

# Attitude Dynamics and Tracking Control of Spacecraft in the Presence of Gravity Oblateness Perturbations

Achim IONITA\*<sup>1</sup>, Ionel POPESCU<sup>2</sup>

\*Corresponding author

\*<sup>1</sup>AEROSPACE Consulting

B-dul Iuliu Maniu 220, Bucharest 061126, Romania

ionita.achim@incas.ro

<sup>2</sup>STRAERO – Institute for Theoretical and Experimental Analysis

of Aeronautical Structures

B-dul Iuliu Maniu 220, Bucharest 061126, Romania

ionel.popescu@straero.ro

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**Abstract:** *The orbital docking represents a problem of great importance in aerospace engineering. The paper aims to perform an analysis of docking maneuvers between a chaser vehicle and a target vehicle in permanent LEO (low earth orbit). The work begins with a study of the attitude dynamics modeling intended to define the strategy that facilitates the chaser movement toward a docking part of the target. An LQR (linear quadratic regulator) approach presents an optimal control design that provides linearized closed-loop error dynamics for tracking a desired quaternion. The control law formulation is combined with the control architecture based on SDRE (State Dependent Riccati equation) technique for rotational maneuvers, including the Earth oblateness perturbation. The chaser body-fixed frame must coincide with the target body-fixed frame at the docking moment. Then the implementation of the control architecture based on LQR technique using the computational tool MATLAB is carried out. In simulation of the docking strategy V-R bar operations are analyzed and the minimum accelerations needs the control of chaser vehicle. The simulation analysis of those maneuvers considered for a chaser vehicle and a target vehicle in LEO orbit is validated in a case study.*

**Key Words:** LEO, orbital rendezvous and docking, attitude dynamics, LQR, SDRE

## ABBREVIATIONS

LEO – Low Earth Orbit

SDRE – State Dependent Riccati Equation

LVLH – Local vertical Local Horizontal frame

SDC – State Dependent Coefficient factorization

GNC – Guidance, Navigation and Control

LQR – Linear Quadratic Regulator

ARE – Algebraic Riccati Equation

ADV – Active Docking Vehicle

CW – Clohessy Wiltshire equations

$J_2$  – Earth Oblateness perturbation  
 ISS – International Space State  
 R&D – Rendezvous and Docking  
 RPOP – Rendezvous and Proximity Operations Program  
 TDV – Target Docking Vehicle

## SYMBOLS AND VARIABLES

$\mu$  – Earth's gravitational constant ( $\text{m}^3/\text{sec}^2$ )  
 $q_1, q_2, q_3, q_4$  – attitude quaternions  
 $\omega_0$  – mean motion (1/sec)  
 $\omega_x, \omega_y, \omega_z$  – relative angular velocities (1/sec)  
 $x_c, y_c, z_c$  – thrusters torque arms (m)  
 $I_x, I_y, I_z$  – moment of inertia ( $\text{kgm}^2$ )  
 $u_x, u_y, u_z$  – forces exerted by the chaser thrusters (N)  
 $m_c$  – mass of the chaser (kg)  
 $a_{J_2}$  – relative effect of the Earth oblateness ( $\text{m}/\text{sec}^2$ )  
 $x, y, z$  – coordinates of the chaser (m)  
 $F_x, F_y, F_z$  – forces exerted by the chaser (N)

## 1. INTRODUCTION

Over the years on-orbit manoeuvres of the spacecraft became priority problems of the space vehicle community. Due to the cost and time disadvantage of manual control an autonomous mission on orbit appears to be favourable. An autonomous chaser vehicle had to be capable of rendezvous and docking manoeuvres and requires a significant development in the various technologies. Rendezvous and docking operations of unmanned vehicles e.g. presently the European ATV, the Japanese HTV and Progress within the International Space Station are automatic but not fully autonomous. The proximity operations and the docking are extremely delicate and precise translational and of course, rotational manoeuvres.

In addition, precise relative position and velocity state estimates are required. The Attitude Control System is a system in charge of maintaining the space orientation of the vehicle. It can do so by means of the passive or active methods. The passive and active methods can sometimes be combined. The active methods use actuators to adjust the satellite attitude in an automated manner, for which they usually need the knowledge of the current attitude. There exist several different actuators for the control of the satellite attitude. However, the attitude control through the thrusters receives increasing attention for some space vehicles. Thrusters require fuel reservoirs which take up a lot of space and have a limited operational time. Various three-axis attitude control approaches are used on the space vehicles with actuators, and more are being researched. Depending on the space vehicles mission and requirements some control strategies perform better than others. The approach strategy used by the generic model of attitude to the docking port located the along the V-bar direction is selected to evaluate the LQR regulator proposed in this work. Autonomous

proximity operations are characterized by controlling a chaser vehicle about predetermined reference trajectory toward the docking part of the target. The system comprises an independent guidance function (CW) and controls function. The last is to provide the control forces that will be executed by the thrusters of the chaser vehicle in order to track the reference trajectory. The attitude manoeuvre is determined through the integration of the state-dependent Riccati equations (SDRE) control formulated using the nonlinear relative dynamics with the weight matrices adjusted at the steady state condition. We intend to maximize the efficiency of the spacecraft which leads to minimizing the error introduced by  $J_2$  contribution in the wrong direction. A simplified CW linear model and a non-linear dynamic model including the effects due to the non-spherical nature of the Earth ( $J_2$  effects) will be developed to be used in designing of the control law. The goal of this paper is to examine the  $J_2$  effects to assess the capability of thrusters as a mean of control. The approach strategy used by the ATV generic model of translating to the docking port located along the V-bar direction is selected to evaluate the LQR regulator proposed in this paper. The efforts to settle some experimental facilities to support the autonomous rendezvous and docking demonstration and testing [1], [2], [3], [4] are well-known. This paper describes a part of R&D algorithm (orbit position) under Matlab/Simulink environment providing all features for analysis and simulations. This paper proposes an autonomous algorithm for the docking of a chaser with a target. The approach strategy is composed of some “V-bar or R-bar approaches” and a circumnavigation maneuver in the closing transfer phase, some periods of station keeping and a “straight line V-bar” approach to the docking port (figure 1).

The guidance and control functions are independently designed and are then integrated in the form of LQR-type control. The ranges of the proximity operations, the closed transfer proposed are extracted from [5, 6, 7, 8] and showed in fig. 1. This paper considers the scenario beginning at  $S_2$  and ending at  $S_3$  (see fig. 1).

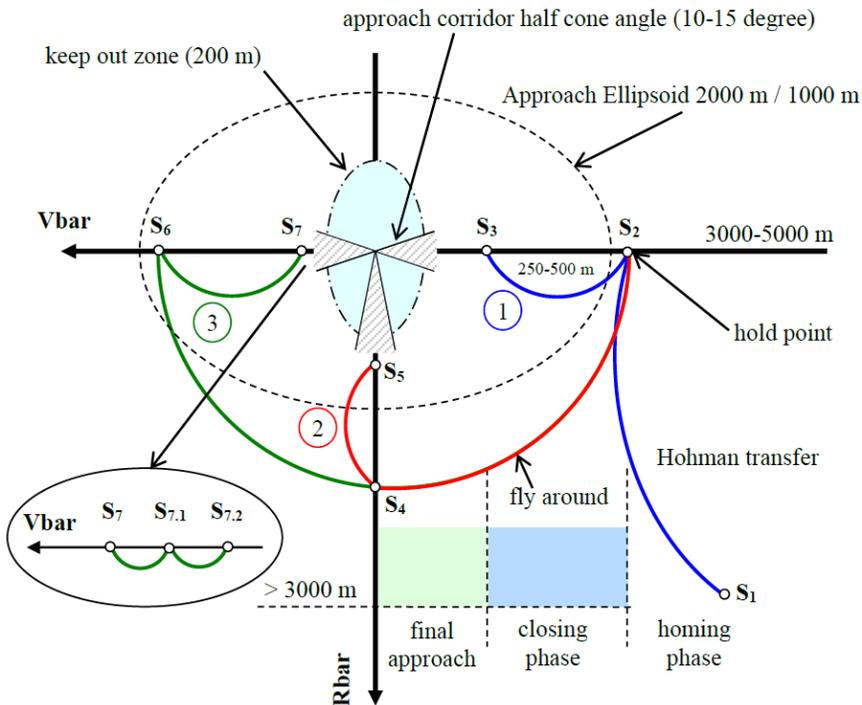


Fig. 1 Proximity operations strategies

(The approach strategy used by the ATV of translating to the docking part located along the V – bar)

## 2. ATTITUDE DYNAMICS MODELING

A nonlinear spacecraft dynamic model including  $J_2$  perturbation is presented to describe the translational motion. The coordinate systems applied here are: local orbital frame centered on the target and chaser and an Earth – Centered Inertial frame [5]. These are shown in fig. 2 below with axes having the following guidance in orbital frame:

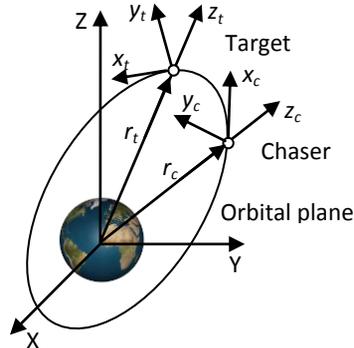


Fig. 2 Chaser and target vehicles orbit. Reference system centred on the target vehicle

$x$  – axis in the same direction and orientation as the orbital velocity vector ( $\bar{V}$ ),  
 $y$  – axis normal to the orbit, with opposite direction of the orbital angular momentum vector ( $\bar{H}$ ),  
 $z$  – axis completes the system, oriented in the radial direction, perpendicular to the plane of horizon, nadir direction ( $\bar{R}$ ).

Initially the attitude dynamic model is a 7 – dimensional system whose state is made of an attitude quaternion  $q$  and an angular velocity vector  $\omega$ . Here the attitude is expressed with respect to  $t$  local orbital reference frame, i.e., the quaternion  $q = [q_1 \ q_2 \ q_3 \ q_4]^T$  represents the rotation from orbital frame to the body fixed frame. The relative angular velocity between the body fixed frame and the orbital reference frame is expressed in body coordinates.

### Attitude representation

The quaternions were introduced by Hamilton in 1843 being the most common attitude parameterization used in space vehicle attitude determination system due to their inherent non – singularity for any rotation. There are multiple ways to represent a rotation between the frames. A number of different representations with less elements have been defined, like Euler parameters and Euler angles.

Another representation, unit quaternions (also called Euler symmetric parameters) are quite popular, for they avoid singularities, require only four parameters and the conversion between them and Direction Cosine Matrix can be done without trigonometric operations.

The unit quaternion is defined in four dimensional vector space,  $q \in R^4$ , and will be denoted as:  $q = [q_1 \ q_2 \ q_3 \ q_4]^T$ .

Nevertheless, one should keep in mind that, under the quaternion representation, four parameters  $q_1, q_2, q_3$  and  $q_4$  are required for representing a three dimensional attitude vector. The connection of the unit quaternion parameters with the rotation vector and amount of rotation around it is given as

$$\begin{aligned}
 q_1 &= e_1 \sin \frac{\alpha}{2} \\
 q_2 &= e_2 \sin \frac{\alpha}{2} \\
 q_3 &= e_3 \sin \frac{\alpha}{2} \\
 q_4 &= \cos \frac{\alpha}{2}
 \end{aligned} \tag{1}$$

The values  $e_1$ ,  $e_2$  and  $e_3$  define the unit vector of rotation, and  $\alpha$  is the angle of rotation. Also the relation  $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$  always holds.

An important operation with the unit quaternions is the relative orientation between two of them. With this operation the error between the desired and the current attitude can be calculated.

### Kinematics

The differential equation of attitude kinematics with quaternion parameters is given by:

$$\dot{q} = \frac{1}{2} \Omega(\omega) q \tag{2}$$

where the  $\Omega$  matrix is defined as:

$$\Omega(\omega) = \begin{vmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{vmatrix} \tag{3}$$

### Rotational motion dynamics

The rotational motion of the chaser is expressed in the body-fixed frame using the well-known Euler's equations of motion. Like the perturbing accelerations in relative translational dynamics, rotational dynamics also experience disturbing torques such as the torque due to gravity-gradient torque.

The effects of the gravitational field are not uniform over an arbitrarily shaped body in space, creating a gravitational torque about the body center of mass. The nonlinear attitude model proposed for simulations has the body axes located along the principal axes of inertia, considering the rotaty frame fixed to the body. In addition the kinematic equations will be described by means of quaternions.

The chaser body will be considered rigid (fig. 3), thus the Euler moment Equation in body fixed coordinates will be used:

$$\dot{\omega} = -I^{-1} \omega \times I \omega + I^{-1} (M_c + M_g) \tag{4}$$

where  $M_g$  is the gravity gradient torque and  $M_u$  is the control moment given by:

$$M_c = \begin{vmatrix} u_x y_c - u_y z_c \\ u_y z_c - u_z x_c \\ u_z x_c - u_x y_c \end{vmatrix} \tag{5}$$

with  $u_x, u_y, u_z$  - that forces exerted by the chaser thrusters. For a chaser with diagonal inertia matrix:

$$I = \begin{vmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{vmatrix} \tag{6}$$

the gravity gradient contribution is given by:

$$M_g = 6\omega_0^2 \begin{vmatrix} (I_z - I_y) \cdot (q_2 q_3 + q_1 q_4) \cdot (-q_1^2 - q_2^2 + q_3^2 + q_4^2) \\ (I_x - I_z) \cdot (q_1 q_3 + q_2 q_4) \cdot (-q_1^2 - q_2^2 + q_3^2 + q_4^2) \\ (I_y - I_x) \cdot (q_2 q_3 - q_1 q_4) \cdot (q_1 q_3 - q_2 q_4) \end{vmatrix} \tag{7}$$

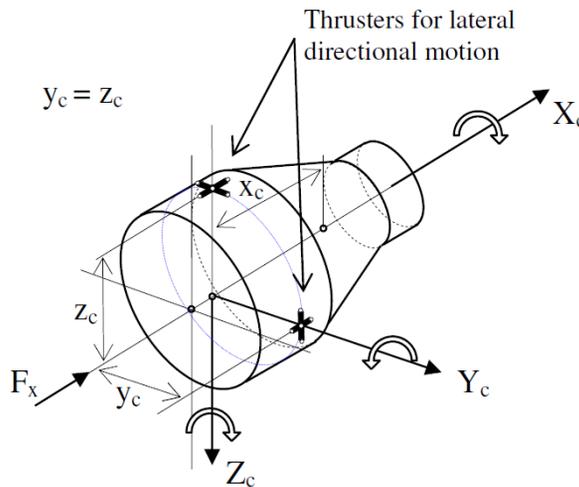


Fig. 3 Generic model proposed for attitude dynamics with controls

### 3. LINEARIZED ATTITUDE DYNAMICS MODELING

In order to apply the LQR / SDRE method to a chaser attitude control, its model needs to linearize. Linearization of the attitude dynamic equation will be done by approximation of rotations with small angles around an imposed attitude (the body – fixed frame coincides with local orbital frame). These hypotheses lead to:

$$q_0 = \begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{vmatrix} \cong \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix} \tag{8}$$

and

$$M_g = 6\omega_0^2 \begin{vmatrix} (I_z - I_y) \cdot q_1 \\ -(I_x - I_z) \cdot q_2 \\ 0 \end{vmatrix} \quad (9)$$

For small perturbation the state vector can be reduced to six variables by dropping the kinematic equation  $\dot{q}_4$  and substituting it with the unit norm constraint. This is convenient to write the state variables as:

$$x = [q_1 \quad q_2 \quad q_3 \quad \omega_x \quad \omega_y \quad \omega_z]^T \quad (10)$$

The linearized attitude dynamics model looks like:

$$\dot{x} = A x + B u \quad (11)$$

where:

$$A = \begin{vmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -8 \frac{I_y - I_z}{I_x} \omega_0^2 & 0 & 0 & 0 & 0 & \left(1 - \frac{I_y - I_z}{I_x}\right) \omega_0 \\ 0 & 6 \frac{I_z - I_x}{I_y} \omega_0^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 \frac{I_x - I_y}{I_z} \omega_0^2 & -\left(1 + \frac{I_x - I_y}{I_z}\right) \omega_0 & 0 & 0 \end{vmatrix} \quad (12)$$

and

$$B = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{y_c}{I_x} & -\frac{z_c}{I_x} & 0 \\ 0 & \frac{z_c}{I_y} & -\frac{x_c}{I_y} \\ -\frac{y_c}{I_z} & 0 & \frac{x_c}{I_z} \end{vmatrix} \quad (13)$$

#### 4. SDRE FORMULATION FOR ATTITUDE RELATIVE DYNAMICS

The SDRE strategies provide an effective and systematic algorithm to synthesize nonlinear feedback control by allowing nonlinearities in the system state [6, 7]. It is a simple extension of the constant valued ARE used to find the optimal feedback control in the Linear Quadratic Regulator problem (LQR).

The procedure of generating the SDRE controller was presented for the first time for the nonlinear optimal regulator problem by Banks and Mhana. Shama and Cloutier [8] studied the nonuniqueness of the state-dependent representation.

Consider the deterministic, infinity-horizon nonlinear optimal regulation problem, where the system is full-state observable, autonomous, nonlinear in state and affine in the input, represented in the form [7]:

$$\dot{x}(t) = f(x) + B(x)u(t) \quad (14)$$

where  $x \in R^n$  is the state vector and  $u \in R^m$  is the input vector.

Through the state-dependent coefficient (SDC) factorization, system designers can represent the nonlinear equations of motion as linear structures with state-dependent coefficients.

Then, the LQR technique can be applied to the state-dependent state-space equations. Thus, the following procedure is similar to the LQR method except that all matrices may depend on the state.

Based on this concept the state-space equations for nonlinear system described in (14) can be expressed as a linear state-space equation using direct SDC factorization as:

$$\dot{x}(t) = A(x) \cdot x + B(x) \cdot u \quad (15)$$

where the factorization for  $f(x) = A(x) \cdot x$  with  $A(x) \in R^{n \times n}$  is possible if and only if  $f(0) = 0$  and  $f(x)$  is continuously differentiable.

The state-dependent dynamic matrix  $A(x)$  is non-unique where  $x > 1$  [2]. The optimal control problem mentioned above is to find a state-feedback control law  $u(x)$  which minimizes the cost functional as:

$$J = \frac{1}{2} \int_0^{\infty} \left( \left[ x^T(t) - x_r^T(t) \right] Q(x) \left[ x^T(t) - x_r^T(t) \right] + u^T(x) R(x) u(x) \right) dt \quad (16)$$

where  $x_r(t)$  is the reference or desired state vector provided by the guidance scheme based on CW state transition matrix (9) and the approach strategy and  $Q(x) \in R^{n \times n}$  is the state weighting matrix satisfying  $Q(x) = Q^T(x) \geq 0$ ,  $R(x) \in R^{m \times m}$  is the input weighting matrix satisfying  $R(x) = R^T(x) > 0$ .

It should be noted that  $Q(x)$  and  $R(x)$  are not only allowed to be constant, but can also be varied as function of states.

Also, it is assumed that  $f(0) = 0$  and  $B(x) \neq 0$ . For a valid solution to the SDRE, the pair  $\{A(x), B(x)\}$  must be point wise stabilizable in the linear sense for all  $x$  in the domain of interests. This SDRE approach for obtaining a suboptimal solution to the nonlinear problem can be summarized with the following steps:

- bring the nonlinear equation into SDC form in equation (15),
- solve the SDRE

$$P(x)A(X) + A^T(x)P(x) - P(x)BR^{-1}(x)B^T(x)P(x) + Q = 0 \quad (17)$$

- the nonlinear feedback controller law becomes:

$$u(x) = -R^{-1}(x)B^T P(x)[x(t) - x_r(t)] \quad (18)$$

- the resulting SDRE controlled trajectory becomes the solution of the quasi – linear

$$\dot{x}(t) = f(x(t)) - BK(x)[x(t) - x_r(t)]$$

- closed loop dynamics:

where the state feedback gain for minimizing equation (15) is:

$$K(x) = R^{-1}(x)B^T P(x) \quad (19)$$

Through the SDC parameterization the nonlinear equation (13) is transformed to the linear – like state – space from equation (14).

The system matrix  $A(x)$  is then:

$$A(x) = \begin{vmatrix} A_{11}(\omega) & A_{12}(q) \\ A_{21}(q) & A_{22}(q, \omega) \end{vmatrix} \quad (20)$$

where:

$$A_{11}(\omega) = \frac{1}{2} \begin{vmatrix} 0 & \omega_z & \omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{vmatrix}, \quad A_{12}(q) = \frac{1}{2} q_4 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A_{21}(q) = -\omega_0^2 I^{-1} \text{skew}(s) I \begin{vmatrix} 2q_2 & 0 & 2q_4 \\ q_1(r-2) & q_2(r-2) & q_3(r-2) \\ -2q_4 & 2q_3 & 0 \end{vmatrix} + \quad (21)$$

$$3\omega_0^2 I^{-1} \begin{vmatrix} -2(I_y - I_z)q_4 & 0 & -2(I_y - I_z)q_2 \\ -2(I_z - I_x)q_3 & 2(I_z - I_x)q_4 & 0 \\ 0 & -4(I_x - I_y)q_1q_4 & 4(I_x - I_y)q_1^2 \end{vmatrix}$$

$$r = \frac{1}{1 - q_4^2}$$

$$A_{22}(q, \omega) = \omega_0 \text{skew}(s) - I^{-1} \text{skew}(p) I - \omega_0 I^{-1} \text{skew}(t) + \omega_0 I^{-1} \text{skew}(s) I$$

$$s = \begin{vmatrix} 2q_3 \\ 1 \\ -2q_1 \end{vmatrix}, \quad p = \begin{vmatrix} \omega_x \\ \omega_y \\ \omega_z \end{vmatrix}, \quad t = \begin{vmatrix} 2q_3 I_x \\ I_y \\ -2q_1 I_z \end{vmatrix} \quad (22)$$

with:

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad skew(a) = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

The control matrix  $B$  is the same as in linearized dynamics. The input controls are independent of the system state, though they still depend on the Earth gravitational field.

The state weight matrix for the performance index in equation (16) is given by:

$$Q = 10^p \cdot I(6 \times 6) \tag{23}$$

and the control weight matrix is given by:

$$R = 10^q \cdot I(3 \times 3) \tag{24}$$

The properly chosen initial matrices without causing the thruster saturation are required. If larger  $Q$  and smaller  $R$  weight matrices are chosen at the initial time, the controller may become saturated resulting in control commands that cannot be executed by the thruster. When the weight matrices are adjusted at steady state, the control forces are modified and tracking is then reduced to the desired value without thruster saturation. This adjustment of the weight matrices is very important in order to generate suitable control forces [9]. The implementation of the closed – loop control as the block diagram of the SDRE control is depicted in figure 4.

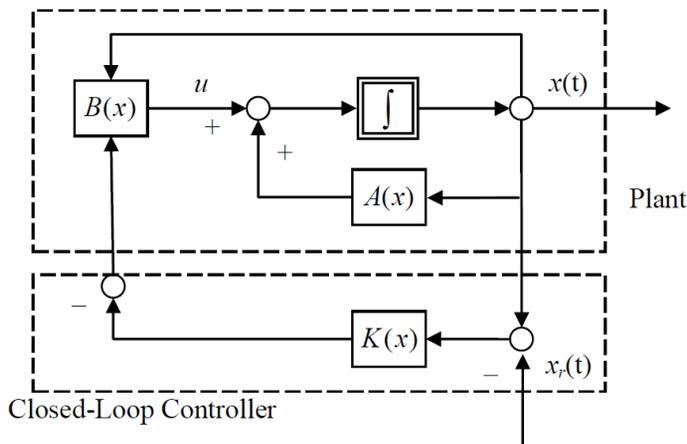


Fig. 4 Block diagram of the SDRE Control

SDRE method for the infinite time nonlinear regulator problem is locally asymptotically stable and locally asymptotically optimal [8].

### 5. SIMULATION RESULTS

To validate the controller the numerical tests have been applied for a rendezvous maneuver with the target vehicle along the V-bar. The initial values for the chaser are:  $m_c=6000$  kg,  $t=2700$  sec,  $x_0=-3250$  m,  $y_0=0$  m,  $z_0=0$  m,  $h_t=400$  km. The maximum value of every thruster was estimated at approx. 400 N along the longitudinal axes.

This work analyzed the controllers developed here in a proximity operation in which  $S_2 - S_3$ ,  $S_3 - S_{31}$ ,  $S_{31} - S_{32}$  and  $S_{32} - S_{33}$ , were studied.

Using the CW guidance scheme the reference trajectory is autonomously commanded to the controllers.

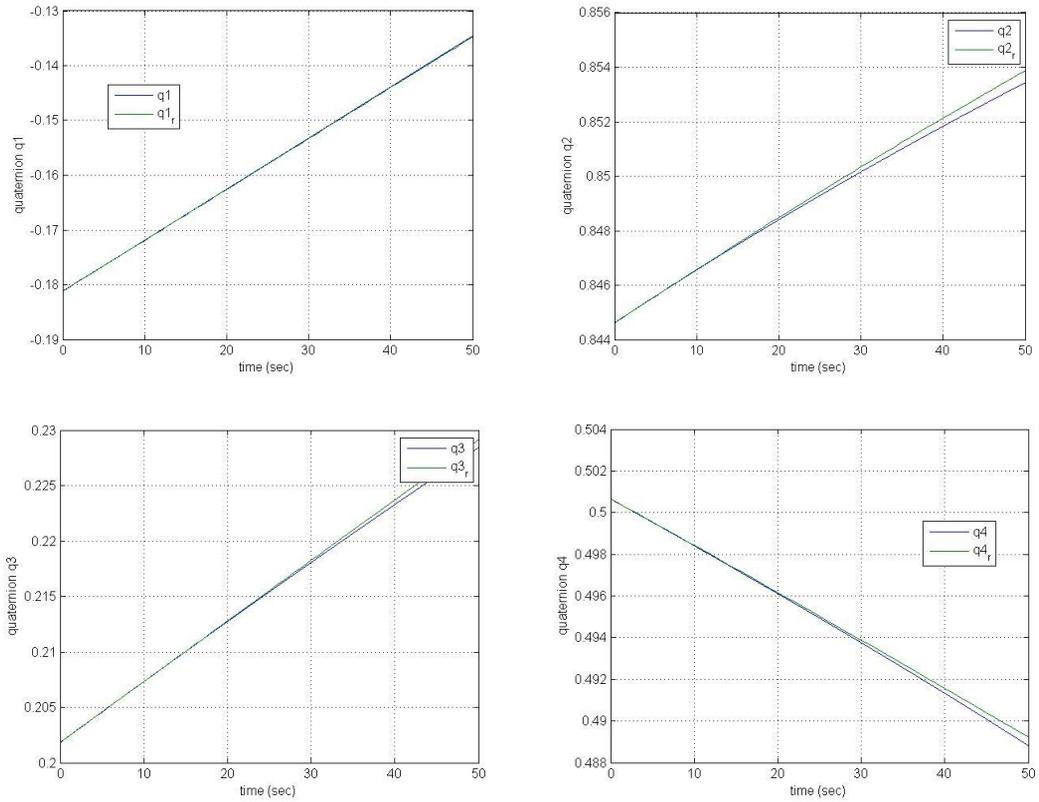


Fig. 5 SDRE Control – Target and Chaser Quaternion Histories

Additional consideration must be given to the attitude kinematic equation because a quaternion must always have a unit norm.

A constraint violation can be eluded by using the Euler – Rodriguez parameters angle (fig. 7).

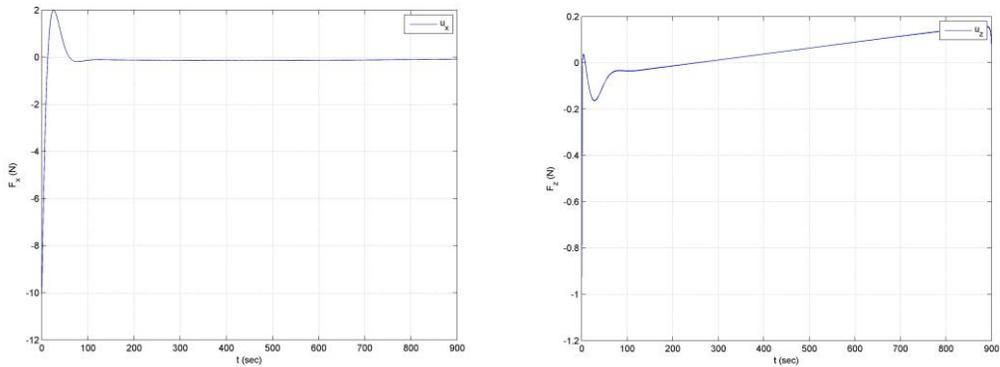


Fig. 6 Control force history

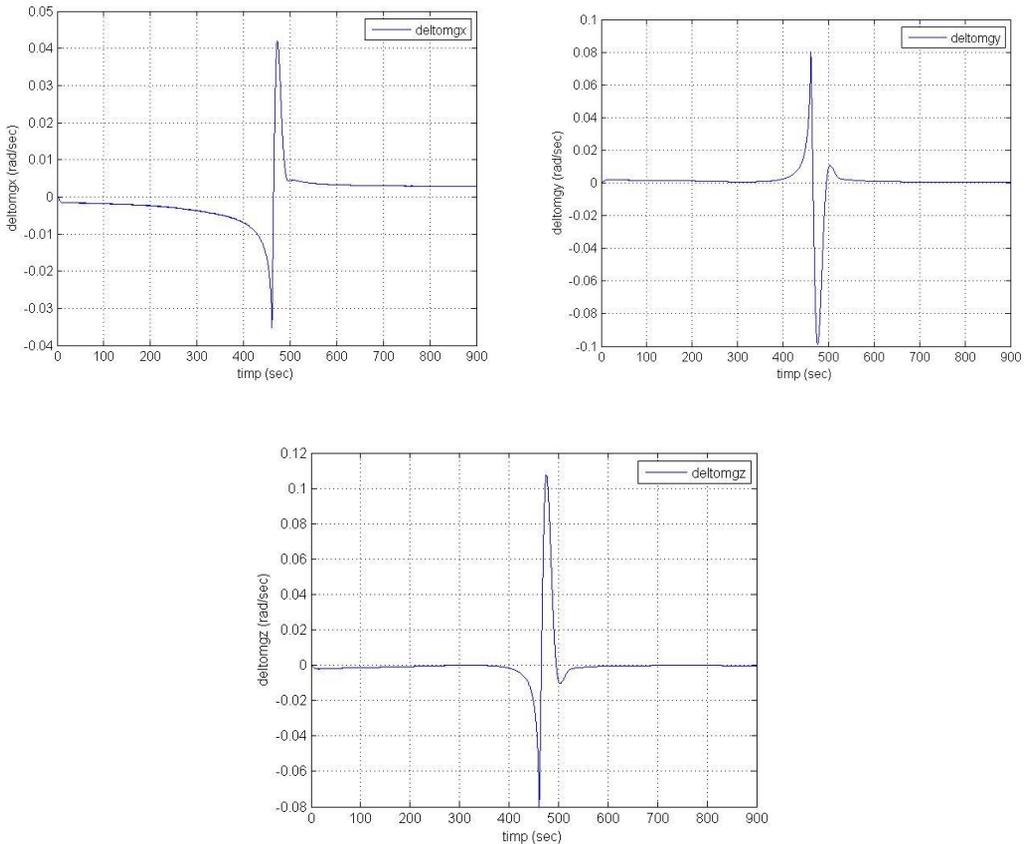


Fig. 7 SDRE control attitude rates error

Figure 7 shows the attitude rates error during the entire simulation. The approach trajectory using SDRE control scheme shows a constant straight line trajectory where the error is less than a prescribed value on the R-bar axes. SDRE controller performs slightly worse for longer time (fig. 5) [9, 10].

The control forces histories produced by the controller are shown in figure 6. The figure also shows that the additional control forces that resulted from the adjustment of weight matrices in SDRE controls increased impulsively before the final straight line approach.

## 6. CONCLUSIONS

The work proposed a control code to be used in docking of spacecraft. This algorithm is composed of an independent guidance function, a control function and a navigation function for the control of the position and attitude control in autonomous missions.

The guidance, navigation and control functions are independently designed and are then integrated in the form of linear LQR-type control and LQG -type controls.

SDRE controller uses the nonlinear model, which is an improvement over LQR when the state is far from the goal state. SDRE is able to quickly reach the region near the desired attitude. However, SDRE showed somewhat worse performance for a close tracking of the reference point. A problem related to the controllability of the LQR has also arisen for the SDRE as well [9]. The attitude manoeuvres are determined through the integration of the

state-dependent Riccati equation control formulated using the nonlinear relative motion dynamics with the weight matrices adjusted at the steady state condition.

The results from simulations are presented to show the impact of  $J_2$  perturbation on the flight path.

The results of this analysis indicate that a coupled orbital and attitude control system using an existing orbital control law and a further attitude control law for the use in spacecraft flying mission ground-based simulation are needed.

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