12].

As it was mentioned in Section 1, the saturation of the actuator and therefore of the input u_2 respectively, constitutes a source of performance limitation greater than even uncertainty modelling itself [13]. Techniques for addressing actuator saturation have been studied since the advent of the modern control theory, while recent activity in this area has been steadily increasing; [14]-[18]. In the present paper we consider the problem of synthesizing the output feedback dynamic compensators for plants with saturating actuators. The two-step approach extends the strategy developed in [4], [5], [19]-[21] and consists in 1) the synthesis of an H_∞ controller for the pair of systems composed by the physical plant (1) and an internal model oriented to the tracking error decreasing in the presence of step input signals; 2) the insertion of an antisaturating compensator based on the constraint to make the state of the saturated system arbitrarily close to the state of the unsaturated system, for every time the actuator saturated. More explicitly, this strategy applyies and extends the solution of the general robust servomechanism problem [22] which supposes two compensators: a) a servo compensator, whose structure is proved to be close to the one designed for the step signals; b) a stabilizing compensator, ensured by the way of H_∞ synthesis procedure. An antiwindup compensation is added to deal with the adverse effects caused by the control saturation; see the scheme in Fig. 1.

To simplify, we will consider the compensators in the following order, first the servocompensator

$$\dot{\eta} = C^*\eta + B^* y \tag{7}$$

then the H_∞ stabilizing compensator of the pair physical plant (1)-internal model (7)

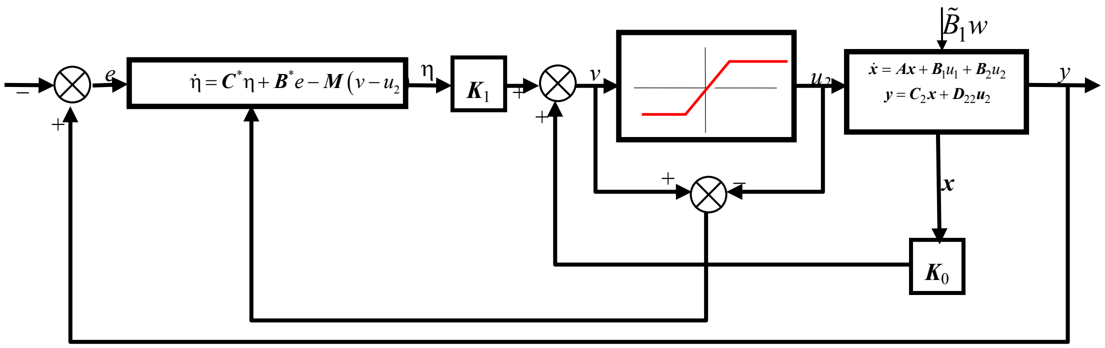

Fig. 1 – The scheme of the solution of robust servomechanism problem with saturation compensation

$$\begin{gathered}\dot{x}_e = A_e x_e + B_{1e} u_1 + B_{2e} u_2,\ x_e := \begin{bmatrix} x \\ \eta \end{bmatrix},\ u_1 := \begin{bmatrix} w \\ \mu \end{bmatrix} \\ A_e := \begin{bmatrix} A & 0 \\ B^* C_2 & C^* \end{bmatrix},\ B_{1e} := \begin{bmatrix} B_1 & 0 \\ 0 & B^* D_{21} \end{bmatrix},\ B_{2e} = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}\end{gathered} \tag{8}$$

given by the software system [11]

$$\begin{gathered}\dot{x}_c = A_c x_c - ZLy \\ u_2 = K_\infty x_c := \begin{bmatrix} k_1 & k_2 & \ldots & k_n & k_\eta \end{bmatrix} x_c\end{gathered} \tag{9}$$

In fact, the input u_2 is susceptible to saturate, that is

$$u_2 = \mathrm{sat} v,\ v = K_\infty x_c = K_0 x + K_1 \eta \tag{9'}$$

and an anti-saturation scheme will be taken into account in the form

$$\dot{\eta} = C^*\eta + B^* y - M(v + u_2) \tag{10}$$

So, the two systems

$$\begin{gathered}\dot{x} = Ax + B_1 w + B_2 u_2,\ \dot{\eta} = C^*\eta + B^* y - M(v - u_2),\ \dot{x}_c = A_c x_c - ZLy \\ u_2 = \mathrm{sat} v,\ v = K_\infty x_c\end{gathered} \tag{11}$$

working in closed loop, will be analyzed, in the absence and the presence of saturating actuators (the function sat(.) is represented in Fig. 1), respectively in the *linear* case $u_2 = v$ and the *nonlinear* saturating case u = sat(v).The concatenation of the systems in (11) will give, at equilibrium *stationary* points, the following linear closed loop system and the associated stationary regime

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}}_c \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B}_2\boldsymbol{K}_0 & \boldsymbol{B}_2 K_1 \\ -\left(\boldsymbol{ZLC}_2\right) & \boldsymbol{A}_c & \boldsymbol{0} \\ B^*\boldsymbol{C}_2 & \boldsymbol{0} & \boldsymbol{C}^* \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_c \\ \eta \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1\boldsymbol{w} \\ -\boldsymbol{ZLD}_{21e}\boldsymbol{w} \\ B^*\boldsymbol{D}_{21e}\boldsymbol{w} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_c \\ \eta \end{bmatrix}_{stl} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B}_2\boldsymbol{K}_0 & \boldsymbol{B}_2 K_1 \\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c & \boldsymbol{0} \\ B^*\boldsymbol{C}_2 & \boldsymbol{0} & C^* \end{bmatrix}^{-1} \begin{bmatrix} -\boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{ZLD}_{21e}\boldsymbol{w} \\ -B^*\boldsymbol{D}_{21e}\boldsymbol{w} \end{bmatrix} \tag{13}$$

In the nonlinear, saturation case, $u_2 \neq v$ a simple calculus gives

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{B}_1\boldsymbol{w} + \boldsymbol{B}_2\text{sat}_i v$$
$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c - \boldsymbol{ZLC}_2\boldsymbol{x} - \boldsymbol{ZLD}_{21e}\boldsymbol{w}$$
$$\dot{\eta} = C^*\eta + B^*\boldsymbol{C}_2\boldsymbol{x} + B^*\boldsymbol{D}_{21e}\boldsymbol{w} - M\left(v - \text{sat}_i v\right)$$

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}}_c \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c & \boldsymbol{0} \\ B^*C_2 & -MK_0 & C^* - MK_1 \end{bmatrix} \begin{bmatrix} x \\ x_c \\ \eta \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1\boldsymbol{w} + B_2\text{sat}_i v \\ -\boldsymbol{ZLD}_{21e}\boldsymbol{w} \\ M\text{sat}_i v + B^*\boldsymbol{D}_{21e}\boldsymbol{w} \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \boldsymbol{x}_i \\ \boldsymbol{x}_{c_i} \\ \eta_i \end{bmatrix}_{st_n} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c & \boldsymbol{0} \\ B^*\boldsymbol{C}_2 & -MK_0 & C^* - M\boldsymbol{K}_1 \end{bmatrix}^{-1} \begin{bmatrix} -\boldsymbol{B}_1\boldsymbol{w} - \boldsymbol{B}_2\text{sat}_i v \\ \boldsymbol{ZLD}_{21e}\boldsymbol{w} \\ -M\text{sat}_i v - B^*\boldsymbol{D}_{21e}\boldsymbol{w} \end{bmatrix}$$

The index i marks the situation of v to be inside the linear region, or larger than the upper limit, or smaller than the lower limit of the saturation. A question thus arises: what is the matrix $\boldsymbol{M}$, generally speaking, which ensures the solving of the optimal control problem? (by exponent T is noted the transpose of a matrix)

$$\min_M J, \quad J := \sum_{i=1}^{2}\left[\boldsymbol{x}_{st_l} - \boldsymbol{x}_{i\,st_n}\right]^{\mathrm{T}}\left[\boldsymbol{x}_{st_l} - \boldsymbol{x}_{i\,st_n}\right] + \left(\eta_{st_l} - \eta_{i\,st_n}\right)^{\mathrm{T}}\left(\eta_{st_l} - \eta_{i\,st_n}\right) \tag{15}$$

The solution of this problem is given by the following

PROPOSITION. *The anti-windup compensation matrix (Fig. 1)*

$$M = \frac{B^*\boldsymbol{C}_2\boldsymbol{P}_{n1}}{1 + \boldsymbol{K}_0\boldsymbol{P}_{n2}}, \begin{bmatrix} \boldsymbol{P}_{n1} \\ \boldsymbol{P}_{n2} \end{bmatrix} := \boldsymbol{P}_n^{-1}\begin{bmatrix} \boldsymbol{B}_2 \\ \boldsymbol{0} \end{bmatrix}, \boldsymbol{P}_n := \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c \end{bmatrix} \tag{16}$$

ensures a minimum of J (with $\boldsymbol{P}_n$ nonsingular), if the internal model $\left(B^*, C^*\right)$ is chosen to fulfill the matrix equations

$$C^{*^{-1}}\left[B^*\boldsymbol{C}_2 \quad \boldsymbol{0}\right]\left(\boldsymbol{P}_l-\boldsymbol{Q}_l C^{*^{-1}}\boldsymbol{R}_l\right)^{-1}\begin{bmatrix}\boldsymbol{B}_1\\ -\boldsymbol{ZLD}_{21e}\end{bmatrix}-\boldsymbol{W}_l B^*\boldsymbol{D}_{21e}-$$

$$-\left(C^*-MK_1\right)^{-1}\left[B^*\boldsymbol{C}_2 \quad -M\boldsymbol{K}_0\right]\boldsymbol{P}_n^{-1}\begin{bmatrix}\boldsymbol{B}_1\\ -\boldsymbol{ZLD}_{21e}\end{bmatrix}+\left(C^*-MK_1\right)^{-1}B^*\boldsymbol{D}_{21e}=0 \qquad (16')$$

$$-\left[B^*\boldsymbol{C}_2 \quad -MK_0\right]\boldsymbol{P}_n^{-1}\begin{bmatrix}\boldsymbol{B}_2\\ 0\end{bmatrix}+M=0$$

PROOF. Let us note

$$\Gamma_l := \begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}_2\boldsymbol{K}_0 & \boldsymbol{B}_2K_1\\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c & \boldsymbol{0}\\ B^*\boldsymbol{C}_2 & \boldsymbol{0} & C^*\end{bmatrix}, \boldsymbol{P}_l := \begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}_2\boldsymbol{K}_0\\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c\end{bmatrix} \qquad (17)$$

$$\Gamma_n := \begin{bmatrix}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0}\\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c & \boldsymbol{0}\\ B^*\boldsymbol{C}_2 & -M\boldsymbol{K}_0 & C^*-MK_1\end{bmatrix}, \boldsymbol{P}_n := \begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ -\boldsymbol{ZLC}_2 & \boldsymbol{A}_c\end{bmatrix}$$

We can apply the matrix inversion Lemma [23]

$$\Gamma = \begin{bmatrix}\boldsymbol{P} & \boldsymbol{Q}\\ \boldsymbol{R} & \boldsymbol{S}\end{bmatrix} \Rightarrow \Gamma^{-1} = \begin{bmatrix}\boldsymbol{X} & -\boldsymbol{XQS}^{-1}\\ -\boldsymbol{S}^{-1}\boldsymbol{RX} & \boldsymbol{W}\end{bmatrix}$$

$$\boldsymbol{X} := \left(\boldsymbol{P}-\boldsymbol{QS}^{-1}\boldsymbol{R}\right)^{-1}, \quad \boldsymbol{W} := \boldsymbol{S}^{-1}+\boldsymbol{S}^{-1}\boldsymbol{RXQS}^{-1} \qquad (18)$$

to the matrices (17), regrouped in (2×2)-blocks. It is worth to mention that the $\boldsymbol{M}$ matrix will be involved only on the ultimate row of the matrix Γ_n^{-1}, thus the performance index J (15) will be optimized (minimized) only relatively to the second product in J. Consequently, we then have that

$$\Gamma_l^{-1} = \begin{bmatrix} --- & ---\\ -C^{*^{-1}}\left(\left[B^*\boldsymbol{C}_2 \quad \boldsymbol{0}\right]\right)\left(\boldsymbol{P}_l-\boldsymbol{Q}_l C^{*^{-1}}\boldsymbol{R}_l\right)^{-1} & \boldsymbol{W}_l\end{bmatrix}$$

$$\boldsymbol{W}_l := C^{*^{-1}}+C^{*^{-1}}\left(\left[B^*\boldsymbol{C}_2 \quad \boldsymbol{0}\right]\right)\left(\boldsymbol{P}_l-\boldsymbol{Q}_l C^{*^{-1}}\boldsymbol{R}_l\right)^{-1}\boldsymbol{Q}_l C^{*^{-1}} \qquad (19)$$

$$\Gamma_n^{-1} = \begin{bmatrix} \boldsymbol{P}_n^{-1} & \boldsymbol{0} \\ -\left(C^{*^{-1}} - MK_1\right)^{-1}\left(\left[B^{*}\boldsymbol{C}_2 \quad -MK_0\right]\right)\boldsymbol{P}_n^{-1} & \left(C^{*} - MK_1\right)^{-1} \end{bmatrix} \tag{20}$$

Further on

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_c \\ \eta \end{bmatrix}_{stl} = \Gamma_l^{-1} \begin{bmatrix} -\boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{ZLD}_{21\mathrm{e}}\boldsymbol{w} \\ -\boldsymbol{B}^{*}\boldsymbol{D}_{21\mathrm{e}}\boldsymbol{w} \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{x}_i \\ \boldsymbol{x}_{c_i} \\ \eta_i \end{bmatrix}_{stn} = \Gamma_n^{-1} \begin{bmatrix} -\boldsymbol{B}_1\boldsymbol{w} - \boldsymbol{B}_2\mathrm{sat}_i v \\ \boldsymbol{ZLD}_{21e}\boldsymbol{w} \\ -\boldsymbol{M}\mathrm{sat}_i v - \boldsymbol{B}^{*}\boldsymbol{D}_{21e}\boldsymbol{w} \end{bmatrix} \tag{21}$$

$$\begin{aligned} \eta_{stl} - \eta_{istn} = {} & C^{*^{-1}}\left[B^{*}\boldsymbol{C}_2 \quad \boldsymbol{0}\right]\left(\boldsymbol{P}_l - \Theta_l C^{*^{-1}}\boldsymbol{R}_l\right)^{-1}\begin{bmatrix} \boldsymbol{B}_1\boldsymbol{w} \\ -\boldsymbol{ZLD}_{21e}\boldsymbol{w} \end{bmatrix} - \boldsymbol{W}_l B^{*}\boldsymbol{D}_{21e}\boldsymbol{w} \\ & + \left(C^{*} - MK_1\right)^{-1}\left[B^{*}C_2 \quad -MK_0\right]P_n^{-1}\begin{bmatrix} -B_1 w - B_2\mathrm{sat}_i v \\ ZLD_{21e}w \end{bmatrix} \\ & + \left(C^{*} - MK_1\right)^{-1}\left(M\mathrm{sat}_i v + B^{*}D_{21e}w\right) \end{aligned} \tag{22}$$

Let us remark the difference with respect to the tracking system considered in [4], [5], where we have the exogeneous signals $r \neq \boldsymbol{0}, w = \boldsymbol{0}.$ To counteract the influence of the perturbation w, the key parameters C^{*} and B^{*} will be used and thus the system of matrix algebraic equations is obtained

$$\begin{aligned} & C^{*^{-1}}\left[B^{*}\boldsymbol{C}_2 \quad \boldsymbol{0}\right]\left(\boldsymbol{P}_l - \boldsymbol{Q}_l C^{*^{-1}}\boldsymbol{R}_l\right)^{-1}\begin{bmatrix} \boldsymbol{B}_1 \\ -\boldsymbol{ZLD}_{21e} \end{bmatrix} - \boldsymbol{W}_l B^{*}\boldsymbol{D}_{21e} - \\ & -\left(C^{*} - MK_1\right)^{-1}\left[B^{*}\boldsymbol{C}_2 \quad -M\boldsymbol{K}_0\right]\boldsymbol{P}_n^{-1}\begin{bmatrix} \boldsymbol{B}_1 \\ -\boldsymbol{ZLD}_{21e} \end{bmatrix} + \left(C^{*} - MK_1\right)^{-1}B^{*}\boldsymbol{D}_{21e} = 0 \\ & -\left[B^{*}\boldsymbol{C}_2 \quad -MK_0\right]\boldsymbol{P}_n^{-1}\begin{bmatrix} \boldsymbol{B}_2 \\ 0 \end{bmatrix} + M = 0 \end{aligned} \tag{23}$$

Let's define the product of matrices

$$\begin{bmatrix} P_{n1} \\ P_{n2} \end{bmatrix} := P_n^{-1}\begin{bmatrix} B_2 \\ 0 \end{bmatrix} \tag{24}$$

If the anti-saturation compensation matrix is given by

$$M = \frac{B^{*}\boldsymbol{C}_2\boldsymbol{P}_{n1}}{1 + \boldsymbol{K}_0\boldsymbol{P}_{n2}} \tag{25}$$

then the second equation in (23) is satisfied. The following step is the substitution of the solution (25) in the first equation (23) and the searching for a pair $\left(C^{*}, B^{*}\right)$ (herein, scalars), to fulfill this equation. Taking into account the proof in [5], an integrator structure of the internal model is preferable

$$C^* \approx B^* \tag{26}$$

This remark ends the proof. $\square$

3. CONCLUDING REMARKS

The main contribution of the paper consists in clarifying an integrated methodology of robust control synthesis with anti-windup feedback compensation. The two-step approach consists in 1) the synthesis of an H_∞ controller for the pair of systems composed by the physical plant (1) and an internal model oriented to the tracking error decreasing in the presence of step input signals and 2) the insertion of an antisaturating compensator based on the constraint to make the state of the saturated system arbitrarily close to the state of the unsaturated system, for every time the actuator saturated. More explicitly, this strategy applies and extends the solution of the general robust servomechanism problem [22] which supposes two compensators: a) a servo compensator, whose structure is proved to be close to the one designed for step signals; b) a stabilizing compensator, ensured by the way of H_∞ synthesis procedure. An antiwindup compensation is added to deal with the adverse effects caused by control saturation, see the scheme in Fig. 1. A detailed proof of the anti-windup compensator is given.

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