Classical and Modern gain estimation approach of PID controller for the pitch control of the RCTA aircraft

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Abstract: *In the current age of globalization, the development of autopilots is superficial as the standard of living is improved through aerial surveillance, defense applications, and door- to-door transportation. To design the PID controller, a mathematical model is created to address the system identification for estimating longitudinal derivatives and to study the handling qualities of aircraft in order to improve piloting performance. This paper exhibits a comparative assessment between the classical closed-loop PID tuning methods like ZN, Modified ZN, Tyreus-luyben, Astrom- Hagglund and the modern control techniques like pole placement, LQR for the pitch controller design. The simulation results are displayed in the time-domain, which demonstrates the effectiveness of the approach used to design a robust controller.*

Key Words: *System Identification, ZN, Modified ZN, Tyreus- luyben, Astrom- Hagglund, Pole placement, LQR*

Nomenclature

a_x, a_y, a_z	*Linear accelerations along x, y, z body axes (m/s^2)*
p, q, r	*roll, pitch, yaw rates (rad-s-1)*
δ_a, δ_e, δ_r	*aileron, elevator, rudder deflection angle*
Φ, θ, ψ	*angle of roll, pitch and yaw (deg)*
α	*angle of attack*
β	*slide slip angle*
V	*Airspeed (m/s)*
u, v, w	*Longitudinal, lateral, and vertical airspeed*
R	*Measurement co-variance Matrix*
J	*Cost Function*
Θ	*vector of unknown parameters*
T	*Thrust*
C_{D_0}	*Coefficient of drag force at zero angle of attack*
C_{D_α}	*Change in Coefficient of drag force with change in angle of attack*
$C_{D_{\delta e}}$	*Change in Coefficient of drag force with change in elevator deflection angle*
C_{L_a}	*Change in Coefficient of lift force with change in angle of attack*
C_{L_0}	*Coefficient of lift force at zero angle of attack*

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C_{L_q}	*Change in Coefficient of lift force with change in pitch rate*
$C_{L_{\delta e}}$	*Change in Coefficient of lift force with change in elevator deflection angle*
C_{m_0}	*Coefficient of pitching moment at zero angle of attack*
C_{m_α}	*Change in Coefficient of pitching moment with change in angle of attack*
C_{m_q}	*Change in Coefficient of pitching moment with change in pitch rate*
$C_{m_{\delta e}}$	*Change in Coefficient of pitching moment with change in elevator deflection angle*
u_o	*Perturbed velocity along X*
ρ	*Air density*
M_q	*Non-dimensional variation of pitching moment with pitch rate*
$M_{\dot{\alpha}}$	*Non-dimensional variation of pitching moment with change in angle of attack*
Z_α	*Non-dimensional variation of Z force with angle of attack*
M_α	*Non-dimensional variation of pitching moment with angle of attack*
$w_{n_{sp}}$	*Frequency of short period*
ξ_{sp}	*Damping ratio of short period*
PID	*Proportional Integral Derivative*
LQR	*Linear Quadratic Regulator*
OEM	*Output Error Method*
ZN	*Ziegler Nicholas*
RCTA	*Research cum trainer aircraft*
UAV	*Unmanned Aerial Vehicle*

1. INTRODUCTION

The rapid development of flying machines like multi-rotor drones and hybrid aircraft is booming. In this context the automatic control design has played a vital role of a catalyst by boosting the interest of researchers in area of application e.g. reconnaissance missions, aerial photography, terrain surveillance, flying accidental military troops etc. An autopilot lessens the pilot workload during various flight regimes at different altitudes and Mach numbers. It also handles unfavourable weather conditions and provides artificial stability. The prerequisites of controller design include in-depth knowledge of control theory, parameter estimation of provided aircraft model at different altitude, Mach numbers [1], [2], and flying handling quality. Khoi Nguyen Dang researched on optimized design of the attitude controller of quadrotor using a system Identification approach. LQR (Linear quadratic regulator) theory was used to design the linear quality servo's to improve the performance characteristics[3]. Valderrama designed an aircraft pitch controller to improve the stability and performance of the UAV(Unmanned aerial vehicle) [4]. ZN (Ziegler Nicholas) methodology was adopted to tune the PID (Proportional Integral Derivative) controller. Various time domain characteristics to study performance of controller are discussed. G. Sudha [5] optimizes various tuning methods for the PID such as Ziegler Nicholas, Tyreus-Luyben, Astrom- Hagglund but the simulation results tuned by ZN method shows the optimum result. The Pitch controller of F-16 aircraft is designed using LQR and linear feedback approach. Pole placement technique is used to determine the value of gain. Simulink block diagram of LQR and linear feedback are validated with the aircraft pitch control system by [6]. The optimal control methodology is adopted to estimate the parameters of the dynamic model by using the Cost function 'J' which is derived using OEM(Output error method) by [7] Gottlicher. The present paper focuses on designing a PID controller for the pitch control of Hansa-III using various closed-loop tuning techniques- ZN, Modified ZN, Tyreus-luyben, Astrom-Hagglund, pole-placement, and LQR based on the study of the specialized literature. The maximum likelihood estimation method was implemented to estimate the stability and control derivative of 4 - seater canard aircraft, firefly. This statistical method has benefits to measure both process and measurement noise as stated by Kim [8]. This technique is widely applicable in time-domain analysis to estimate derivatives using the vehicle flight data. D. H. Shim [9] researched by developing the Non-linear transport aircraft Simulink model applying M. L and EKF. Mathematical modeling is developed to design LQR Controller having optimal weighing matrices that be used for tracking aircraft trajectory. This approach can handle the noise and make the system robust. Maximum Likelihood is a widely used statistical technique to minimize the error and make the system dynamically stable as suggested by the researcher [10]. It uses linearized perturbation state equation to study the dynamic response characteristics of longitudinal motion to assess flight handling [11]. One of the most crucial problem in system identification is data compatibility as explained by Grauer [12]. Data compatibility is a part of parameter estimation and is used to check the data accuracy by making bias free and error free flight data. The transfer function of the pitch controller is calculated using longitudinal state equations. The Simulink model is constructed to perform simulations and optimize the performance parameters of a PID Controller.

2. IDENTIFICATION AND MODELLING OF HANSA-III

Hansa-III is two-seater trainer aircraft (RCTA) manufactured by NAL, Bangalore, India. Three designs were developed by NAL in which Hansa-II was built as prototype and Hansa-III was finalized for production. In order to fetch the flight data, multi-variant sensors are instrumented in the aircraft for flight data acquisition. The aircraft structure is fully composite having a low wing configuration with tricycle landing gear arrangement. It consists of a Rotax-914 F3 engine coupled with Hoffmann propeller [10] as shown in fig. 1.

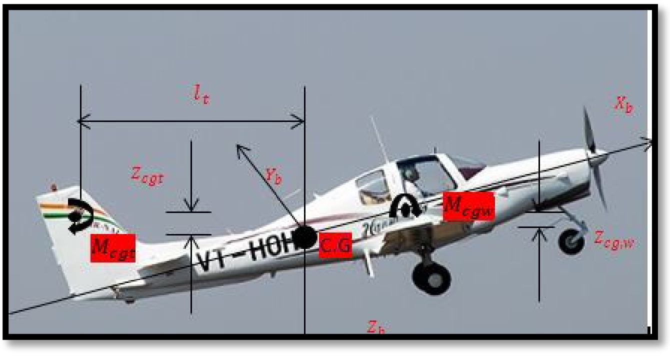

Fig. 1 A three-view sketch of Hansa-III research aircraft

The System Identification is an indispensable tool that identifies parameters of the physical system based on observations by developing a mathematical model of the dynamic system. Zadeh stated, “Identification is a tool to identify systems on basis of Inputs, outputs and test condition” [13] as explained in fig. 2 The unified concept stated by the German Aerospace Centre (DLR) of Quad M to define five elements is a key to define the System Identification [14], [15].

1. **Maneuverers**: The control inputs in terms of voltage (3-2-1-1) input, doublet, and pulse were fed, in addition converted to respective motion and control variables using an appropriate calibration chart to experience the despite output.

2. **Measurement**: A high-quality sensor measures the process and measurement noise of the system, known as a data compatibility check.

3. **Model**: An algorithm of differential equations derived from Newtonian law of motion in terms of acceleration, state and control variable is used for the flight vehicle system Identification [16].
4. **Methodology**: The procedure adopted on the Input/output interface to acquire desired output of the flight vehicle.

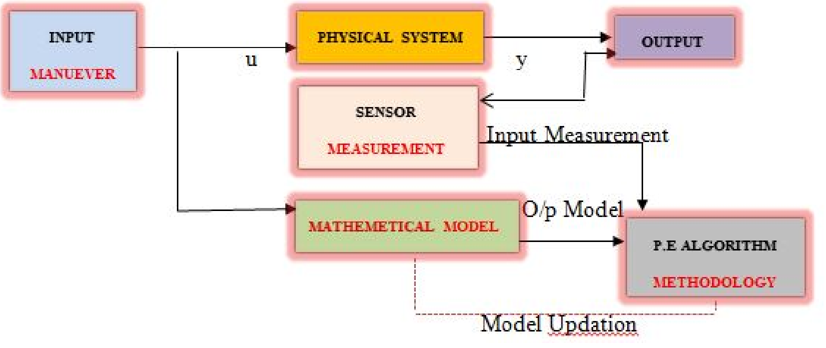

Fig. 2 Concept of System Identification

Step I. The raw flight data at a low angle of attack is recorded in terms of voltage (3-2-1-1) input from flight laboratory of Indian Institute of Technology, Kanpur and converted to respective motion. The raw data are calibrated in form of V, p, q, r θ, ψ, δ_e, δ_a, δ_r, φ, a_x, a_y, a_z, α, and β to locate the sensor position [10].
Step II. Recorded data consist of systematic errors like scale factor, bias factor, time delay, and zero shifts cause data incompatible so sensors are required to avoid measurement errors. The flight path reconstruction, also known as a data compatibility check, ensures the data accuracy fetched from the data acquisition system and forms the model consistent and error-free.
Step III. Mathematical model of aircraft is formulated by the algorithm of equations of motion [17]. The following set of state equations, observation equations estimates aircraft transfer function for designing PID Controller.
Governing Longitudinal Equation in Wind Axis [18], [19]

$$\alpha_{11}\dot{v} = -\left\{\frac{\bar{q}.s}{m.}\right\} C_D + g\,sin(\alpha-\theta) + \left\{\frac{T}{m.}\right\}\cos\alpha \tag{1}$$

$$\dot{\alpha} = -\left\{\frac{\bar{q}.s}{m.v}\right\} C_L + q + \frac{g}{V} cos(\alpha-\theta) - \left\{\frac{T}{m.v}\right\}\sin\alpha \tag{2}$$

$$\dot{\theta} = q \tag{3}$$

$$\dot{q} = \left(\frac{\bar{q}.s.c}{I_y}\right).C_m + \left\{\frac{T}{I_{YY}}\right\} l_{tz} \tag{4}$$

In order to analyse the aircraft dynamics, the aircraft is modelled in terms of mathematical equations as aerodynamic stability and control derivatives shown below

$$C_L=\{C_{L_0} + C_{L_\alpha}.\alpha + C_{L_q}.\frac{q\bar{c}}{2U_1} + C_{L_{\delta_e}}.\delta_e\} \tag{5}$$

$$C_D=\left\{C_{D_0} + C_{D_\alpha}.\alpha + C_{D_q}.\frac{q\bar{c}}{2U_1} + C_{D_{\delta_e}}.\delta_e\right\} \tag{6}$$

$$C_m=\left\{ C_{m_0} + C_{m_\alpha}.\alpha + C_{m_q}.\frac{q\bar{c}}{2U_1} + C_{m_{\delta_e}}.\delta_e \right\} \tag{7}$$

A few assumptions are required before modelling the system.

1. At cruise state; Thrust setting angle = 0 Flight-path Y = constant.
2. Elevator control input excites short period dynamics; hence flight velocity is constant.

Referring longitudinal State Equation (2-4) and applying following assumptions mentioned in above section as

$$\dot{\alpha} = -\left\{\frac{\bar{q}.s}{m.v}\right\} C_L + q \tag{8}$$

$$\dot{\theta} = q \tag{9}$$

$$\dot{q} = \left(\frac{\bar{q}.s.c}{I_y}\right).C_m \tag{10}$$

Applying equation (5-7) to estimate non-dimensional derivatives as longitudinal stability and control predominantly excite short period mode, is expressed as

$$\dot{\alpha} = q - \frac{\rho V S_w}{2m}\left\{C_{L_0} + C_{L_\alpha}.\alpha + C_{L_q}.\frac{q\bar{c}}{2U_1} + C_{L_{\delta_e}}.\delta_e\right\} \tag{11}$$

$$\dot{\theta} = q \tag{12}$$

$$\dot{q} = \frac{\rho V^2 S_w \bar{c}}{2I_y}\left\{C_{m_0} + C_{m_\alpha}.\alpha + C_{m_q}.\frac{q\bar{c}}{2U_1} + C_{m_{\delta_e}}.\delta_e\right\} \tag{13}$$

Table 1: Geometrical parameters[20]

Geometrical Parameters	Value	Geometrical Parameters	Value
Wing		**Horizontal Tail**	
Planform area	12.47(m^2)	Planform area	2.04(m^2)
Aspect ratio	8.8	Aspect ratio	6.35
MAC	1.21(m)	MAC	0.59(m)
Root Chord	1.3(m)	Root Chord	0.78(m)
Tip Chord	0.8	Tip Chord	0.354(m)
Taper ratio	6(deg)	Taper ratio	0.454
Aircraft		**Aerodynamic derivatives**	
Aircraft span	10.47(m)	$(C_{L_{\alpha,w}})_{ss}$	4.5
Mass	750(kg)	$(C_{L_{\alpha_t}})_{ss}$	1.48
Velocity	36(m/s)	$(C_{m_{\alpha,f}})_{ss}$	0.3
Moment of Inertia I_Y	907(kg-m^2)	$(C_{L_{\alpha_t}} \frac{d\epsilon}{d\alpha})_{ss}$	0.22
Moment arm	3.624(m)		
Density	0.96(kg/m^3)		
Moment of Inertia I_Y	925(kg-m^2)		
Moment arm	3.624(m)		

Step IV. Maximum likelihood is enforced for longitudinal parameter estimation of the dynamic model of Hansa-III using real flight data [10]. The statistical technique is applied in the time domain to estimate the stability and control derivatives by minimizing the cost function J [14], [21]

$$J(\Theta,R) = L(z \mid \Theta,R) = ½ \sum_{k=1}^{N}[z(tk) - y(tk)]^{R-1}[\, z(tk) - y(tk)] + N/2 \ln [\det(R)] + Nn_y/2 \ln(2\prod)$$

The unknown parameter vector Θ determines the value of the non-dimensional longitudinal derivatives given

$$\Theta = [C_{L_0}\, C_{L_a} C_{L_q} C_{L_{\delta e}} C_{D_0}\;\; C_{D_\alpha} C_{D_{\delta e}} C_{m_0} C_{m_\alpha} C_{m_q} C_{m_{\delta e}}\,]^T \tag{14}$$

Table 2: Longitudinal Parameters Value using ML Technique[22]

S. No	Derivatives	Value
1.	C_{L_0}	0.2254
2.	C_{L_α}	6.45
3.	C_{L_q}	37.2
4.	$C_{L_{\delta_e}}$	0.0196
5.	C_{m_0}	0.078
6.	C_{m_α}	-0.4259
7.	C_{m_q}	-11.61
8.	$C_{m_{\delta_e}}$	-0.8665

The simplifying longitudinal state equation by substituting geometrical and longitudinal parameters as presented in Table 1, 2 is represented as

$$\dot{\alpha} = 0.8207 \,.\, q \; - \; 1.851 \,.\, \alpha + \; 0.00562 \,.\, \delta_e - 0.0646 \tag{15}$$

$$\dot{q} = -\, 2.01 \,.\, q - \; 4.403 \,.\, \alpha - 8.95 \,.\, \delta_e + 0.806 \tag{16}$$

$$\dot{\theta} = q \tag{17}$$

The generalized State equation in Matrix form can be written as:

$$\dot{x} = \mathrm{A}x + \mathrm{B}u \tag{18}$$

$$\mathrm{y} = \mathrm{C}x + \mathrm{D}u \tag{19}$$

The state space matrices A, B, C, and D from equation (18-19) are compared with equation (15-17) to define the plant matrix A, the control matrix B, the output matrix C, and the null matrix D as reflected in equation (20-21)

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -1.851 & 0.8207 & 0 \\ -4.403 & -2.01 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -0.0056 & -8.95 \\ -0.0646 & -0.806 \\ 0 & 0 \end{bmatrix} [\delta] \tag{20}$$

$$[\theta] = [0 \quad 0 \quad 1] \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + [0][\delta] \tag{21}$$

The flight control designer has the formidable task to design the controller of an airplane of good dynamic qualities as flying handling qualities are directly linked to the dynamic response of the aircraft [11]. The damping and frequency of both short period and long period plays a vital role while studying the pilot handling hence, an approximation of short period mode can be obtained by dropping the X-force equation and reducing the state matrix.

$$\mathrm{A} = \begin{bmatrix} -1.851 & 0.8207 \\ -4.403 & -2.01 \end{bmatrix}.$$

The eigenvalues of the state matrix A can be obtained by solving

$$|\lambda I - A| = 0 \tag{22}$$

The characteristic equation for the determinant is

$$\lambda^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right)\boldsymbol{\lambda} + M_q \frac{Z_\alpha}{u_0} - M_\alpha = 0 \tag{23}$$

Short period roots in term of damping and natural frequency are calculated using equation (22-23)

$$w_{n_{sp}} = \sqrt{\frac{Z_\alpha M_q}{u_0} - M_\alpha} = \sqrt{7.333} = 2.70,\ f_{n_{sp}} = 0.429 \quad (24)$$

$$\xi_{sp} = -\frac{\left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{u_0}\right)}{2w_{n_{sp}}} = 0.71 \quad (25)$$

The damping and natural frequency of short-period mode can be determined in terms of derivatives. The flying handling quality of an airplane can be defined by stability and control characteristics. The handling quality is experienced by pilot depending on the category and class of aircraft. The flight phase is classified into three categories A, B, and C as displayed in Table 3. The information mentioned in table3 illustrate that Hansa-III is B category, Class-I Aircraft [17]. The value of ξ and f_n calculated using equation (22-23) provides information about handling quality as per Cooper-Harper Scale rating as discussed in fig. 3

Table3: Short period mode flying quality

	Category A and C		Category B	
Class	ξ_{sp} (min)	ξ_{sp}(max)	ξ_{sp} (min)	ξ_{sp} (max)
I	0.35	1.30	0.3	2.0
II	0.25	2.00	0.2	2.0
III	0.15	-	0.15	-

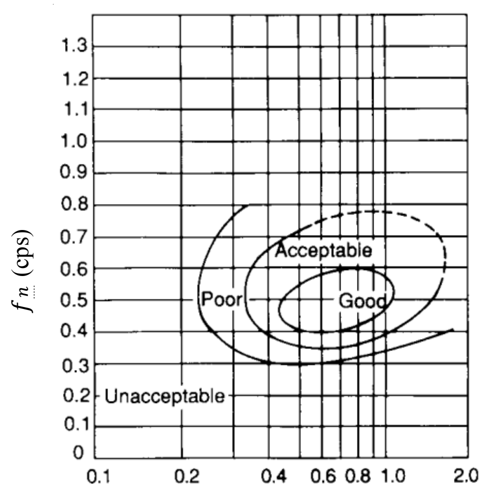

Fig. 3 Relationship between ξ_{sp}, $f_{n_{sp}}$ and level of flying qualities of short period mode [17]

$f_{n_{sp}} = 0.429$; $\xi_{sp} = 0.71$

The value of ξ and f_n demonstrates that Hansa-III has a good flying quality and has a pilot-scale rating of 2. Thus, a minimum pilot effort is required to attain the desired performance. The above explanation provides the information that the automatic controllers can be designed as the aircraft satisfy all criteria to attain stability. The methodology followed to design a PID Controller requires the transfer function. The transfer function of short period mode can be represented by using formulae discussed below

$$\text{T.F} = \left|\frac{C\ Adj\ (Is-A)B}{Is-A}\right| + D \quad (26)$$

The transfer function for the pitch angle to the elevator deflection angle is represented as G(s)

$$G(s) = \frac{\theta(s)}{\delta(s)} = \frac{-\{8.95s+16.5313\}}{s^3+3.861s^2+7.33141s} \tag{27}$$

<u>Open loop Transfer function</u>

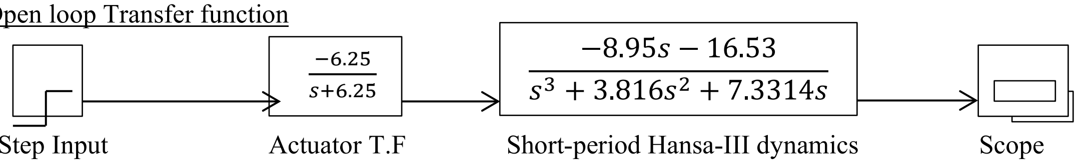

Fig. 4 Open Loop Control system

The Open loop control system as illustrated in Fig. 4 is independent of response in action of control. The transfer function estimated for the pitch angle to the elevator deflection angle is represented as G3(s)

$$G3(s) = \frac{\theta(s)}{\delta(s)} = \frac{-\{8.95s+16.5313\}}{s^3+3.861s^2+7.33141s} \tag{28}$$

<u>Closed loop Transfer function</u>

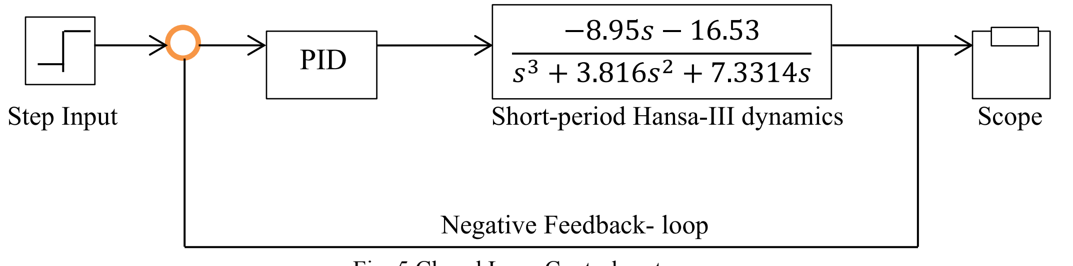

Fig. 5 Closed Loop Control system

The controller of the closed loop control system shown in fig. 5 depends on the output response termed as feedback control system. The transfer function G4(s) is the output response of the input fed to control system

$$G4(s) = \frac{\theta(s)}{\delta(s)} = \frac{\{55.94s+103.3\}}{s^4+10.07s^3+31.18+101.8s+103.3} \tag{29}$$

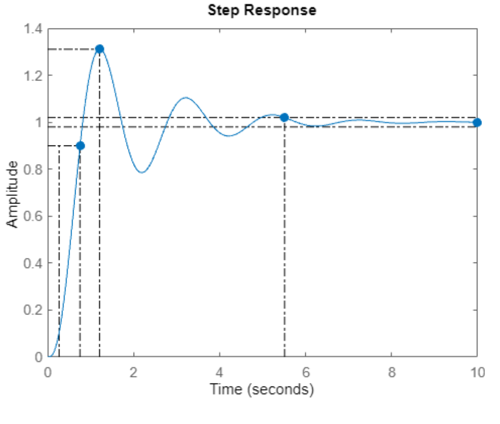

Fig. 6 Closed loop step response

The Step response of closed loop transfer function G4 interpret the system stability after a time-span of 8 seconds as shown in figure 6.

3. PID CONTROLLER

PID stands for proportional, integral, and derivative. This controller boosts the system stability and reduces the steady-state error. It is used in modern industry as automatic process control for the flight control system. The terms P, I, D effectively control the system dynamics by calculating the error between the measured value and desired value. The gain is tuned as per system design requirements of these three terms. The feedback controller is designed to control the desired output accurately operating a PID controller [17]. Control law u(t) is expressed as

$$u(t) = K_P\, e(t) + K_I \int e(t)dt + K_D \frac{d}{dt} e(t) \quad (30)$$

where, K_P refers to proportional gain, K_I is the integral gain, and K_D is the derivative gain. The Laplace transform of above equation in the continuous S-domain is given by equation (31)

$$U(s) = [K_P + \frac{K_I}{s} + K_D s]\, E(s) \quad (31)$$

The Transfer function of PID Controller is

$$G_{PID} = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s = [\frac{K_P\, S + K_I + K_D S^2}{S}] \quad (32)$$

3.1 Closed-loop Tuning Techniques

In this paper the classical and modern tuning techniques of the PID Controller are discussed independently as two different cases in order to reach the objective as presented in a Table 4.

Table 4: Closed-loop Tuning Techniques

Classical Approach		Modern Approach
ZN	Modified ZN	Pole- placement
Tyreus-luyben	Astrom-Haggulund	LQR

Case I: Classical PID Tuning Methods

3.1.1 Ziegler Nicholas

ZN method refers to open-loop and closed-loop control systems. It is first proposed in 1942 based on the system time response [23], [24]. This trial-error method proposed the use of ultimate gain K_{pu} and period of oscillation at ultimate gain Tu when the system is neutrally stable. The numeric value of K_P, K_I, K_D to tune PID Controller is estimated using the following relationship as discussed in Table 5 [5]

Classical closed loop PID Tuning Methods

Table5: Classical PIDTuning Parameters

SNo	Methods	K_P	K_I	K_D
1.	ZN	$0.6K_{pu}$	$1.2K_{pu}/T_u$	$0.075K_{pu}T_u$
2.	Modified ZN	$0.33K_{pu}$	$0.5T_u$	$0.33T_u$
3.	Tyreus- Luyben	$0.3125K_{pu}$	$2.2T_u$	$0.1587T_u$
4.	Astrom- Hagglund	$0.32K_{pu}$	$0.94T_u$	0

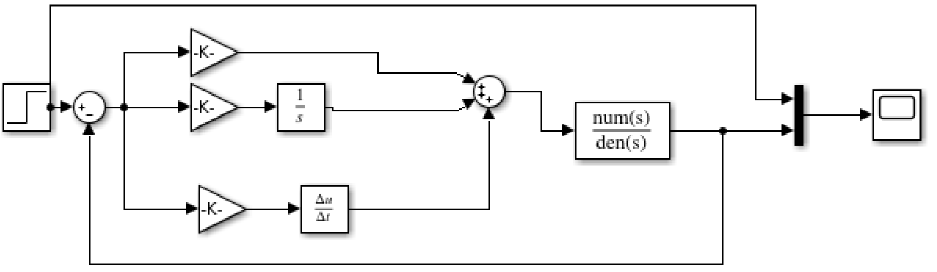

Fig. 7 Simulink block diagram of PID

The block diagram shown in figure 7 is used to estimate the gain parameters of PID while adopting various tuning techniques discussed in Table 5 using software SIMULINK. The procedure applied to determine the value of K_{pu} and T_u is discussed below:

Step 1. Initializing K_I and K_D to be zero and iterate numeric value of K_P to attain marginal stability curve in Scope

Step 2. Estimating the value of K_{pu} and T_u from neutrally stable curve as displayed in figure 8,9

Step 3. The gain of K_p becomes K_{pu} when the system achieves neutral oscillation and T_u reflects the time- period of oscillations between one cycle occurs at an ultimate gain.

Step 4. Gain value estimated are $K_{pu} = 1.3400$, $T_u = 1.5040$

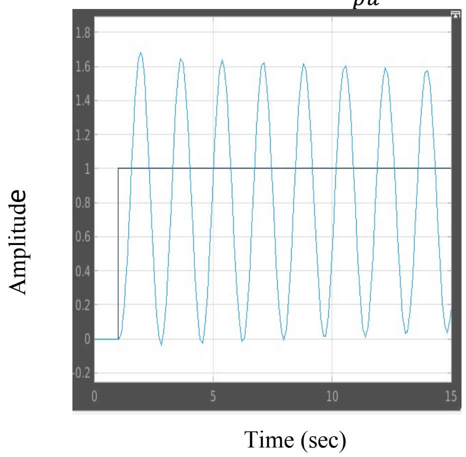

Fig. 8 Neutral Oscillations

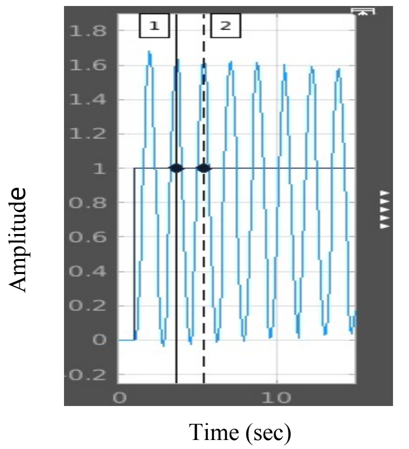

Fig. 9 Ultimate time period T_u

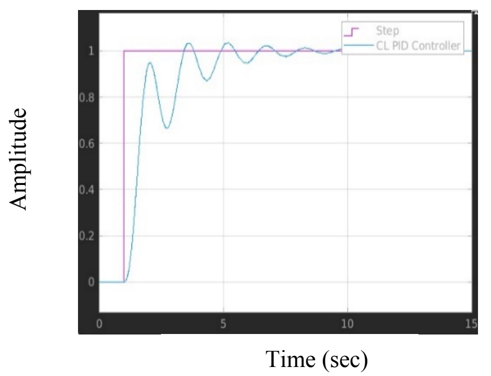

Fig. 10 Step response of aircraft dynamics with PID Controller

Table 6: ZN PID tuning parameters

S.No	Controller	K_P	K_I	K_D
1.	Classic PID	0.8040	1.0691	0.1512
2.	PD	0.2015	-	1.0720
3.	PI	0.6030	48.117	-

The unit step response in fig. 10 shows that the decayed oscillatory motion with damped amplitude of pitch controller signifies the stability. The gain parameters of PID presented in Table 6 illustrate that all types of controller have a high value of K_I which overall affects the system performance and leads to good steady-state response.

3.1.2 Modified ZN

Undesirable large overshoot value changes rapidly which can be predicted by using a Modified version of Ziegler Nicholas. This trial and error closed-loop method is similar to CHR (Chien-Hrones-Reswick) PID tuning applied to regulate the desired value of overshoot [23], [25], [26]. Modified gain value using this technique is presented in table 7.

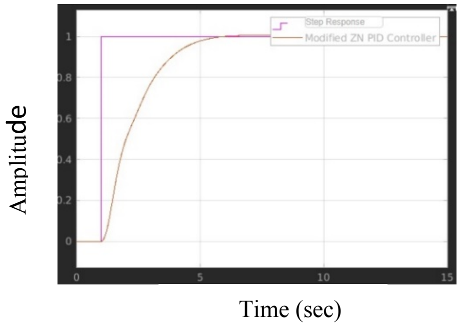

Fig. 11 Step Input response of Modified ZN Method

Table7: Modified ZN PID step response tuning parameters

S.No	K_P	K_I	K_D
1.	$0.33K_{pu}$	$0.5T_u$	$0.33T_u$
2.	0.4422	0.7520	0.4963

The unit step response in fig. 11 shows aperiodic non-oscillatory motion of the pitch controller with good steady state response and highly stable.

3.1.3 Tyreus-Luyben

This approach is similar to the ZN method and time-consuming but gives better performance results [27]. It depends on two parameters K_{pu} and T_u for tuning of gain value. This technique only proposes setting for PID and PI Controller [28]

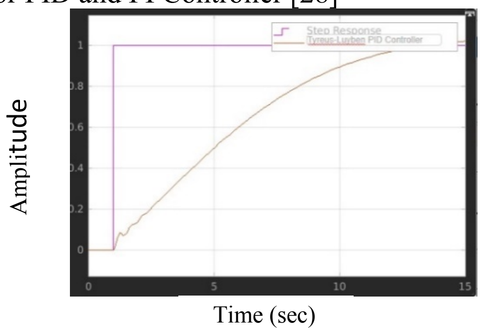

Fig. 12 Step Input response of Tyreus-Luyben Method

Table 8: Tyreus-Luyben PID step response tuning parameters

S.No	K_P	K_I	K_D
1.	$0.3125K_{pu}$	$2.2T_u$	$0.1587T_u$
2.	0.4188	0.2376	3.3088

The unit step response in fig. 12 shows aperiodic non-oscillatory motion of the pitch controller with high value of proportional gain that makes overall system to produce constant steady state error and decreases system sensitivity as discussed in Table 8.

3.1.4 Astrom- Hagglund

This auto-tuning approach recommended by Astrom and Hagglund in 1995 proposes settings for PID Controller without derivative filter [5]. It controls the system to meet the desired specification by regulating the value of overshoot.

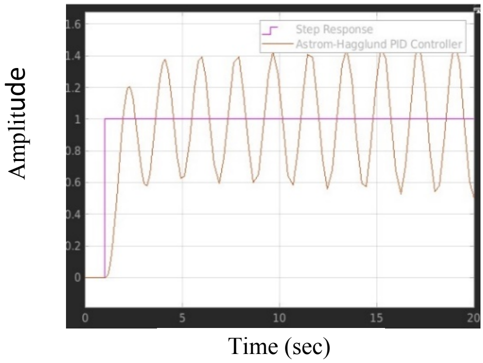

Fig. 13 Step Input response of Astrom- Hagglund Method

Table 9: Astrom- Hagglund PID step response tuning parameter

S.No	K_P	K_I	K_D
1.	$0.32K_{pu}$	$0.94T_u$	0
2.	0.4422	1.4138	0

The unit step response in fig. 13 presents the oscillatory motion with un-damped amplitude of pitch controller and states the system instability. The gain parameters of PID presented in Table 9 illustrate the good steady-state response.

Case II: Modern Control Methods

Recent advancement in technology involves novel approaches to design control systems termed modern control theory. Classical methods are limited to SISO systems while modern control theory encompasses the scope of MIMO, time-variant, linear or non-linear systems. The high-order systems are replaced by first-order differential equations to reduce the system complexity. Optimization techniques are easily applicable to solve optimal control problems using this approach [17]. Two methodologies such as pole placement and LQR are proposed to estimate the gain matrix for designing PID Controller. The block diagram of the state feedback control system is shown in figure 14.

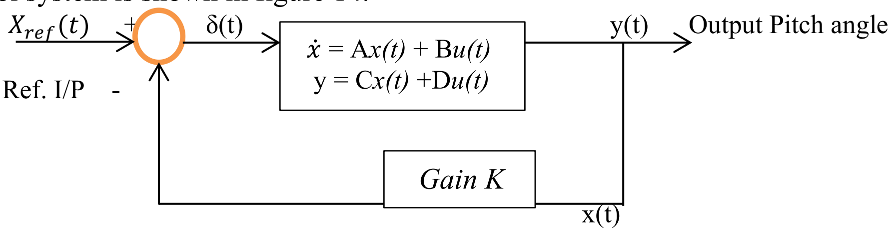

Fig. 14 Feedback Control Design

3.1.5 Pole placement

It is a State-space model approach that calculates gain matrix K to obtain the system stability as shown in fig. 15. Specific eigenvalues/ pole location is the desired feature of the state feedback design. This methodology positions closed-loop poles in the desired location by meeting the design requirements through a state feedback gain matrix [29], [6].The controller modifies matrix A to change the plant dynamics as eigenvalues of matrix A signify poles of the system and its location governs the system stability. The methodology adopted in pole placement to choose closed-loop poles using the Butterworth polynomial equation is shown below

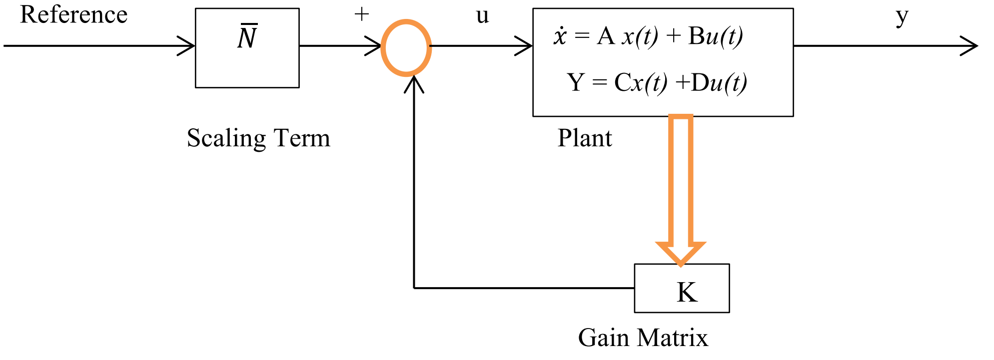

Fig. 15 Pole Placement and LQR

$$\text{Butterworth Filter} = \left(\frac{s}{w_O}\right) = (-1)^{\frac{n+1}{2n}}\left[\frac{e^{j(2k+1)\pi}}{-1}\right]^{\frac{n+1}{2n}} \tag{33}$$

where k = 0, 1, 2---------, w_O= natural frequency, n= system order (no of closed loop poles) now substituting $w_O = 2.70$, n = 3 and k = 0, 1, 2, 3, 4, 5 and simplifying Butterworth filter equation to estimate desired closed loop poles s_1, s_2, s_3 = -1.35 ± 2.338j, -1.3.The value of gain K is determined using Matlab function 'acker' thus Gain K = acker (A, B, S); K = [-0.2612 0.0157 0.572].

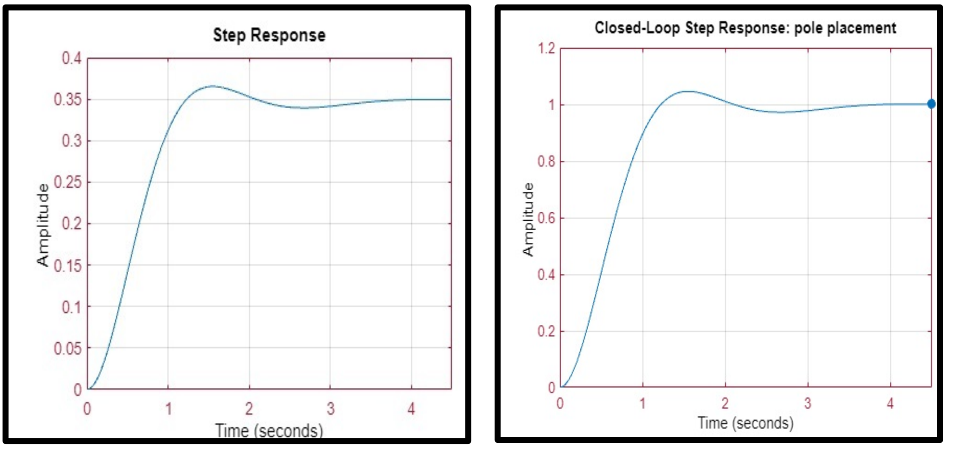

Fig. 16 Pitch angle step response: Pole placement

The pitch angle response using pole assignment technique presented in fig. 16 represents zero steady state error while applying scale factor$\bar{N}$ = 0.5728 to compensate the steady state error from 0.349 to 0.01.

3.1.6 Linear Quadratic Regulator (LQR)

LQR is an optimal modern control approach that solves the optimization problem by keeping the cost function minimal subjected to a given set of constraint [30]. This regulator has good set point tracking performance [31]. This approach is similar to pole placement as the implementation of gain K is similar as per fig. 15 but the procedure of choosing value of gain K is different. Optimal gain K is estimated by choosing closed-loop characteristics using the cost function [6] [32].

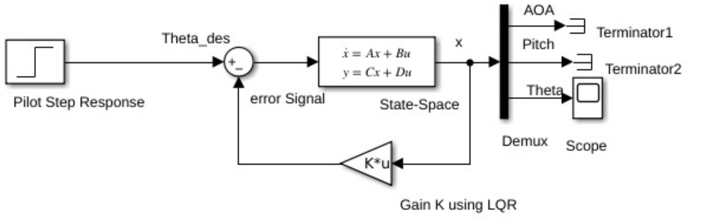

Fig. 17 LQR Simulink Block diagram

The block diagram shown in fig. 17 estimates the pitch angle response by evaluating the gain matrix K using the LQR approach.
In designing the LQR Controller, two parameters Q and R are required to determine the value of gain K. Q and R weighing square matrices are associated with state and control input of the system. In SISO systems, R is left unity and Q weighs the most important state of the system response [33]. The LQR control problem is solved by minimizing the cost function "J"

$$J = \int_0^{\infty} \{X^T Q x + U^T R u\} \, dt \tag{34}$$

where, X(t) = n× 1 state vector, u(t) = m× 1 control vector, Q = n×n symmetric positive semi-definite matrix, R = m×m symmetric positive semi-definite matrix henceforth J will be positive Initializing x= 400, $Q = x*C^T*C$

C= [0 0 1] Q = [0 0 0;0 0 0;0 0 x] R= [1]

The optimal control law is given by by $\eta = - k^T x$ where $k^T = R^{-1}B^TS$ thus substituting the value of B, Q, and R to solve the algebraic Ricatti equation for S as K refers to $R^{-1}B^TS$. Gain K is also obtained using lqr Matlab function as [K] = lqr (A, B, Q, R) thus the estimated value of optimal gain K = [-0.4717 1.88 20.00].

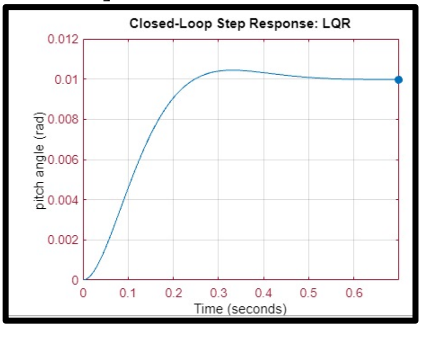

Fig. 18 pitch angle step response: LQR

The pitch angle response using LQR presented in fig. 18 shows that the gain value founds unity which concludes that K itself stabilizes the system and steady-state error approaches 0.01 as per design requirement.

4. RESULTS AND DISCUSSIONS

The study incorporates the implementation of classical and modern PID controller techniques to optimize PID parameters for the pitch control of Hansa-III aircraft.

Case I: The result of distinct types of classical PID tuning methods are compared in form of gains and displayed in Table 10. All gains have a specific function like K_p improves steady-state tracking accuracy, decreases system sensitivity on parameter variation, and produces the constant steady-state error. K_d leads to the system stability but has a poor steady-state response whereas K_i has a good steady-state response and leads to un-stability. T.F of PID contains two zeros in the numerator and one pole at origin in the denominator which makes the overall system highly stable. As per the Table 10, Astrum -Hagglund methodology does not have a derivative filter which leads to the aircraft instability. Thus the response of the system is an undamped oscillatory motion.

Table 10: Comparison of PID Tuning Methods

S. No	Tuning Methods	K_P	K_I	K_D
1.	Ziegler Nicholas	0.8040	1.0691	0.1512
2.	Modified ZN	0.4422	0.7520	0.4963
3.	Tyreus- Luyben	0.4188	0.2376	3.3088
4.	Astrom- Hagglund	0.4422	1.4138	0

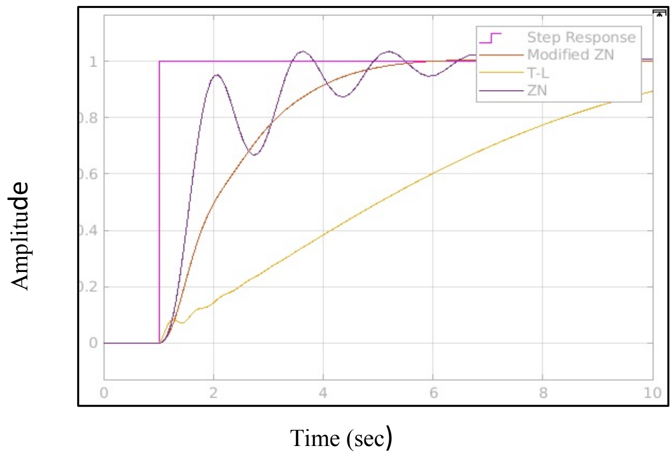

Fig. 19 Comparison ZN, Modified ZN, TL for step input

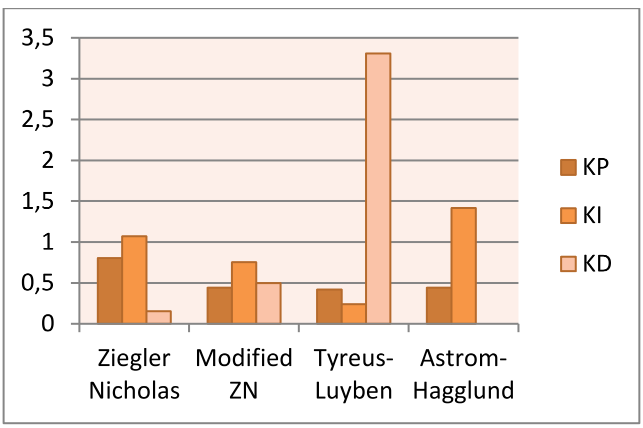

Fig. 20 Bar representation of ZN, Modified ZN, Tyreus- Luyben for Step Input

Tyreus- Luyben exhibit a large value of gain K_d which influences the overall system parameters by increasing the aircraft stability but difficult to attain steady-state value. This method is unable to accomplish the time-domain design requirements so does not display the optimum result. ZN and Modified ZN while comparing gain values as illustrated in fig. 19, 20 show that both controllers have the characteristic of stability and approaches S.S.E rapidly but modified ZN shows the best optimal result as this tuning controller satisfies the controller design requirements by approaching the steady state value close to zero.
Case II: Modern control methodologies such as pole-placement, LQR controllers are designed successfully. The results obtained using the pole-placement and LQR are presented in Table11. They are analyzed showing that LQR controller settles rapidly as settling time which is 0.44s as compared to pole-placement with excellent property of eliminating steady-state error to zero without using scale effect. The value of peak overshoot provides information about the deviation of the response in peak time with respect to final response is 0.332 for LQR as compared with pole placement which illustrates that LQR deviates with less amount and provides stability. This controller is robust-free, has good performance characteristics, and is highly efficient against disturbances.

Table 11: Time domain Performance characteristics

S. No	Closed loop Time-domain response	Design Specification	Pole-placement without Scaling effect	LQR without Scaling effect
1.	Steady state error	<1%	0.349	0.01
2.	Peak overshoot	<5%	4.59	0.332
3.	Settling Time	<5sec	3.08	0.44
4.	Rise Time	<2sec	0.793	0.16

5. CONCLUSIONS

The main objective of this paper is to do a comparative study of the classical and modern control techniques for the pitch control of Hansa-III. In this paper, the design and optimization of gain parameters of the PID using ZN, Modified ZN, Tyreus- Luyben, Astrom- Hagglund, Pole-placement, LQR is presented. Classical tuning techniques and modern control approaches are analyzed and compared independently. Modified ZN shows the best optimal result for Case I as this tuning controller meets the design requirements. Case II concluded that among modern control approaches the best transient and steady-state response for the pitch control is obtained for the PID Controller when tuned using LQR. Tuned gain values of the PID eliminate the disturbances, oscillations and provide stability to the aircraft.

FUTURE PROSPECTS

The high value of steady-state error affects the final value of the plant during operation, which is compensated by the use of compensators [33]. The addition of poles, zeros, and a combination of both modifies the transient response of the system and drives steady-state error as zero. It is suggested to use compensators while designing a controller for eliminating the large steady-state errors.

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