Unsteady dynamics of a sandwich plate under the influence of a cylindrical wave in an elastic medium

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Abstract: *The interaction of a sandwich plate with a damped cylindrical wave in the ground has been investigated. A sandwich plate is considered as a model of a barrier in the ground, described by a system of equations by V. N. Paimushin, placed in the ground dividing it into two parts. The plane problem formulation is considered. The boundary conditions correspond to the hinge attachment of the barrier, and the initial conditions are zero. A cylindrical damped wave is considered as an external influence. To describe the ground movement, the equations of the elasticity theory, the Cauchy relations and the physical principle, or equivalent displacements in potentials and the Lame equations are used. The problem is solved in a related formulation, where the movement of the plate and its surrounding media is considered together. All components of the equations of motion of the plate and media are decomposed into trigonometric series and the Laplace transform is applied to them. As the conditions for the contact of the plate and the ground, the equality of normal displacements at the boundary of the medium and the plate is assumed. It is also assumed that the pressure amplitudes and normal stresses coincide. After determining the constants from the contact conditions, the displacement values and the values of normal and tangential stresses are found, after which their originals are found.*

Key Words: *non-stationary dynamics, cylindrical wave, elastic medium, plate, integral transformations, vibration absorption.*

1. INTRODUCTION

The constant intensification of urban development and the introduction of infrastructure into the existing urban environment raises the question of protecting both the population and buildings from negative anthropogenic impact. The main source of adverse external influences on the foundations of buildings are technology-related vibrations. Such sources include engineering equipment, industrial installations, and vehicles (low-depth underground railway, heavy trucks, railway trains, trams) that create large dynamic loads during operation [1]. There are two types of impacts on the foundations of buildings – a stationary action created by regular sources of vibration and impulse excitation. However, these types of impacts and, accordingly, calculations for them are regulated by a much smaller number of specifications [2]. Due to the fact that these situations do not often occur in practice, this issue has been studied to a much lesser extent. There are two approaches to the organisation of vibration protection of the foundations of buildings and structures [3]: vibration protection, which is laid in the design and includes vibration damping devices; and an approach based on the creation of vibration-absorbing barriers [4], [5]. Vibration shields are most fully considered in [6].

Nonstationary problems have recently become widespread, and various types of interaction between external loads and plates and shells, both homogeneous and anisotropic, are studied. One of the methods for solving such problems is the method for determining the influence functions, described in [7], [8], [9], [10], [11], [12]. In addition, at the moment, inverse problems for nonstationary loads are widely studied, such as problems for a Timoshenko-type beam of finite length under the influence of a nonstationary load, and issues related to the identification of defects in an elastic rod [8], [9], [10]. The case of a nonstationary effect of a rigid indenter on an elastic half-plane is considered [11], [12]. In papers [13], [14], [15], the problems of nonstationary dynamics and the features of constructing the influence function for anisotropic plates and shells are considered.

This study deals with the nonstationary interaction of a cylindrical wave induced in the ground with a vibration-absorbing barrier in the form of a sandwich plate. Notably, the aforementioned studies are mainly focused on the direct impact of the load on the object under study, but this study considers a related problem that takes into account the position of the wave source in the medium, as well as the vibrations that occur in the ground. An elastic medium is used as a ground model, which is acceptable for small oscillation amplitudes. The selected plate model allows varying the geometric parameters and material properties of the bearing layers and the core, thus obtaining a barrier with optimal vibration-absorbing properties.

2. MATERIALS AND METHODS

This study investigates the non-stationary effect of a plane pulse on a complex barrier, where the design features and the shape of the incoming wave are taken into account. A sandwich plate is considered as a model of a barrier in the ground. It is placed in the soil, dividing it into two parts – the media "1" and "2", described by the system of equations by V.N. Paimushin [16]. The isotropic elastic medium "1" has a density ρ_1, with the speed of sound wave propagation – c_1. The isotropic elastic medium "2" has a density ρ_2, with the speed of sound wave propagation – c_2. The plate is located in the Cartesian coordinate system $Oxyz$, and it is assumed that the plane Oxy for the plate is the median, and the axis Oz is directed to the depth of the medium "2". The plane problem formulation is considered. The boundary conditions correspond to the hinge attachment of the barrier, and the initial conditions are zero. The incoming wave is a damped cylindrical wave with a pressure amplitude p_* at the front. The initial conditions are zero. As a result of its interaction with the plate in the media "1" and "2", the transmitted and reflected waves are induced [17], [18], [19], [20].

Movements in media "1" and "2" are defined, as well as movements at any point in the medium "2". To solve this problem, the Fourier series expansion and the Laplace transform are used [21], [22], [23], [24], [25].

The object of the study is a sandwich plate with a symmetrical structure consisting of two bearing layers and a core between them. The bearing layers of the plate are elastic and isotropic, with a modulus of elasticity of the first kind E and a Poisson's ratio ν, and have a thickness $2t_1$. The core material is orthotropic, of a honeycomb structure, with a elasticity modulus E_z and a Poisson's ratio ν_z, and has a thickness $2h$. The core has a compression module E_3 and transverse shear modules $G_1$and G_2 both in the axes Ox and directions Oy, respectively. The amplitudes of tangential displacements along the axes Ox and Oy are denoted by $u_1^{(k)}$ and $u_2^{(k)}$, respectively, and by the normal displacement $w^{(k)}$ of the k-th carrier layer. q^1 and q^2 – the amplitudes of transverse shear stresses of constant thickness in the core, directed along the axes Ox and Oy.

Then the equations of motion of the plate have the form (1)-(7) (the presence of a coordinate after the decimal point corresponds to differentiation by it, and the point corresponds to differentiation by time τ) [26], [27], [28], [29].

$$\rho_c \ddot{u}_1^c = L_{11}(u_1^c) + L_{12}(u_2^c), \rho_c \ddot{u}_2^c = L_{21}(u_1^c) + L_{22}(u_2^c), \tag{1}$$

$$\rho_a \ddot{u}_1^a = L_{11}(u_1^a) + L_{12}(u_2^a) + 2q^1, \tag{2}$$

$$\rho_a \ddot{u}_2^a = L_{21}(u_1^a) + L_{22}(u_2^a) + 2q^2 \tag{3}$$

$$\rho_c \ddot{w}_c - \underline{m_c \Delta \ddot{w}_c} + \underline{\rho_{wq}\left(\ddot{q}_{,x}^1 + \ddot{q}_{,y}^1\right)} = -D\Delta_2^2 w_c + 2k_1\left(q_{,x}^1 + q_{,y}^1\right) + p_1 - p_2, \tag{4}$$

$$\rho_{aw} \ddot{w}_a - \underline{m_a \Delta \ddot{w}_a} = -D\Delta_2^2 w_a - 2c_3 w_a + p_1 + p_2, \tag{5}$$

$$\underline{\rho_{q1}\ddot{q}^1} - \underline{\rho_{wq1}\ddot{w}_{c,x}} = u_1^a - k_1 w_{c,x} - k_2\left(q_{,x}^1 + q_{,y}^2\right)_{,x} + k_{31}q^1 \tag{6}$$

$$\underline{\rho_{q2}\ddot{q}^2} - \underline{\rho_{wq2}\ddot{w}_{c,y}} = u_2^a - k_1 w_{c,y} - k_2\left(q_{,x}^1 + q_{,y}^2\right)_{,y} + k_{32}q^2 \tag{7}$$

Since a plane formulation of the problem is considered, the system of equations by V.N. Paimushin [16] takes the following form (8)-(11), where:

$$Bu_{1,xx}^a(x,t) - \rho_a \ddot{u}_1^a(x,t) + 2q_1(x,t) = 0 \tag{8}$$

$$-Dw_{c,xxxx}(x,t) - \rho_c \ddot{w}_c(x,t) + 2k_1 q_{1,x}(x,t) + p_1 - p_2 = 0, \tag{9}$$

$$-Dw_{a,xxxx}(x,t) - \rho_{aw} \ddot{w}_a(x,t) - 2c_3 w_a(x,t) + p_1 + p_2 = 0 \tag{10}$$

$$u_1^a(x,t) - k_1 w_{c,x}(x,t) - k_2 q_{1,xx}(x,t) + k_{31} q_1(x,t) = 0 \tag{11}$$

$$B = \frac{2Et}{1-\nu^2} \tag{12}$$

$$D = \frac{Bt^2}{3} \tag{13}$$

$$u_i^c = u_i^{(1)} + u_i^{(2)}, u_i^a = u_i^{(1)} - u_i^{(2)} (i = 1,2), w_c = w_0{}^{(1)} + w_0{}^{(2)}, w_a = w_0{}^{(1)} - w_0{}^{(2)} \tag{14}$$

$$k_1 = t + h, k_2 = \frac{h^2}{3c_3}, k_{3i} = \frac{2h}{G_i}, (i = 1,2), c_3 = \frac{E_3}{2h}. \tag{15}$$

Core compression module:

$$E_3 = \frac{4dE_z}{3(1-\nu_z{}^2)a\,sin(\varphi)} \tag{16}$$

The case of a transversally soft core is considered, where the transverse shear modules of the core are equal to each other. In this case:

$$G_1 = G_2 = G \tag{17}$$

$$\rho_{wq1} = \rho_{wq2} = \rho_{wq} = \frac{2\rho h^3}{3G}, \rho_{q1} = \rho_{q2} = \rho_q = \frac{2\rho_{wq}}{G}, k_{31} = k_{32} = k_3 = \frac{2h}{G} \tag{18}$$

The transverse shear modulus of the core G and the shear modulus of the core material G_z are defined as:

$$G = G_z \frac{2d(1 + cos^2(\varphi))}{3a\, sin(\varphi)} \tag{19}$$

$$G_z = \frac{E_z}{2(1 + \nu_z)} \tag{20}$$

Introducing the following dimensionless quantities:

$$\overline{w} = \frac{w}{l}; \overline{u} = \frac{u}{l}; \tau = \frac{c \cdot t}{l}; \overline{x} = \frac{x}{l}; \overline{q_1} = \frac{l(1 - \nu^2)}{Et_1} q_1; \overline{p_1} = \frac{l(1 - \nu^2)}{Et_1} p_1; \overline{p_2} = \frac{l(1 - \nu^2)}{Et_1} p_2 \tag{21}$$

All the functions included in the system of equations (8)-(11) are decomposed into trigonometric series that satisfy the boundary conditions, taking into account (21). (8)-(11) are applied to the series expansion (22)-(27) and the Laplace transform in time:

$$M = \sum_{n=1}^{\infty} M_n \, sin\, \lambda_n \, \overline{x} \tag{22}$$

$$M = (\overline{w_c}, \overline{w_a}, \overline{p_1}, \overline{p_2})^T \tag{23}$$

$$M_n = (\overline{w_{cn}}, \overline{w_{an}}, \overline{p_{1n}}, \overline{p_{2n}})^T \tag{24}$$

$$N = \sum_{n=1}^{\infty} N_n \, cos\, \lambda_n \, \overline{x} \tag{25}$$

$$N = \left(\overline{u_1^{(2)}}, \overline{q_1}\right)^T \tag{26}$$

$$N_n = \left(\overline{u_{1т}^{(2)}}, \overline{q_{1n}}\right)^T \tag{27}$$

$$2\lambda_n{}^2 \overline{u_{1n}^{(2)L}} + 2m_6 s^2 \overline{u_{1n}^{(2)L}} + \overline{q_{1n}^L} = 0 \tag{28}$$

$$-m_1 \lambda_n^4 \overline{w_{cn}^L} - (2 + m_2) m_6 s^2 \overline{w_{cn}^L} - 2m_3 \lambda_n \overline{q_{1n}^L} + \overline{p_{1n}^L} - \overline{p_{2n}^L} = 0 \tag{29}$$

$$-m_1 \lambda_n^4 \overline{w_{an}^L} - \left(2 + \frac{m_2}{3}\right) m_6 s^2 \overline{w_{an}^L} - \frac{4}{3m_5} \overline{w_{an}^L} + \overline{p_{1n}^L} + \overline{p_{2n}^L} = 0 \tag{30}$$

$$-2\overline{u_{1n}^{(2)L}} - m_3 \lambda_n \overline{w_{cn}^L} + (\lambda_n{}^2 m_4 + m_7) m_5 \overline{q_{1n}^L} = 0 \tag{31}$$

The following parameters, including the physical and geometric characteristics of the plate in equations (28)-(31) are denoted as:

$$m_1 = \frac{2{t_1}^2}{3l^2} \tag{32}$$

$$m_2 = \frac{\rho h}{\rho_b t_1} \tag{33}$$

$$m_3 = \frac{t_1 + h}{l} \tag{34}$$

$$m_4 = \frac{h^2}{2l^2} \tag{35}$$

$$m_7 = m_7 = \frac{6}{(1-\nu_z)(1+cos^2(\varphi))} \tag{36}$$

$$m_5 = \frac{E}{E_z}\frac{(1-{\nu_z}^2)}{(1-\nu^2)}\frac{at_1 h\, sin(\varphi)}{dl^2} \tag{37}$$

$$m_6 = (1-\nu^2) \tag{38}$$

From the system of equations (28)-(31), the normal $\overline{w_{0n}^{(1)L}}$ and $\overline{w_{0n}^{(2)L}}$, and tangent $\overline{u_{1n}^L} = -\overline{u_{2n}^L}$ displacements at the boundaries of the plate and layers “1” and “2” are determined:

$$\overline{w_{0n}^{(1)L}} = \frac{1}{2}\left(\overline{w_{cn}^L} + \overline{w_{an}^L}\right) = \frac{1}{2}\frac{-\left(\overline{p_{2n}^L} - \overline{p_{1n}^L}\right)I_1(s^2,\lambda_n^4)}{I_2(s^2,\lambda_n^2)I_3(s^2,\lambda_n^6) + I_4(s^2,\lambda_n^4)} + \frac{1}{2}\frac{3m_5\left(\overline{p_{1n}^L} + \overline{p_{2n}^L}\right)}{I_5(s^2,\lambda_n^4)} \tag{39}$$

$$\overline{w_{0n}^{(2)L}} = \frac{1}{2}\left(\overline{w_{cn}^L} - \overline{w_{an}^L}\right) = \frac{1}{2}\frac{-\left(\overline{p_{2n}^L} - \overline{p_{1n}^L}\right)I_1(s^2,\lambda_n^4)}{I_2(s^2,\lambda_n^4)I_3(s^2,\lambda_n^4) + I_4(s^2,\lambda_n^4)} - \frac{1}{2}\frac{3m_5\left(\overline{p_{1n}^L} + \overline{p_{2n}^L}\right)}{I_5(s^2,\lambda_n^4)} \tag{40}$$

$$\overline{u_{1n}^L} = \frac{m_3\lambda_n\left(\overline{p_{2n}^L} - \overline{p_{1n}^L}\right)}{Q_1(s^2,\lambda_n^8) + Q_2(s^4,\lambda_n^2) + Q_3(s^4,\lambda_n^0)} \tag{41}$$

where the following notation is introduced:

$$I_1(s^2,\lambda_n^4) = \left(m_5\left(m_6 s^2 + {\lambda_n}^2\right)\left({\lambda_n}^2 m_4 + m_7\right) + 1\right) \tag{42}$$

$$I_2(s^2,\lambda_n^2) = \left(m_6 s^2 + {\lambda_n}^2\right) \tag{43}$$

$$I_3(s^2,\lambda_n^6) = \left(m_1{\lambda_n}^4 + (2+m_2)m_6 s^2\right)\left({\lambda_n}^2 m_4 + m_7\right)m_5 \tag{44}$$

$$I_4(s^2,\lambda_n^4) = (2{m_3}^2 + m_1){\lambda_n}^4 + 2{m_3}^2 m_6{\lambda_n}^2 s^2 + (2+m_2)m_6 s^2 \tag{45}$$

$$I_5(s^2,\lambda_n^4) = (6+m_2)m_5 m_6 s^2 + 3m_5 m_1\lambda_n^4 + 4 \tag{46}$$

$$Q_3(s^4,\lambda_n^0) = 2s^2 m_6(1 + s^2 m_7 m_5 m_6)(2+m_2) \tag{47}$$

$$Q_2(s^4, \lambda_n^2) = 2m_6 s^2\big(2{m_3}^2 + (m_4 m_5 m_6 s^2(2 + m_2) + (2 + m_2)m_5 m_7 + m_1 m_7)\big){\lambda_n}^2 \tag{48}$$

$$Q_1(s^2, \lambda_n^8) = 2m_1 m_4 m_5 {\lambda_n}^8 + 2m_1 m_5(m_7 + m_4 m_6 s^2){\lambda_n}^6 + +(2m_5 m_6 s^2((2 + m_2)m_4 + m_1 m_7) + 2{m_3}^2 + m_1){\lambda_n}^4 \tag{49}$$

The model of the ground is an isotropic elastic medium, described by the equations of the elasticity theory. The closed system of equations describing its plane motion has the form (the forces of gravity are omitted) [30], [31]:

- equations of motion:

$$\rho \ddot{u}_1 = \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} \tag{50}$$

$$\rho \ddot{u}_2 = \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} \tag{51}$$

$$\rho \ddot{w} = \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} \tag{52}$$

- Cauchy relations:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x}, \tag{53}$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{54}$$

$$\varepsilon_{33} = \frac{\partial w}{\partial z}, \tag{55}$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial y}, \tag{56}$$

$$\varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial y} + \frac{\partial w}{\partial z}\right), \tag{57}$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y}\right), \tag{58}$$

$$\theta = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial w}{\partial z}, \tag{59}$$

- physical principle:

$$\sigma_{11} = \lambda\theta + 2\mu\varepsilon_{11}, \tag{60}$$

$$\sigma_{13} = 2\mu\varepsilon_{13} \tag{61}$$

$$\sigma_{33} = \lambda\theta + 2\mu\varepsilon_{33} \tag{62}$$

$$\sigma_{22} = \lambda\theta + 2\mu\varepsilon_{22} \tag{63}$$

$$\sigma_{23} = 2\mu\varepsilon_{23} \tag{64}$$

where: u and w – the displacements along the axes Ox and Oz, respectively; σ_{ij} and ε_{ij} – the components of the stress and strain tensors; θ – the volume expansion coefficient; $\rho$and λ, μ – the density and elastic constants of the Lame ground; the points here and further denote the time derivatives t. The system (50)-(64) is equivalent to the equations in displacements (Lame equations):

$$\rho \ddot{u}_1 = (\lambda + \mu)\frac{\partial \theta}{\partial x} + \mu \Delta u_1 \tag{65}$$

$$\rho \ddot{u}_2 = (\lambda + \mu)\frac{\partial \theta}{\partial y} + \mu \Delta u_2 \tag{66}$$

$$\rho \ddot{w} = (\lambda + \mu)\frac{\partial \theta}{\partial z} + \mu \Delta w \tag{67}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{68}$$

Another variant of the equivalent system with respect to the scalar potential φ and the components ψ of the vector potential of displacements, where: c_1 and c_2 – the propagation velocities of the waves of tension-compression and shear):

$$\ddot{\varphi} = c_1^2 \Delta \psi, \overleftrightarrow{}\overleftrightarrow{}\ \ddot{\psi} = c_2^2 \Delta \psi, \overleftrightarrow{}\overleftrightarrow{} \tag{69}$$

$$c_1^2 = \frac{\lambda + 2\mu}{\rho} \tag{70}$$

$$c_2^2 = \frac{\mu}{\rho} \tag{71}$$

$$u_1 = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z} \tag{72}$$

$$u_2 = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial z} \tag{73}$$

$$w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \tag{74}$$

Introducing the following dimensionless quantities:

$$\overline{u_1} = \frac{u_1}{l}; \overline{w} = \frac{w}{l}; \overline{x} = \frac{x}{l}; \overline{z} = \frac{z}{l}; \tau = \frac{c \cdot t}{l}; \overline{\sigma_{11}} = \frac{\sigma_{11}}{E_{gr}}; \overline{\sigma_{13}} = \frac{\sigma_{13}}{E_{gr}}; \overline{\sigma_{33}} = \frac{\sigma_{33}}{E_{gr}}; \bar{\psi} = \frac{\psi}{l^2}; \overline{\varphi} = \frac{\varphi}{l^2} \tag{75}$$

All functions included in the equations of ground movement are decomposed into trigonometric series [32]:

- potentials:

$$L = \sum_{n=1}^{\infty} L_n \sin \lambda_n \overline{x} \tag{76}$$

$$L = \left(\overline{\varphi}^{(i)}, \varepsilon_{11}^{(i)}, \overline{\sigma}_{11}^{(i)}, \overline{\sigma}_{33}^{(i)}\right)^T \tag{77}$$

$$L_n = \left(\overline{\varphi}_n^{(i)}, \varepsilon_{11n}^{(i)}, \overline{\sigma}_{11n}^{(i)}, \overline{\sigma}_{33n}^{(i)}\right)^T \tag{78}$$

$$K = \sum_{n=0}^{\infty} K_n \cos \lambda_n \overline{x}, \tag{79}$$

$$K = \left(\overline{\psi}^{(i)}, \varepsilon_{13}^{(i)}, \varepsilon_{33}^{(i)}, \theta^{(i)}, \overline{\sigma}_{13}^{(i)}\right)^T \tag{80}$$

$$K_n = \left(\overline{\psi}_n^{(i)}, \varepsilon_{13n}^{(l)}, \varepsilon_{33n}^{(l)}, \theta_n^{(l)}, \overline{\sigma}_{13n}^{(i)}\right)^T \tag{81}$$

Taking into account the expansions into trigonometric series that satisfy the boundary conditions and the Laplace transform performed, the equations of ground movement, Cauchy relations, and the physical principle in the coefficients of the series are written as follows:

- equations of motion:

$$s^2 \overline{u}_{1n}^{(j)L} = \lambda_n \overline{\sigma}_{11n}^{(j)L} + \frac{\partial \overline{\sigma}_{13n}^{(j)L}}{\partial \overline{z}} \tag{82}$$

$$s^2 \overline{w}_n^{(j)L} = \lambda_n \overline{\sigma}_{13n}^{(j)L} + \frac{\partial \overline{\sigma}_{33n}^{(j)L}}{\partial \overline{z}} \tag{83}$$

- Cauchy relations:

$$\overline{\varepsilon}_{11n}^{(j)L} = -\lambda_n \overline{u}_{1n}^{(i)L} \tag{84}$$

$$\overline{\varepsilon}_{33n}^{(j)L} = \frac{\partial \overline{w}_n^{(j)L}}{\partial \overline{z}} \tag{85}$$

$$\overline{\varepsilon}_{13n}^{(j)L} =; \overline{\theta}_n^{(j)L} = \frac{\partial \overline{\varphi}_n^{(j)L}}{\partial \overline{x}} + \frac{\partial \overline{w}_n^{(j)L}}{\partial \overline{z}} \tag{86}$$

- physical principle:

$$\overline{\sigma}_{11n}^{(j)L} = \lambda \frac{\partial \overline{w}_n^{(j)L}}{\partial \overline{z}}; \overline{\sigma}_{13n}^{(j)L} = \mu \lambda_n \overline{w}_n^{(j)L} \tag{87}$$

$$\begin{aligned}\overline{\sigma}_{33}^{(j)L} &= \lambda \frac{\partial \overline{u}_1^{(j)L}}{\partial \overline{x}} + (\lambda + 2\mu) \frac{\partial \overline{w}^{(j)L}}{\partial \overline{z}} \\ &= \frac{\nu_{gr}}{(1+\nu_{gr})(1-2\nu_{gr})} \frac{\partial \overline{u}_1^{(j)L}}{\partial \overline{x}} + \frac{(1-\nu_{gr})}{(1+\nu_{gr})(1-2\nu_{gr})} \frac{\partial \overline{w}^{(j)L}}{\partial \overline{z}} = \\ &= -\alpha \lambda_n \overline{u}_{1n}^{(j)L} + \gamma \frac{\partial \overline{w}_n^{(j)L}}{\partial \overline{z}}; \alpha = \frac{\nu_{gr}}{(1+\nu_{gr})(1-2\nu_{gr})}; \gamma = \frac{(1-\nu_{gr})}{(1+\nu_{gr})(1-2\nu_{gr})}\end{aligned} \tag{88}$$

Equations of motion in potentials:

$$\frac{\partial^2 \overline{\varphi}_n^L}{\partial \overline{z}^2} - \beta_{1n}{}^2 \overline{\varphi}_n^L = 0; \beta_{1n}{}^2 = \lambda_n{}^2 + s^2 \tag{89}$$

$$\frac{\partial^2 \overline{\psi}_n^L}{\partial \overline{z}^2} - \beta_{2n}{}^2 \overline{\psi}_n^L = 0; \beta_{2n}{}^2 = \lambda_n{}^2 + \frac{s^2}{\eta} \tag{90}$$

$$\overline{u}_{1n} = \lambda_n \overline{\varphi}_n - \frac{\partial \overline{\psi}_n}{\partial \overline{z}} \tag{91}$$

$$\overline{w}_n = \frac{\partial \overline{\varphi}_n}{\partial \overline{z}} - \lambda_n \overline{\psi}_n \tag{92}$$

The obtained equations (82)-(92) determine the values of displacements, stresses, and deformations in any of the media at known potential values.

3. RESULTS AND DISCUSSIONS

To find the dynamic and kinematic parameters of the medium, it is necessary to determine the values of the vector and scalar potentials. The equations of motion of the medium with respect to the scalar potential φ and the components ψ [32] of the vector potential of displacements after the corresponding expansion into series and the application of the Laplace transform will take the form (89), (90). The organic condition for the ground at infinity can be written as follows. The solutions of equations (93) satisfying the condition (89), (90) have the form:

$$\overline{\varphi_n}^L(\overline{z}, s) = O(1);\ \text{for}\ \overline{z} \to +\infty \tag{93}$$

$$\overline{\varphi_n}^{(j)L} = C_{1j} e^{-\beta_{1n}\overline{z}} \tag{94}$$

$$\overline{\psi_n}^{(j)L} = C_{2j} e^{-\beta_{2n}\overline{z}} \tag{95}$$

where: C1j, C2j – integration constants; j=1, 2. Therefore, the values of the scalar and vector potentials for the media "1" and "2" are found:

$$\overline{\varphi_n^{(1)L}} = C_{11} e^{-\beta_n \overline{z}} \tag{96}$$

$$\overline{\varphi_n^{(2)L}} = C_{12} e^{-\beta_n \overline{z}} \tag{97}$$

$$\overline{\psi_n^{(1)L}} = C_{21} e^{-\beta_n \overline{z}} \tag{98}$$

$$\overline{\psi_n^{(2)L}} = C_{22} e^{-\beta_n \overline{z}} \tag{99}$$

To solve this problem, it is necessary to set the incoming wave, which is a cylindrical damping wave.

The equation of motion of the medium in potentials is written in the same way as (89), (90), and the boundedness condition (93). A cylindrical wave propagating from a source located at a point $O_1(0,0,-d)$. A cylindrical coordinate system with centre in O_1, parallel axis Oz, and radius is introduced:

$$r_1 = \sqrt{x^2 + (z+d)^2}, \overline{r_1} = \frac{r_1}{l} \tag{100}$$

Assuming that $\varphi_a = \varphi_a(r_1)$, from (90), (91) the following equation is obtained with respect to this function (the strokes denote the derivative by r_1):

$$\frac{1}{r_1}\frac{\partial}{\partial r_1}\left(r_1\frac{\partial}{\partial r_1}\overline{\varphi_a}\right)+k_1^2\overline{\varphi_a}=0 \tag{101}$$

Its general solution has the form:

$$\overline{\varphi_a}=A_\varphi H_0^{(2)}(k_1r_1)+B_\varphi H_0^{(1)}(k_1r_1), k_1=i \tag{102}$$

where: $H_\nu^{(1)}(\zeta)$ and $H_\nu^{(2)}(\zeta)$ – Hankel functions of order ν; A_φ and B_φ are arbitrary constants. The radiation condition (82), (83), where should be put satisfies the solution:

$$\overline{\varphi_a}=A_\varphi H_0^{(1)}(k_1r_1) \tag{103}$$

Then the following equation for the potential is obtained:

$$\overline{\varphi}=A_\varphi H_0^{(1)}(k_1r_1)\cdot e^{-\tau} \tag{104}$$

Substituting this result in (82)-(92) and applying the Laplace transform, the following equations for displacements and stresses on the plate surface are obtained:

$$u_*^L|_{\overline{z}=0}=\frac{p_*\overline{x}\overline{d}}{N}\frac{1}{r_{10}}H_1^{(1)}(k_1r_{10})\frac{1}{s+1} \tag{105}$$

$$w_*^L|_{\overline{z}=0}=\frac{p_*\overline{d}^2}{N}\frac{1}{r_{10}}H_1^{(1)}(k_1r_{10})\frac{1}{s+1} \tag{106}$$

$$\sigma_{11*}|_{\overline{z}=0}=\frac{p_*\overline{d}}{r_{10}^2N}\left[(\gamma-\alpha)r_{10}H_1^{(1)}(k_1r_{10})-k_1r_{110}^2H_2^{(1)}(k_1r_{10})\right]\frac{1}{s+1} \tag{107}$$

$$r_{110}=\sqrt{\gamma\overline{x}^2+\alpha\overline{d}^2} \tag{108}$$

$$\sigma_{13*}|_{\overline{z}=0}=\frac{ip_*\overline{d}}{N(1+\nu_{gr})}\frac{\overline{x}\overline{d}}{r_{10}^2}H_2^{(1)}(k_1r_{10})\frac{1}{s+1} \tag{109}$$

$$r_{10}=\sqrt{\overline{x}^2+\overline{d}^2} \tag{110}$$

$$k_1=i \tag{111}$$

$$\sigma_{33*}|_{\overline{z}=0}=\frac{p_*\overline{d}}{r_{10}^2N}\left[(\alpha+\gamma)r_{10}H_1^{(1)}(k_1r_{10})-k_1r_{330}^2H_2^{(1)}(k_1r_{10})\right]\frac{1}{s+1} \tag{112}$$

$$r_{330}=\sqrt{\alpha\overline{x}^2+\gamma\overline{d}^2} \tag{113}$$

$$r_{330}=\sqrt{\alpha\overline{x}^2+\gamma\overline{d}^2} \tag{114}$$

$$r_{10}=\sqrt{\overline{x}^2+\overline{d}^2} \tag{115}$$

$$r_{110} = \sqrt{\gamma \bar{x}^2 + \alpha \bar{d}^2} \tag{116}$$

To determine the integration constants in (96)-(99), it is necessary to write down the contact conditions of the plate and the ground, similarly to [33]:

- pressures and stresses at the boundaries with media “1” and “2”:

$$\overline{p_{1n}^{L}}(\bar{z},s) = \left(-\overline{\sigma_{33n}^{(1)L}}(\bar{z},s) + \overline{p_{*n}}(\bar{z},s)\right)\Big|_{\bar{z}=0} \tag{117}$$

$$\overline{\sigma_{33*}}|_{\bar{z}=0} = \overline{p_{*n}} \tag{118}$$

$$\overline{p_{2n}^{L}}(\bar{z},s) = -\overline{\sigma_{33n}^{(2)L}}(\bar{z},s)\Big|_{\bar{z}=0} \tag{119}$$

$$\bar{\sigma}_{13n}^{(1)}(\bar{z},s)\Big|_{z=0} = \bar{\sigma}_{13*n}(\bar{z},s)|_{z=0} \tag{120}$$

- normal displacements:

$$\overline{w_{0n}^{(1)L}}(\bar{z},s) = \left(-\overline{w_{n}^{(1)L}}(\bar{z},s) + \overline{w_{n*}^{L}}(\bar{z},s)\right)\Big|_{z=0} \tag{121}$$

$$\overline{w_{0n}^{(2)L}}(\bar{z},s) = \overline{w_{n}^{(2)L}}(\bar{z},s)\Big|_{z=0} \tag{122}$$

$$\overline{u_{0n}^{(1)L}}(\bar{z},s) = \left(-\overline{u_{n}^{(1)L}}(\bar{z},s) + \overline{u_{*}}(\bar{z},s)\right)\Big|_{z=0} \tag{123}$$

Taking into account the integration constants obtained in (60)-(64), the values of normal and tangential displacements, as well as stresses in the media “1” and “2”, will take the form:

- stresses in media “1” and “2”:

$$\overline{\sigma_{33n}^{(1)L}} = \left(-\alpha{\lambda_n}^2 + \gamma{\beta_{1n}}^2\right)C_{11} \cdot e^{\beta_{1n}\bar{z}} + (\alpha-\gamma)\lambda_n\beta_{2n}C_{21} \cdot e^{\beta_{2n}\bar{z}} \tag{124}$$

$$\overline{\sigma_{33n}^{(2)L}} = \left(-\alpha{\lambda_n}^2 + \gamma{\beta_{1n}}^2\right)C_{12} \cdot e^{-\beta_{1n}\bar{z}} + (\gamma-\alpha)\lambda_n\beta_{2n}C_{22} \cdot e^{-\beta_{2n}\bar{z}} \tag{125}$$

$$\overline{\sigma_{13n}^{(1)}} = \frac{1}{\left(1+\nu_{gr}\right)}\left\{2\lambda_n\beta_{1n}C_{11}e^{\beta_{1n}\bar{z}} - C_{21}\left({\beta^2}_{2n} + {\lambda^2}_n\right)e^{\beta_{2n}\bar{z}}\right\} \tag{126}$$

$$\overline{\sigma_{13n}^{(2)}} = -\frac{1}{\left(1+\nu_{gr}\right)}\left\{2\lambda_n\beta_{1n}C_{12}e^{-\beta_{1n}\bar{z}} + C_{22}\left({\beta^2}_{2n} + {\lambda^2}_n\right)e^{-\beta_{2n}\bar{z}}\right\} \tag{127}$$

- stresses in media “1” and “2”:

$$\overline{w_{n}^{(1)L}} = \beta_{1n}C_{11} \cdot e^{\beta_{1n}\bar{z}} - \lambda_n C_{21} \cdot e^{\beta_{2n}\bar{z}} \tag{128}$$

$$\overline{w_{n}^{(2)L}} = -\beta_{1n}C_{12} \cdot e^{-\beta_{1n}\bar{z}} - \lambda_n C_{22} \cdot e^{-\beta_{2n}\bar{z}} \tag{129}$$

$$\overline{u_{1n}^{(1)L}} = \lambda_n C_{11} \cdot e^{\beta_{1n}\bar{z}} - \beta_{2n}C_{21} \cdot e^{\beta_{2n}\bar{z}} \tag{130}$$

$$\overline{u_{1n}^{(2)L}} = \lambda_n C_{12} \cdot e^{-\beta_{1n}\bar{z}} + \beta_{2n} C_{22} \cdot e^{-\beta_{2n}\bar{z}} \tag{131}$$

Substituting in the contact conditions (102) and (103) the values of the displacements of the bearing layers of the plate (39)-(41) and the displacements and stresses in the media “1” and “2” (104)-(116), the values of the constants are obtained:

$$C_{11} = \frac{R_1(s^8)}{R_2(s^{11})} \tag{132}$$

$$C_{21} = \frac{R_3(s^7)}{R_4(s^{11})} \tag{133}$$

$$C_{12} = \frac{R_5(s^{24})}{R_6(s^{25})} \tag{134}$$

$$C_{22} = \frac{R_7(s^{13})}{R_8(s^{14})}; \tag{135}$$

$$\lambda_n = \pi n; \beta_{1n}^2 = \lambda_n^2 + s^2; \beta_{2n}^2 = \lambda_n^2 + \frac{s^2}{\eta^2} \tag{136}$$

Substituting (132)-(136) in (128)-(131), the values of normal and tangent displacements in the medium “2” are obtained:

$$\overline{w_n^{(2)L}} = -\beta_{1n}(s)\frac{R_5(s^{24})}{R_6(s^{25})} \cdot e^{-\beta_{1n}\bar{z}} - \lambda_n \frac{R_7(s^{13})}{R_8(s^{14})} \cdot e^{-\beta_{2n}\bar{z}} \tag{137}$$

$$\overline{u_{1n}^{(1)L}} = \lambda_n \frac{R_1(s^8)}{R_2(s^{11})} \cdot e^{\beta_{1n}\bar{z}} - \beta_{2n}(s)\frac{R_3(s^7)}{R_4(s^{11})} \cdot e^{\beta_{2n}\bar{z}} \tag{138}$$

$$\overline{u_{1n}^{(2)L}} = \lambda_n \frac{R_5(s^{24})}{R_6(s^{25})} \cdot e^{-\beta_{1n}\bar{z}} + \beta_{2n}(s)\frac{R_7(s^{13})}{R_8(s^{14})} \cdot e^{-\beta_{2n}\bar{z}} \tag{139}$$

Since the solution turns out to be extremely cumbersome, a tracking notation is adopted. $R_i(s^k)$ – the polynomials that arise when finding the values of the displacements. This form of writing would allow evaluating the structure of the resulting equations.

The inverse Laplace transform must be applied to the obtained results (137)-(139). As is known from [34], [35], the inversion of the Laplace transform is easily performed in the case of a bounded domain and no branching point. However, in this case the domain is unbounded and the functions (102) contain branching points, thus, it becomes obvious that analytical inversion is impossible.

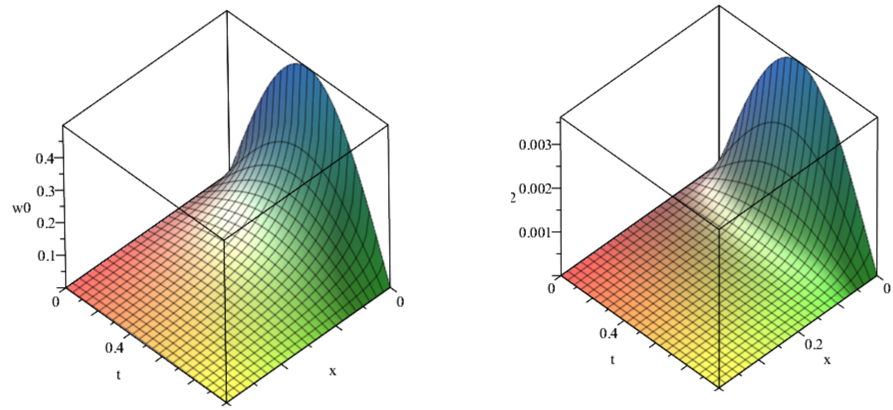
To reverse the function, the Durbin method [36] is used, where variables are replaced, based on the representation of exponential functions in the form of trigonometric functions and taking into account the features of these functions. As a result, any function is drawn by the following formula:

$$f(t) = \frac{2e^{\sigma t}}{\pi}\int_0^{\Omega} Re\, f^L\,(\sigma + i\omega)\, cos\,\omega\, t\, d\omega \tag{140}$$

where the integral is calculated numerically. As an example, a plate with the following parameters is considered: length l = 1 m, the thickness of the bearing stratum t = 0.15 mm, the thickness of the core h = 1.4 mm (Figure 1). Material of the bearing layers – steel 12X18N10T: $E = 1.8 \cdot 10^{10} kg/m^2, \rho_b = 7900 kg/m^3, \nu = 0.29$, core material – Amg2-N: $E_z = 7.1 \cdot 10^9 kg/m^2, \rho_z = 2690 kg/m^3; \nu_z = 0.32, a_z = 6 \cdot 10^{-3} m; dz = 0.05 \cdot 10^{-3} m; \varphi = 120$. Where a_z – the length of the core wall, dz – the thickness of the core wall, φ – the angle between the walls of the core.

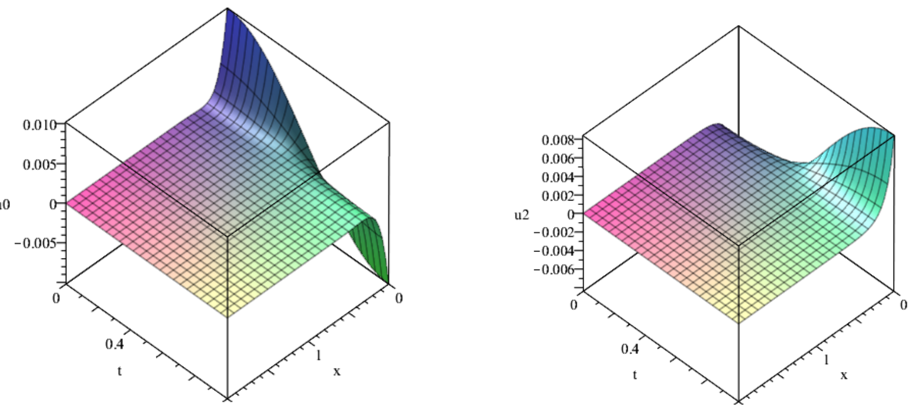
The ground has the following parameters [37]: density – $\rho_{gr} = 1600 kg/m^3$, elasticity modulus – $E_{gr} = 10^9 kg/m^2$.

As a result of the calculations, a graph of normal displacements and stresses at the boundary of the plate with the media "1" and "2" as a function of time τ is obtained (Figure 2).



a b

Fig. 1 – Normal displacements in the a) incoming wave $\overline{w_*}$; *b*) transmitted wave in the medium "2" $\overline{w^{(2)}}$



a b

Fig. 2 – Tangential displacements in the a) incoming wave $\overline{u_*^{(2)}}$;

b)Tangential displacements in the medium "2" $\overline{u_1^{(2)}}$

4. CONCLUSIONS

As a result of the interaction of the damped cylindrical wave with the plate in the media "1" and "2", the transmitted and reflected waves are induced. As a result of solving the equations of motion in potentials, the scalar and vector potentials of displacements in the media "1" and

“2” are determined. In addition, from the equations of motion of the medium in the potentials, the stresses and displacements in the incoming cylindrical wave are determined. As the conditions for the contact of the plate and the ground, the equality of normal displacements at the boundary of the medium and the plate is assumed. It is also assumed that the pressure amplitudes and normal stresses coincide. After determining the constants from the contact conditions, it becomes possible to determine the values of the scalar and vector potentials of the displacement field, through which the displacements at any point of the medium located behind the plate are unambiguously expressed. Next, the inverse Laplace transform is performed and the sums of the Fourier series for the displacement of stresses in both media are found. The solution of the problem in a related form allows taking into account not only the vibration-absorbing properties of the plate itself, but also the behaviour of the soil, and the features of the incoming cylindrical wave. The results obtained allows one to determine the values of the integration constants based on the contact conditions, knowing which it becomes possible to determine the kinematic and dynamic parameters at any point of the media surrounding the plate and, accordingly, to evaluate its vibration-absorbing properties.

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