Calculation of Induced Velocity in Normal Operating Conditions for Vertical Climb and Hover, Autorotation Regime, Vortex Ring State, and Forward Flight Regime for the Sikorsky S-70 Black Hawk Helicopter

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Abstract: *This paper investigates the variations of induced velocity in the main flight regimes of the Sikorsky S-70 Black Hawk helicopter, an essential parameter in the aerodynamic performance analysis of the main rotor. Based on ideal theory, the study relies on the momentum theorem and analytical methods used in fluid dynamics. The aim is to establish the mathematical expressions of the induced velocity in the helicopter's primary flight regimes. The analysis of these regimes highlights velocity variations and their impact on the helicopter's performance. The results obtained can be used to optimize rotor design, leading to improved energy efficiency.*

Key Words: *induced velocity, aerodinamics, ideal theory, momentum theorem, helicopters, flight regime*

1. INTRODUCTION

Modern helicopters, such as the Sikorsky S-70 Black Hawk, operate in a wide range of flight regimes, each affecting the in-flight performance of the main rotor. The velocity of the airflow generated through the rotor plane, known as induced velocity, is one of the key parameters influencing the rotor's performance characteristics [1]. Additionally, it is a crucial factor in the power consumption required to maintain a balanced and safe flight.

This paper aims to determine and analyze the induced velocity in the four main operating regimes [1] of the Sikorsky S-70 Black Hawk:

- Normal operating regime, which includes vertical climb and hover flight;
- Autorotation regime, where the engine receives most of its energy from the upward aerodynamic flow;
- Vortex ring state, where aerodynamic instabilities occur, typically during vertical descent at certain speeds lower than twice the induced velocity;
- Forward flight regime, where the rotor operates in an oblique airflow, and the air stream through the rotor plane stabilizes.

In this study, the mathematical expressions will be determined of the induced velocity for each of these flight regimes.

The results obtained will provide a deeper understanding of rotor efficiency, which can be applied to optimize the overall efficiency of the helicopter.

2. IDEAL THEORY

Before calculating the induced velocity, it is necessary to establish the results of the ideal theory [1], which consists of the simple application of the momentum theorem to the streamtube formed by the mass of air driven by the rotor's operation, under general assumptions. The induced velocity is considered constant in the rotor plane and is calculated in multiple operating regimes of the rotor by applying the momentum theorem [1]:

- The rotor is assimilated to a permeable, infinitely thin disc with an infinite number of blades, imparting acceleration to the air passing through it [1];
- The airflow driven by the rotor blades forms a streamtube with a circular cross-section, extending beyond the rotor disc [1];
- The air is considered inviscid and incompressible, not being entrained into rotational motion by the blades [1].

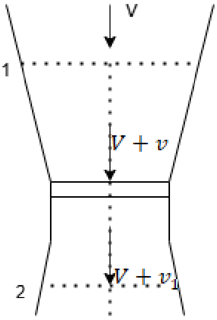

Figure 1. The streamtube passing through the rotor disc

The streamtube passing through the rotor disc (Figure 1) is considered, with a circular cross-section, extending infinitely in the flow direction with the velocities V, and a circular cross-section extending infinitely (opposite to the flow direction) [2] with velocity $V+v_1$. The induced velocity in the rotor plane is $V+v$,where v is induced velocity.The mass flow rate through the rotor disc (with an area $A=\pi R^2$ is $\rho A(V+v)$. The thrust force is considered equal to the difference in momentum between exit sections 1 and 2, the latter representing the variation of momentum over time [1].

$$T=\rho A(V+v)(V+v_1)-\rho A(V+v)V=\rho A(V+v)v_1\,. \tag{1}$$

3. CALCULATION OF INDUCED VELOCITY

Next, a more detailed analysis of the rotor's behavior will be carried out to determine how the results of Equation (1) apply to different flight regimes. In this regard, the three main flight regimes can be mentioned: hover flight, vertical flight, and forward flight.

3.1 Vertical Flight and Hover

Considering that in both hover flight and vertical flight,climbing or descending, the helicopter rotor operates in an axially symmetric airflow, these cases will be calculated separately due to differences in the airflow characteristics [1].

In hover flight and vertical climb, the air column driven by the rotor does not contain vortex regions. This indicates that the velocity of an air particle at any point within the air column is composed of two components: the induced velocity and the velocity due to the climb (both directed downward) [1]. These velocities are parallel and have the same direction.

In the case of vertical descent, the relative velocity of the air with respect to the rotor is directed from bottom to top, while the induced velocity is also directed upward. When the rotor's descent speed approaches the magnitude of the induced air velocity, vortex regions begin to form. As the descent speed increases beyond twice the induced velocity, the flow becomes stable again. However, in this context, the mass of air passes through the rotor from bottom to top [1], [2].

It can be concluded by defining the three operating regimes of the rotor in vertical flight: the normal regime (characteristic of vertical climb or hover flight), the vortex ring state (vertical descent at speeds lower than twice the induced velocity), and the autorotation regime (characteristic of descent at speeds greater than twice the induced velocity) [1].

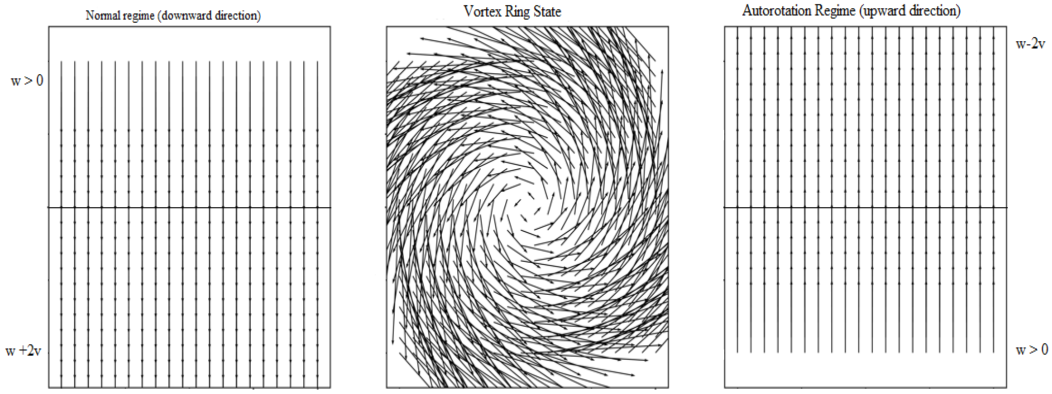

Figure 2. Air Column in Different Flight Regimes

- **Normal Regime**

In the normal regime (vertical climb or hover flight), the momentum theorem is applied in the form defined by Equation (1):

$$\mathrm{T} = 2\rho A(w+v)v\,. \tag{2}$$

The vertical climb flight is considered, where w represents the climb velocity, under the condition that: $w > 0$.

The induced velocity is obtained as:

$$v = -\frac{w}{2} + \sqrt{\frac{w^2}{4} + \frac{T}{2\rho A}} \tag{3}$$

Since $v > 0$, only the positive root of the radical is considered.

Applying the formula defined in Equation (3) for the **Sikorsky S-70 Black Hawk** helicopter:

It is noted that:

w – vertical climb velocity of the aircraft;

T – rotor thrust (N);

ρ- air density (kg/m³);

A – rotor disc area (m²).

* Determination of Rotor Disc Area (A). The main rotor diameter of the Sikorsky S-70 Black Hawk is 16.36 meters [3]. The rotor disc area is calculated using the classical formula:

$$A = \pi R^2 \tag{4}$$

where R is the rotor radius, half of the diameter. Thus,

$$A = \pi \text{ x} \left(\frac{16{,}36}{2}\right)^2 = 210{,}28\, m^2$$

* Determination of Rotor Thrust (T). The maximum takeoff weight of the helicopter is approximately 10,660 kg [3]. The thrust required to sustain this mass in steady climb flight is:

$$T = mg = 10660x9{,}81 = 104994{,}6\, N$$

* Air density (ρ) is calculated at sea level and at a standard temperature of 15°C, with an air density of 1.225 kg/m³.

*Considering w = 5 m/s ($w > 0$), results:

$$v = -\frac{5}{2} + \sqrt{\frac{5^2}{4} + \frac{104994.6}{2 \text{ x } 1.225 \text{ x } 210.28}} = 11.97\, m/s$$

In **hover flight**, where w = 0, the induced velocity is:

$$v_0 = \sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{104994.6}{2 \text{ x } 1.225 \text{ x } 210.28}} = 14.25\, m/s$$

Or, by introducing the thrust coefficient: $T = \frac{\rho}{2}(R\Omega)^2 A C_T$,

$$v_0 = \frac{R\Omega}{2}\sqrt{C_T}\,. \tag{5}$$

In order to apply the new formula to the **Sikorsky S-70 Black Hawk** helicopter, it is necessary to determine the thrust coefficient C_T and the angular velocity of the rotor. Ω (rad/s) (the other components have been previously calculated).

* To determine the angular velocity at a rotation speed of 258 RPM (as specified by the manufacturer) [3], the following equation is used:

$$\Omega = \frac{2\pi \text{ x N}}{60} = \frac{2\pi \text{ x } 258}{60}$$

From which it follows that: $\Omega = 27 \text{ rad/s}$.

The thrust coefficient C_T is obtained using the equation:

$$C_T = \frac{\text{T}}{\left(\frac{1}{2}\right)\rho(R\Omega)^2 A} \tag{6}$$

Substituting the previously determined elements into the formula, namely: rotor radius (8.18 m), rotor angular velocity (27 rad/s), air density (1.225 kg/m³), rotor disc area (210.28 m²), and rotor thrust (104994.6 N) /we obtain/gives/obtains,

$$C_T = \frac{104994.6}{\left(\frac{1}{2}\right) 1.225(8.18\, x\, 27)^2 210.28} = 0.01665$$

Substituting into Equation (5),

$$v_0 = \frac{8.18\ x\ 27}{2}\sqrt{0.01665} = 14.25\ m/s$$

A result similar to the previously calculated induced velocity is obtained. If it is approximated that the rotor thrust is the same in climb and in hover, the induced velocity in climb, according to Equation (3), is written as:

$$v = -\frac{w}{2} + \sqrt{\frac{w^2}{4} + v_0^2} = 11.97\ m/s$$

The induced velocity in hover flight (vertical climb regime) for the Sikorsky S-70 Black Hawk rotor is v_0= 14.25 m/s, while in vertical climb flight, the induced velocity is 11.97 m/s. This confirms that the rotor is more efficient in vertical climb than in hover flight. The induced velocity in climb is lower than in stationary hover flight, which was expected, as the surrounding air assists the airflow through the rotor.

- **Autorotation Regime**

This time, the vertical descent speed is greater than twice the induced velocity in hover flight.$(2\gamma_0)$. The steady nature of the flow and the absence of turbulent regions allow the momentum theorem to be applied once again. This time, the air passes through the rotor from bottom to top, being decelerated from velocity w at the top to velocity[1] w - 2v at the bottom.

$$T = -2\rho A(w + v)v \tag{7}$$

The impulse theorem can be written in the form of Equation (7), with the specification that that: $w < 0, v > 0(|w| > 2v)$, the rotor's airflow is directed upward [1]. The induced velocity follows from Equation (7) as:

$$v = -\frac{w}{2} + \sqrt{\frac{w^2}{4} - \frac{T}{2\rho A}} \tag{8}$$

Or, with the above approximation,

$$v_0 = \sqrt{\frac{T}{2\rho A}}, \qquad v = -\frac{w}{2} \pm \sqrt{\frac{w^2}{4} - v_0^2} \tag{9}$$

Agreeing on both real roots, $(w < -2v_0\ )$, but only one has physical meaning[1]:

$$v = -\frac{w}{2} - \sqrt{\frac{w^2}{4} - v_0^2} \tag{10}$$

Respecting the condition,$w < 0, v > 0(|w| > 2v)$, it is assume that
$w = \ -2.5\ x\ v_0 = 35.63\ m/s$ and substituting the previously calculated data for the **Sikorsky S-70 Black Hawk** helicopter, it follows that:

$$v = -\frac{-35.63}{2} - \sqrt{\frac{(-35.63)^2}{4} - 14.25^2} = 7.12\ m/S$$

The induced velocity in autorotation regime for the Sikorsky S-70 Black Hawk is approximately 7.12 m/s. In autorotation, the air flows through the rotor from bottom to top, which sustains the blade rotation. The rapid descent provides the necessary energy for the rotor to maintain sufficient lift.

- **Vortex Ring State (Turbulent Regime)**

This flight regime occurs during vertical descent at speeds lower than twice the induced velocity. In this case, the momentum theorem no longer applies [1]. A mean induced velocity over the rotor can be considered, necessary for generating the thrust force on the rotor, but it can only be determined experimentally. However, there is a specific case where the descent velocity is equal and opposite in direction to the induced velocity [1].
$w = -v$, What does it mean when air no longer passes through the rotor.

$$\rho A(w+v)=0$$

This is the ideal autorotation regime (stable vortex ring state). In this case, the rotor thrust force, having become impermeable, must be equal to the drag force of a circular plate with the same area as the rotor disc [1].

$$T=\frac{\rho}{2}v^2 AC_R \tag{11}$$

So:

$$v=\sqrt{\frac{2T}{C_R\rho A}}=\frac{2v_0}{\sqrt{C_R}},\left(v_0=\sqrt{\frac{T}{2\rho A}}\right) \tag{12}$$

Taking an average value for the drag coefficient of the flat plate [1], $*C_R = 1{,}28$, or the **Sikorsky S-70 Black Hawk**, it follows that:

$$v=1{,}77\ v_0=25.2\ m/s$$

In conclusion, if the descent velocity is equal and opposite to the induced velocity in stationary hover flight, $w = -v$, the airflow no longer passes through the rotor. This results in a complete loss of rotor thrust, leading to the helicopter entering free fall. At the obtained induced velocity $v \approx 25.2$ m/s, the rotor aerodynamics indicate a region close to a turbulent airflow state. In the vortex ring state, the rotor's efficiency is extremely low.

3.2 Forward Flight

In this flight regime, the rotor operates in an oblique airflow, introducing variability in the velocity field affecting the blades, not only based on their radial position but also their azimuthal position [1]. The airflow through the rotor is uniform and free of turbulence.
The momentum theorem is applied to the axial flow,

$$T=2\rho A V_1 v\,. \tag{13}$$

* Note: Equation (18) is an analogy proposed by Glauert [1], relating the thrust of a rotor with radius R to the lift of an elliptical wing with a span of b = 2R, which can be expressed as:

$$P=2\rho\pi\left(\frac{b}{2}\right)^2 \mathrm{V}v\,, \tag{14}$$

where V is the forward velocity of the wing, and v is the induced velocity.

According to Equation (13), the mass flow rate, $\rho A V_1$, is the one corresponding to the resultant relative velocity of the air with respect to the rotor V_1, taking into account: $A_{.} = \pi R^2$.

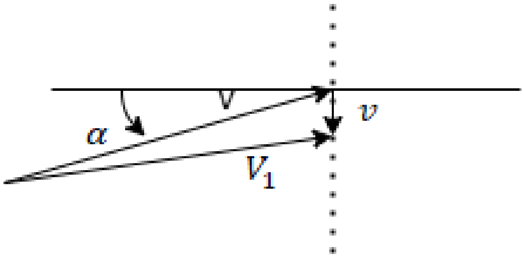

Figure 3. Representation of V, α, and v

From Figure 3, it can be concluded that V is the forward velocity of the helicopter, α is the incidence angle of the rotor, and v is the induced velocity. The resultant velocity is then determined as:

$$V_1 = \sqrt{(V\sin\alpha - v)^2 + (V\cos\alpha)^2}\,.$$

Therefore, Equation (13) is written as:

$$T = 2\rho A V\sqrt{(V\sin\alpha - v)^2 + (V\cos\alpha)^2}\,. \tag{15}$$

The induced velocity will be obtained by solving Equation (15), which is brought to a fourth-degree algebraic form [1]. In addition to knowing the thrust T and the forward velocity V, the incidence angle of the rotor α must be determined. The functional parameters are used of the rotor, which are functions of the flight speed and the rotor incidence angle:

$$\lambda = \frac{V\sin\alpha - v}{R\Omega}, \text{ the permeability coefficient and} \tag{16}$$

$$\mu = \frac{V\cos\alpha}{R\Omega}, \text{ the advance ratio.} \tag{17}$$

Reformulating Equation (15) with the two variable parameters, he following is obtained,

$$T = 2\rho A v\sqrt{\lambda^2 + \mu^2}R\Omega\,, \tag{18}$$

The induced velocity is obtained by introducing the thrust coefficient C_T,

$$v = \frac{C_T}{\sqrt{\lambda^2 + \mu^2}}R\Omega\,. \tag{19}$$

Applying the equation for the **Sikorsky S-70 Black Hawk** helicopter, it is necessary to determine the following unknowns: the permeability coefficient λ and the advance ratio μ.
*Previously, the following values were determined: $C_T = 0.01665$, $R = 8.18\, m$ and $\Omega = 27$ rad/s.
*Note: A forward velocity of V = 50 m/s will be considered.
The permeability coefficient λ is calculated by applying Equation (16).
*According to the aircraft manufacturer's manual, the cyclic pitch angle towards the front is: $\alpha = 3°$ [3]. To determine the permeability coefficient λ, the degrees are converting into radians;

$$T = \frac{180}{\pi} \times 3 = 0.05236\, rad\, ,$$

The result is:

$$\lambda = \frac{V \sin \alpha - v}{R\Omega} = \frac{50\; x \sin(0.05236) - 14.25}{R\Omega} = -0.0527\, .$$

Having all these data, applying the formula to determine the advance ratio μ becomes:

$$\mu = \frac{V \cos \alpha}{R\Omega} = \frac{50\; x \cos(0.05236)}{8.18\; x\; 27} = 0.2261\, .$$

Having all the data from the problem, by substituting into Equation (19), is obtained:

$$v = \frac{C_T}{\sqrt{\lambda^2 + \mu^2}} R\Omega = \frac{0.01665}{\sqrt{(-0.0527)^2 + 0.2261^2}} (8.18\; x\; 27) = 15.84 m/s.$$

The induced velocity in the rotor plane during forward flight for the Sikorsky S-70, moving at a speed of 50 m/s, is 15.84 m/s. It is observed that the induced velocity in forward flight is higher than that in hover flight due to the combined effect of thrust and airflow through the rotor. The permeability coefficient λ is negative, indicating a slight reduction in induced velocity due to the vertical component of the velocity. The advance ratio μ is stable and relatively low, indicating a typical cruise configuration.

4. CONCLUSIONS AND RESULTS

The result reflects the aerodynamic differences between the flight regimes, from which it can be observed that:

Table 1 - Results

Flight regime	The induced velocity in the rotor plane obtained	Direction of the air column	Turbulent zones
Vertical climb flight	11.97 m/s	Downward oriented	NO
Hover flight	14.25 m/s	Downward oriented	NO
Autorotation	7.12 m/s	Upward oriented	NO
Vortex ring state (vertical descent)	25.2 m/s	Downward oriented	YES
Forward flight	15.84 m/s	Obliquely oriented	NO

The vertical climb flight, where the velocity is lower than in hover and the air moves from bottom to top, reduces the need for additional airflow generated by the rotor. The aerodynamic flow is stable, and the rotor operates more efficiently than in hover flight.

Hover flight, with a higher result in the normal flight regime (14.25 m/s), explains the high energy consumption needed to keep the helicopter in a stationary position. The induced velocity is maximum because the airflow has no initial upward motion component.

Autorotation occurs when the engines stop working. The induced velocity is significantly lower, meaning that the rotor is partially supported by the airflow through the blades. The induced velocity formula in this flight regime shows that the induced velocity is reduced compared to hover flight because the air plays a significant role in generating lift.

The vortex ring state is characterized by turbulent airflow passing through the rotor. It is observed that in the vortex ring state, the induced velocity increases significantly compared to hover flight, indicating the instability of the airflow.

Forward flight has a different characteristic of oblique flow that complicates the aerodynamic flow but eliminates turbulence. The induced velocity is higher than in hover flight but lower than in the vortex ring state, as the rotor is supported by the airflow. The calculation formula indicates that, depending on the rotor angle ($\alpha = 3°$ in the case of the Sikorsky S-70 Black Hawk), the induced velocity varies, but it remains stable and free of turbulence.

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