LAMINAR STABILITY ANALYSIS IN BOUNDARY LAYER FLOW

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Abstract
This study presents a numerical study concerning the flow control by suction and injection. The case study is over a symmetrical airfoil with suction and injection slots. The angle of attack is 3 degree with the Mach number 0.12.

Introduction

This paper aims to study the stability of incompressible laminar boundary layer by numerical simulation carried out on an aerodynamic profile with suction and injection flow.

The e^n - method purpose in this paper for the study of laminar stability is based on small - disturbance theory in witch a small sinusoidal disturbance is imposed on a given steady laminar flow to see whether the disturbance will amplify or decay in time. If the disturbance decays, the flow will stay laminar, if the disturbance amplifies sufficiently the flow become turbulent.

The small - disturbance theory does not predict the details of the nonlinear process by which the flow changes from laminar to turbulent. It establishes which shapes of velocity profiles are unstable, identifies those frequencies that amplify fastest and indicates how the parameters governing the flow can be changed to delay transition process.

Mathematical Model

Let be $\overline{u},\overline{v},\overline{p}$ a solution of the incompressible laminar boundary layer equations. Over this solution assign a disturbance u',v',p'.

$$u=\overline{u}+u';v=\overline{v}+v';p=\overline{p}+p' \tag{1}$$

Since the basic solution $\overline{u},\overline{v},\overline{p}$ and sizes *u,v,p* satisfy the boundary layer equations, following equations are obtained for disturbance:

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0 \tag{2}$$

$$\begin{cases}\frac{\partial u'}{\partial t}+u'\frac{\partial \overline{u}}{\partial x}+u\frac{\partial u'}{\partial x}+v'\frac{\partial \overline{u}}{\partial y}+v\frac{\partial u'}{\partial y}=-\frac{1}{\rho}\frac{\partial p'}{\partial x}+\nu\Delta u' \\ \frac{\partial v'}{\partial t}+u'\frac{\partial \overline{v}}{\partial x}+u\frac{\partial v'}{\partial x}+v'\frac{\partial \overline{v}}{\partial y}+v\frac{\partial v'}{\partial y}=-\frac{1}{\rho}\frac{\partial p'}{\partial y}+\nu\Delta v'\end{cases} \tag{3}$$

If we use the following characteristic dimensions:

$L-$ length characteristic; u_0- speed characteristic ;

ρu_0- dynamic pressure; Re - Reynolds number.

The Reynolds number is defined as the ratio of inertial and friction forces which are manifested in the flow of fluid.

$$\mathrm{Re}=\frac{F_i}{F_f}=\frac{\rho\cdot V\cdot L}{\mu}=\frac{V\cdot L}{\nu} \tag{4}$$

The introduction of dimensionless quantities defined by:

$$\tilde{x}=\frac{x}{L};\quad \tilde{y}=\frac{y}{L};\tilde{\overline{u}}=\frac{\overline{u}}{u_0};\tilde{u}'=\frac{u'}{u_0};\quad \tilde{v}'=\frac{v'}{u_0};\quad \tilde{p}'=\frac{p'}{\rho u_0};\tilde{t}=\frac{t\cdot u_0}{L} \tag{5}$$

allows eq. (2) and (3) to be written as:

$$\frac{\partial \tilde{u}'}{\partial \tilde{x}}+\frac{\partial \tilde{v}'}{\partial \tilde{y}}=0 \tag{6}$$

$$\begin{cases}\frac{\partial \tilde{u}'}{\partial \tilde{t}}+\tilde{\overline{u}}\frac{\partial \tilde{u}'}{\partial \tilde{x}}+\tilde{v}'\frac{\partial \tilde{\overline{u}}}{\partial \tilde{y}}=-\frac{\partial \tilde{p}'}{\partial \tilde{x}}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{u}'}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{u}'}{\partial \tilde{y}^2}\right) \\ \frac{\partial \tilde{v}'}{\partial \tilde{t}}+\tilde{u}\frac{\partial \tilde{v}'}{\partial \tilde{x}}=-\frac{\partial \tilde{p}'}{\partial \tilde{y}}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{v}'}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{v}'}{\partial \tilde{y}^2}\right)\end{cases} \tag{7}$$

Equations (6) and (7) form a set of coupled partial - differential equations with solutions describing how disturbances originate near surface and spread out through the boundary layer and beyond as they are convected along the local streamlines. To study the properties of these equations, we apply the standard procedure of stability theory, namely separation of variables.

If we assume that the small disturbance is a sinusoidal traveling wave and represents a two - dimensional disturbance as:

$$\Psi(x, y, t) = \phi(y)e^{i(\alpha x - \omega t)} \quad (8)$$

and introducing a stream function $\Psi(x, y, t)$ such that:

$$u' = \frac{\partial \Psi}{\partial y}; \quad v' = -\frac{\partial \Psi}{\partial x} \quad (9)$$

where:

$\phi(y)$ -is the disturbance amplitude;

$\alpha -$ is the dimensionless wave number;

$$\alpha = \alpha_r + i\alpha_i ; \; \alpha = \frac{2\pi L}{\lambda_x};$$

$\lambda_x -$ is the wavelength in x direction;

$\omega -$ is the dimensional frequency disturbance;

$$[\omega] = \text{rad} ; \omega = \omega_r + i\omega_i ; \omega = \frac{\omega^* L}{u_0};$$

$\omega^* -$ is the dimensionless frequency disturbance;

Introducing eq.(8) and (9) into eq.(6) and (7), we obtain the fallowing fourth - order ordinary differential equation for the amplitude $\phi(y)$:

$$\phi^{iv} - 2\alpha^2\phi'' + \alpha^4\phi = i\,\mathrm{Re}(\alpha u - \omega)(\phi'' - \alpha^2\phi) - i\,\mathrm{Re}\,\alpha u''\phi \quad (10)$$

Equation (10) is named Orr-Sommerfeld equation and is the fundamental equation for the incompressible stability theory.

The solutions of eq.(10) corresponded to small disturbance waves and are called Tollmien - Schlichting waves.

The boundary conditions are homogeneous and form in relation to the normal wall:

$$\begin{cases} y = 0 \rightarrow u' = v' = 0 \rightarrow \phi = \phi' = 0 \\ y = \delta \rightarrow u' = v' = 0 \rightarrow \phi = \phi' = 0 \end{cases} \quad (11)$$

Tuncer Cebeci suggests in [1], [3] the following boundary conditions:

$$\begin{cases} y = 0 \rightarrow u' = v' = 0 \rightarrow \phi = \phi' = 0 \\ y = \delta \rightarrow \begin{cases} \left(\dfrac{d\phi}{dy} + \xi_1\right)\left(\dfrac{d\phi}{dy} + \xi_2\right) = 0 \\ \left(\dfrac{d\phi}{dy} + \xi_2\right)\left(\dfrac{d^2\phi}{dy^2} - \xi_1^2\right) = 0 \end{cases} \end{cases} \quad (12)$$

where:

$\xi_1^2 = \alpha^2$; $\xi_2^2 = \xi_1^2 + i\,\mathrm{Re}(\alpha u - \omega)$; $\xi_3 = i\,\mathrm{Re}\,\alpha u''$

$\mathrm{Re}(\alpha) > 0; \mathrm{Re}(\xi_1) > 0; \mathrm{Re}(\xi_2) > 0$

The solution of eq. (10) and its boundary conditions (12) exist only for certain combinations of Reynolds number Re and the parameters of the disturbance α, ω. This is an eigenvalues problem in witch values of $\mathrm{Re}, \alpha, \omega$ are the eingenvalues and $\phi(y)$ are eigenfunctions. The parameters $\mathrm{Re}, \alpha, \omega$ must satisfy an equation of the form:

$$F(\mathrm{Re}, \alpha, \omega) = 0 \quad (13)$$

Solutions of eq. (10) will be determinate by using the spatial amplification theory, since the amplitude of disturbance at a point is independent of time, this change is only with the distance. The eigenvalues of eq. (10) are presented in $(\alpha, \mathrm{Re}), (\omega, \mathrm{Re})$ diagrams that describe the three states of a disturbance at a given Reynolds number as damped, neutral or amplified. The locus where $\alpha_i = 0$ called the curve of neutral stability and separates the stable region from the unstable region.

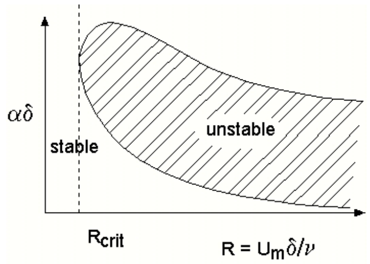

fig.1Curve of neutral stability

The point on this curve at which Reynolds number has its smallest value is of special interest, because at values of Re less than this value, all disturbances are stable. This smallest Reynolds number is known as the critical Reynolds number.

Numerical model

To solve eq. (10) with boundary condition (12) we use Keller's method [1],[3], which is a two point finite-difference scheme to express them as a first-order system. The first-order equations are approximated on an arbitrary rectangular net, with centered-difference derivates and averages at the midpoints of the net rectangle difference equation.

The resulting system of equation witch is implicit and nonlinear is linearized by Newton-Raphson method and solved by the block elimination method.

We put the system of equation (6) and (7) in the following form:

$$\begin{cases} \phi` = f \\ f` = s + \xi_1^2 \phi \\ s` = g \\ g` = \xi_2^2 s - \xi_3 \phi \end{cases} \quad (14)$$

The boundary condition becomes:

$$\begin{cases} y = 0 \to \phi = f = 0 \\ y = \delta \to \begin{cases} s + (\xi_1 + \xi_2) f + \xi_1 (\xi_1 + \xi_2) \phi = 0 \\ g + \xi_2 s = 0 \end{cases} \end{cases} \quad (15)$$

The eq. (14) and (15) are written in point $y_{j-\frac{1}{2}}$ and thus the following system is obtain:

$$\begin{cases} \phi_j - \phi_{j-1} - c_3 (f_j + f_{j-1}) = (r_1)_j = 0 \\ f_j - f_{j-1} - c_3 (s_j + s_{j-1}) - c_1 (\phi_j + \phi_{j-1}) = (r_3)_{j-1} = 0 \\ s_j - s_{j-1} - c_3 (g_j + g_{j-1}) = (r_2)_j = 0 \\ g_j - g_{j-1} - c_4 (s_j + s_{j-1}) - c_2 (\phi_j + \phi_{j-1}) = (r_4)_{j-1} = 0 \end{cases} \quad (16)$$

where:

$$c_3 = \frac{h_{j-1}}{2}; \quad c_1 = \xi_1^2 c_3; \quad c_2 = -(\xi_3)_{j-\frac{1}{2}} c_3; \quad (17)$$

$$c_4 = -(\xi_2^2)_{j-\frac{1}{2}} c_3; \quad h_{j-1} = y_j - y_{j-1}$$

The boundary condition becomes:

$$\begin{cases} j = 0 \to \begin{cases} \phi_0 = (r_1)_0 = 0 \\ f_0 = (r_2)_0 = 0 \end{cases} \\ j = J \to \begin{cases} f_J + \bar{c}_3 s_J + \bar{c}_1 \phi_J = (r_3)_J = 0 \\ g_J + \bar{c}_4 s_J = (r_4)_J = 0 \end{cases} \end{cases} \quad (18)$$

where:

$$\bar{c}_3 = \frac{1}{\xi_1 + \xi_2}; \quad \bar{c}_1 = \xi_1; \ \bar{c}_4 = \xi_2 \quad (19)$$

The system of equation (16) has trivial solutions for all values of j.

The Orr-Sommerfeld equation comes up in the form:

$$f_0(\alpha, \omega, \mathrm{Re}) = 0 \quad (20)$$

To find the solutions of this equation the following algorithm is applied:

1.The parameters $\alpha_r, \omega, \mathrm{Re}$ are evaluated for a given value of α_i .

2. The Reynolds number is fixed and the equation is solved for α_r, ω .

Results

For laminar stability analysis NACA 0015 airfoil was used and numerical simulations were made at an air speed velocity $V = 42\left[\frac{m}{s}\right]$; Reynolds number $\mathrm{Re} = 3 \cdot 10^6$; chord length $c = 1[m]$. Injection speed was $v_{inj} = 0.02\left[\frac{m}{s}\right]$ and suction speed was $v_{suc} = -0.02\left[\frac{m}{s}\right]$.

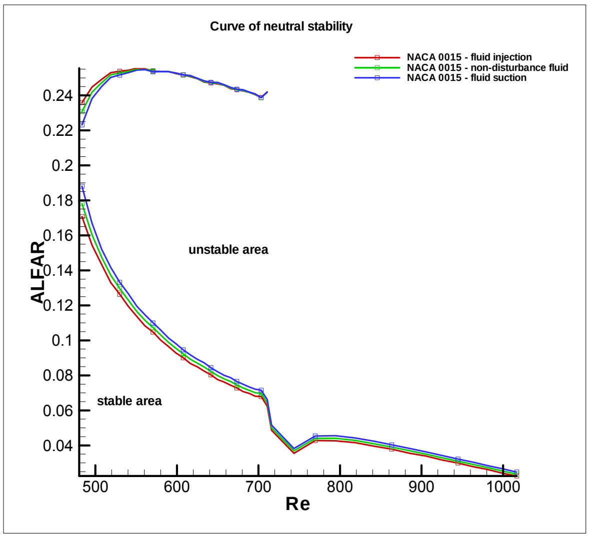

fig.2 Curve of neutral stability. The variation of wave number with Reynolds number.

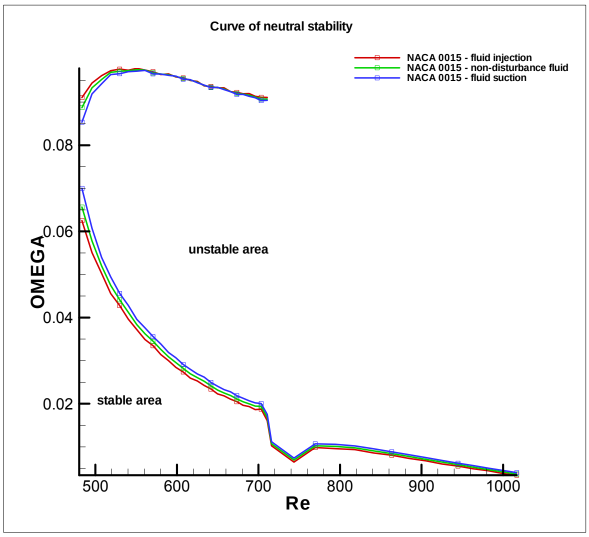

fig.3 Curve of neutral stability. The variation of frequency perturbation with Reynolds number.

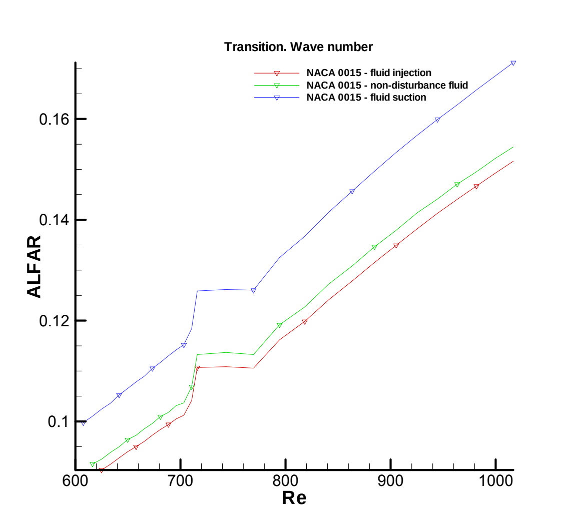

fig.4 Transition. The variation of wave number with Reynolds number.

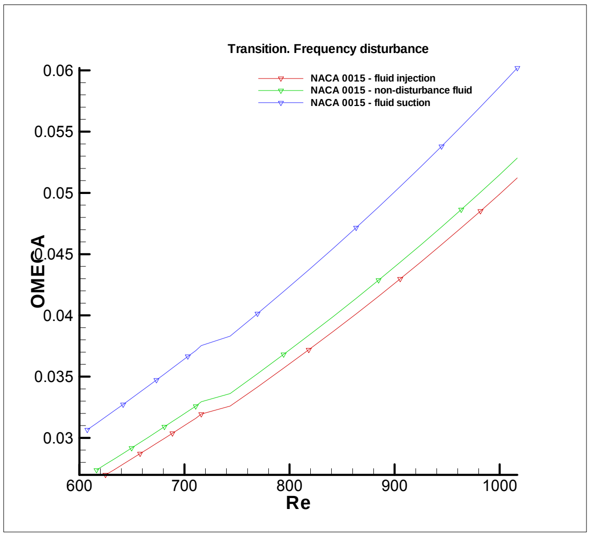

fig.5 Transition. The variation of frequency perturbation with Reynolds number.

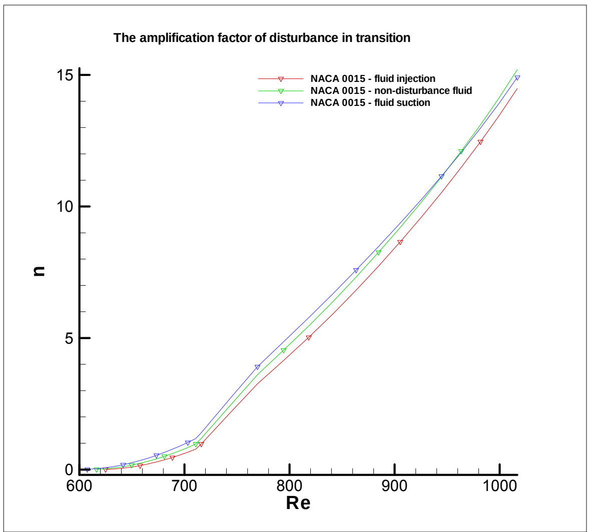

fig.6 The variation of amplification factor with Reynolds number.

Conclusion

From diagrams is seeing that the injection fluid have a destabilizing effect of the transition time, while the suction fluid has a stabilizing effect. The suction fluid delays transition and the portion of the chord where the flow staying laminar is greater than with injection.

The amplification factor where the transition occurs is about 9.

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