New Search Space Reduction Algorithm for Vertical Reference Trajectory Optimization

Alejandro MURRIETA-MENDOZA[1], Jocelyn GAGNÉ[1],
Ruxandra Mihaela BOTEZ*[,1]

*Corresponding author
[1]LARCASE Laboratory of Applied Research in Active Control, Avionics and Aeroservoelasticity, École de Technologie Supérieure, Université du Québec, Montréal, H3C1K3, Québec, Canada
alejandro.murrieta-mendoza.1@etsmtl.net, jocelyn.gagne@cae.com, ruxandra.botez@etsmtl.ca*

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Abstract: *Burning the fuel required to sustain a given flight releases pollution such as carbon dioxide and nitrogen oxides, and the amount of fuel consumed is also a significant expense for airlines. It is desirable to reduce fuel consumption to reduce both pollution and flight costs. To increase fuel savings in a given flight, one option is to compute the most economical vertical reference trajectory (or flight plan). A deterministic algorithm was developed using a numerical aircraft performance model to determine the most economical vertical flight profile considering take-off weight, flight distance, step climb and weather conditions. This algorithm is based on linear interpolations of the performance model using the Lagrange interpolation method. The algorithm downloads the latest available forecast from Environment Canada according to the departure date and flight coordinates, and calculates the optimal trajectory taking into account the effects of wind and temperature. Techniques to avoid unnecessary calculations are implemented to reduce the computation time. The costs of the reference trajectories proposed by the algorithm are compared with the costs of the reference trajectories proposed by a commercial flight management system using the fuel consumption estimated by the FlightSim® simulator made by Presagis®.*

Key Words: *Flight Planning, Optimization, Reference vertical trajectory.*

I. INTRODUCTION

One of the most common emissions found in fuel consumption is carbon dioxide (CO_2), which contributes to global warming. The aeronautical industry has slowly incremented its CO_2 share so that it is now responsible for close to 2% of all the CO_2 released to the atmosphere[1]. Ambitious goals to further reduce emissions have been set by the industry, such as reducing the CO_2 to 50% of 2005 level by 2050[2]. Hydrocarbons and Nitrogen Oxides (NOx) are released to the atmosphere when fossil fuel is burned[3-5], the first damages the respiratory and nervous system and the latter depletes the ozone layer, and both contribute to global warming. From the current economic perspective, fuel is normally one of the largest airline expenses. It is also expected that in the future pollution amounts will incur a proportional cost to airlines. Reducing the quantity of fuel required to fly a given trajectory would result in less pollution released to the atmosphere, contributing to the aeronautical industry's pollution-reducing goal, and it will have a positive effect on aircraft operator's profits. Different technologies have been implemented and/ or proposed to reduce fuel consumption, such as improvements in engines, aircraft weight reduction by using composites and modern avionics, adding winglets (a fold in the tips of wings to reduce induced drag, thereby improving aircraft performance) to wings[6], as well as the use of bio fuel[7] and the possibility of air-air refueling[8]. Green[9] provides a summary of these techniques along with their environmental contributions. Flight trajectory planning has been identified as a means to reduce fuel consumption and optimize flight costs and management. This expectation has been validated by Jenset *et al*[10-12], whose studies showed that aircraft do not always fly at their optimal speeds and altitudes. Developing algorithms that optimize the flight reference trajectory is thus a realistic approach to reducing fuel consumption.

An aircraft's flight reference trajectory (or the flight plan) is normally calculated a priori by an airline ground team with powerful computers; this trajectory is then submitted to Air Traffic Management (ATM) for approval. This flight plan is introduced to an avionics device, the Flight Management System (FMS), by the crew before getting airborne or via an ACARS data link. However, not having to rely on ground-based computers to calculate an alternative optimal trajectory would be a valuable option. The task of reference trajectory optimization is performed by the FMS[13]. Researchers have been particularly interested in FMSs; investigating their evolution and their role in the future, including the role they play in various aircraft tasks[14, 15]. Specifically, studies have focused on their performance of current functions, measuring the capacity of the FMS to intercept and follow waypoints[16, 17] and the capacity of this device to follow a time imposed trajectory during descent[18], one of the functions required for the next generation of FMSs.

Current generations of FMS are able to manage the speeds and altitudes of a given vertical reference trajectory for a fixed lateral reference trajectory. This ability means that they could compute the altitudes and speeds that most reduce the flight cost, in other words, the optimal vertical reference trajectory. The vertical reference trajectory for the descent phase has been of particular interest, mostly as a tool to target noise reduction, which has been linked to serious health issues such as stress and hearth problems[19], and to reduce pollution near populated areas. The most popular technique that has proven to reduce noise and fuel consumption is the Continuous Descent Approach (CDA). This technique consists of descending following a constant descent angle, normally 3º, while setting the engines at IDLE (lowest thrust). Traditional descent pattern has been done by following descent steps: the flight level, or altitude, is reduced and a small cruise is traveled before descending some more. This repeatedly varies the engine power causing high fuel consumption. However, the CDA requires good synchronization with traffic and ATC. Not descending correctly might require the aircraft to abort the landing approach, something known as missed approach (or go-around), a costly procedure in terms of fuel flow and pollution released to the atmosphere[20, 21], as well as incurring a time delay. The CDA has motivated other studies to estimate parameters such as fuel flow[22], the influence of weight, speed and wind at the Top of Descent (ToD), as well as the behavior of the FMS when the descent speed is below or above the target, and to verify the accuracy of the FMS prediction of the final arrival time[23]. The CDA has been tested in simulations, and under certain conditions it has been experimentally tested and even functional in many airports, including Miami[24], Kentucky[25], Houston[26], Amsterdam[27], Bucharest[28], Zabgreb[29], London[30]. The other main flight phases that can be optimized are the climb and cruise phase. However, the climb phase is normally heavily constrained by ATM, and so there is not much opportunity to optimize it. On the other hand, cruise is normally the longest flight phase, and thus it is the phase that requires the largest share of fuel. Liden[31], using the energy model to compute the fuel burn, was one of the first researchers to study trajectory optimization with the FMS. In this research, the effects of tailwind, headwind and no wind on the flight cost were studied in the cruise regime by varying the speed, but no step climbs were performed. Another popular model to simulate aircraft behavior and fuel burn are the mass-point equations of motion (EoM). Hagelauer e*t al.*[32] implemented soft dynamic programming to solve the FMS trajectory optimization problem by using optimal control of a hybrid system with the speed as the control variable. A database-oriented model was emulated by discretizing the EoM of the Airbus A-340, and the altitude was considered as a discrete set of values. To reduce calculation time, neural networks were utilized to perform fuel flow evaluations. Sridhat *et al.*[33] developed an algorithm solving a non-linear optimal control with path constraints problem to calculate a wind-optimal trajectory while avoiding zones that facilitate the formation of contrails. Ripper *et al.*[34] studied different graph search algorithms to solve the trajectory problem for general aviation aircraft. For the trajectory graph search, the space where the aircraft can fly is discretized, so an ensemble of grid points is created within which the aircraft can vary its altitude and lateral position. Cobano *et al.*[35] developed a trajectory planning algorithm in a conflict detection and resolution system from a 4D trajectory approach. This algorithm is interesting because of its use of the Particle Swarm Optimization and because it always provides a solution. However, they used an aircraft model that does not correspond to the models used in industry or in the literature. Although equations of motion have produced good results, they have the drawback of demanding (substantial) computational resources for their solution; making it complicated or impossible to implement EoM in a FMS, which normally has a low power processor. Commercial FMSs normally use a performance model in the form of a database and a Navigation database (NDB). The performance model contains the experimental information of a given aircraft and the NDB is a list of the published waypoints an aircraft can use to follow a trajectory. Different optimization algorithms have been developed using a performance model in the form of a database.

Dancila *et al.*[36] proposed an algorithm using a database to find the most economical altitude in cruise, and validated this algorithm against three different aircraft. Dancila *et al* used their own method[37] to predict the fuel burn in a given cruise segment. Felix *et al*[38,39] proposed a vertical reference trajectory optimization algorithm with the same performance model database. Felix found the optimal speed schedule and the optimal cruise altitude using the Golden Section Search for flight distances of less than 500 nm. For flight distances larger than 500 nm, this algorithm evaluated the possible step climbs in every waypoint defined along the route. Nevertheless, this algorithm did not take wind into account, and it required a complete analysis of all the available pair Indicated Air Speed (IAS)/Mach climbs and of all the Mach/IAS descents making it time consuming. An alternative reduction search algorithm was developed[40] by selecting candidate cruise optimal solutions. The branch and bound method was used to cut planes and reduce the computation time to optimize the vertical trajectory of reference[41, 42]. Genetic algorithms have been implemented to select the best waypoints with which to execute a step-climb[43, 44]. Two algorithms for the FMS trajectory optimization were developed: one to avoid No-Flight-Zones (NFZ) - zones where the aircraft should not fly - and the other to find the optimal trajectory of an aircraft by combining the optimization techniques of descent and taboo search[45,46]. Another important factor to be considered is the meteorological influence. Temperature and wind play a significant role in flight optimization. High temperatures reduce engine performance, thus it is important to avoid altitudes with higher temperatures, and to seek out colder altitudes. Head winds reduce the ground speed; under these winds the engines need to increase their thrust just to maintain the desired speed, otherwise the flight would take longer than expected. Tail winds tend to "push" an aircraft, making it fly at a higher ground speed. Weather conditions are important for decision making. For example, the decision to perform a step climb is affected by the wind and temperature at the different altitudes. Houghton[47] studied jet streams and how they can be identified while airborne. There are a number of weather models that have been implemented, such as the National Oceanic and Atmospheric Administration (NOAA) weather[48]. Felix *et al.*[49] considered weather factors (winds) for a lateral reference trajectory and found the most economical routes using genetic algorithms. At each cruise waypoint, the aircraft considered a possible deviation in order to profit from tailwinds or to avoid strong headwinds. Genetic algorithms were later used to find the most economical vertical and lateral reference trajectories[50, 51]. The artificial bee colony algorithm has been used to optimize the lateral reference trajectory with the novelty of using a dynamic grid[52]. Dijsktra's algorithm was used to solve this problem by focusing on reducing the flight time[53]. As observed in the literature, one way to solve the trajectory cost reduction problem is by optimizing one of the flight phases. For long flights, cruise is normally the targeted phase due to its length and thus the larger number of optimization opportunities that exist (as well as its being the phase where most of the fuel is consumed). In short flights, climb normally represents the most expensive part of a trajectory. As stated above, descent has been of great concern for researchers. However, optimizing only one phase does not guarantee that the overall optimal trajectory profile will be found. This paper proposes an optimization algorithm to be implemented in the FMS which considers all the flight phases during flight, including step climbs. This algorithm takes into account take-off weight, the weather forecast and the flight time. The weather information is obtained by using the information provided by Environment Canada. This algorithm fetches the weather forecast from the exact geographical position of the aircraft at the exact time the aircraft is there. The fuel burn information is given in the form of a database. The algorithm follows the geodesic trajectory or great circle (shortest path between two points in a sphere) between the initial and the final points. The algorithm can propose altitude changes at will since it assumes a free vertical flight mode. This is one of the future aspects of airspace being proposed by SESARS and NextGEN, where aircraft will be able to directly negotiate their individual trajectories with air control[54]. Testing and validation of this algorithm is performed using an aerodynamic model in FlightSim® by Presagis®, and the profiles it generates are compared against the profiles generated by a commercial FMS. When the word "optimal" is used in this document, it refers to those conditions that create a trajectory that will have the lowest costs. This paper is structured as follows: First, the numerical model of the aircraft is explained, followed by a description of a typical commercial flight. The algorithm is presented in detail in section III. The weather model from Environment Canada is then explained, as well as how the algorithm uses it. Next, the algorithm's results are evaluated with an aerodynamic model and the profiles it generates are compared to those generated by a commercial FMS. The paper ends with a conclusion and some of the anticipated future work.

II. METHODOLOGY

Aircraft Performance Model

An aircraft performance model is given in the form of a database. This model is usually created from experimental flight data from a commercial aircraft. However, this numerical performance model database can also be created using a flight simulator.[55-57]

The performance model database is divided into sub-databases with information about aspects of each of the three main flight phases, climb, cruise and descent: indicated airspeed (IAS) climb, acceleration, Mach climb, cruise, Mach descent, descent deceleration and IAS descent. Table 1 describes the inputs and outputs of the different sub-databases. ISA dev stands for the ISA temperature deviation, which is the difference from the ISA temperature at a given altitude and the temperature at the "measured" altitude.

Table. 1 Sub-databases from the numerical performance model

Sub-database	**Inputs**	**Outputs**
IAS Climb	IAS (knots) Gross weight (kg) ISA deviation temperature (ºC) Altitude (ft)	Fuel burn (kg) Horizontal traveled distance (nm)
Climb acceleration	Gross weight Initial IAS (knots) Altitude when acceleration begins (ft) Delta speed to accelerate (knots)	Fuel burn (kg) Horizontal traveled distance (nm) Altitude needed (ft)
Climb Mach	Mach number Gross weight (kg) ISA deviation temperature (ºC) Altitude (ft)	Fuel burn (kg) Horizontal traveled distance (nm)
Cruise Mach	Mach number Gross weight (kg) ISA deviation temperature (ºC) Altitude (ft)	Fuel flow (kg/hr)
Descent Mach	Mach number Gross weight (kg) ISA deviation temperature (ºC) Altitude (ft)	Fuel burn (kg) Horizontal traveled distance (nm)
Descent Deceleration	Gross weight Initial IAS (knots) Altitude when deceleration begins (ft) Delta speed to accelerate (knots)	Fuel burn (kg) Horizontal traveled distance (nm) Altitude needed (ft)
Descent IAS	IAS (knots) Gross weight (kg) ISA deviation temperature (ºC) Altitude (ft)	Fuel burn (kg) Horizontal traveled distance (nm)

Using this database, algorithms have been developed to compute the fuel burn, the flight time, and the flight cost[58, 59]. These methods are based on the Lagrange interpolation shown in Eq (1):

$$p_1(x) = \frac{x - x_1}{x_0 - x_1} f_0 + \frac{x - x_0}{x_1 - x_0} f_1 \tag{1}$$

where x_0 corresponds to the lower input limit, f_0 corresponds to the lower input limit output, x_1 corresponds to the upper limit, f_1 corresponds to the upper limit output, and x corresponds to the desired value. Interpolations are executed first for the required ISA deviation temperatures, and then using those results, interpolations for the weight are performed. Interpolations between speeds and altitudes are not executed. Altitudes are always given in multiples of 1,000 ft in accordance with the regulations. Speeds are given in multiples of 10 knots or 0.05 Mach numbers.

Flight Description

The flight reference trajectory to be optimized is the vertical reference for a fixed lateral reference. From this point onwards, the trajectory to be optimized will be referred to as the "flight plan". The flight performed in this algorithm begins over the selected departure airport at 2,000 ft and climbs at a fixed speed of 250 IAS to 10,000 ft. This speed restriction is due to the FAA 14 CFR Section 91.117 that forbids aircraft to fly faster than 250 IAS in altitudes below 10,000 ft. The initial altitude of 2,000 ft was chosen due to data availability in the performance model and to avoid any problems with SID (Standard Instruments Departure) variation in different airports. Following this initial IAS climb, an acceleration to the desired IAS is performed. This IAS is kept constant at an altitude called the "crossover altitude". The crossover altitude is the altitude where the IAS equals the desired Mach number. From this point the climb continues in Mach until the desired cruise altitude is attained, called the Top of Climb (TOC). The cruise is flown following a constant Mach number until the beginning of descent, called the Top of Descent (TOD). According to the literature, the ideal cruise is one that gradually gains altitude, mainly for two reasons; as fuel is burned, the weight goes down, making the aircraft more efficient at higher altitudes, and at high altitudes the air density decreases (the air is "thinner"), reducing the drag. Due to airspace restrictions, performing a constant-climb cruise is difficult to perform due to the large airspace an aircraft would need in order to fly safely. To avoid this problem while taking advantage of higher altitudes, small changes in altitudes of normally 2,000 ft or even 4,000 ft followed a constant altitude cruise can be performed. This change of altitude is called a "step climb". For the descent phase the stages of flight in climb are repeated but in the inverse, acceleration is changed for a deceleration including the speed restrictions in place from 10,000 ft to 2,000 ft.

<u>Weather Model Description</u>

The forecast model is the open source model provided by Environment Canada. This model provides information in the form of a 0.6º x 0.6º grid for many different isobaric altitudes (pressure altitudes). As the aircraft gains altitude, the pressure in the atmosphere diminishes, thus setting standard pressures to define altitudes is a normal practice among meteorologists and within aeronautics. The forecast is also given in 3-hour time blocks. A series of interpolations (two linear followed by a single bilinear) are executed to obtain the weather information at the exact location of the aircraft for a given time. Figure 1displays the aircraft in space and time to visualize the required interpolations.

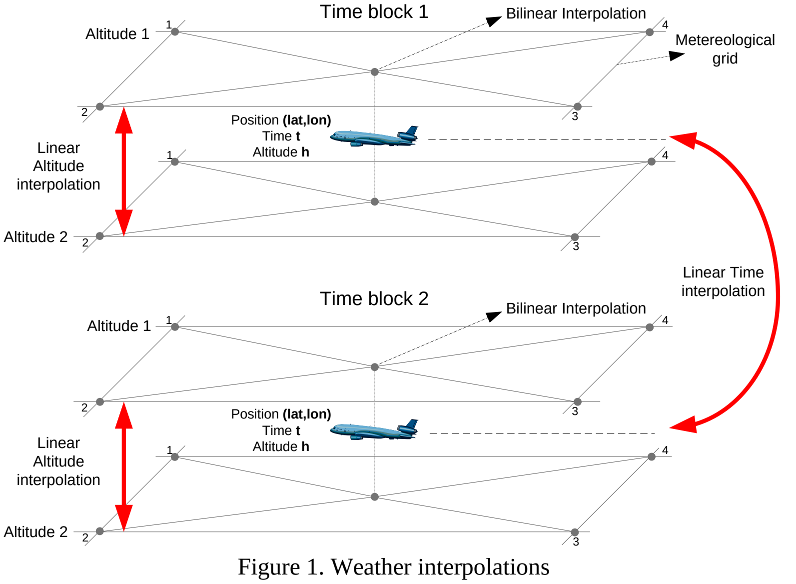

Figure 1. Weather interpolations

The bilinear interpolation is executed following Eq. (2)

$$v(lon, lat) = \frac{P_1(i,j)v_1(i,j) + P_2(i,j)v_2(i,j) + P_3(i,j)v_3(i,j) + P_4(i,j)v_4(i,j)}{P_1(i,j) + P_2(i,j) + P_3(i,j) + P_4(i,j)} \tag{2}$$

where V_n is the meteorological information of interest at each of the points, P_x represents the weight of each node, i is the horizontal axis, and j represents the vertical axis. The node has more influence the closer it is to the aircraft. The weight function is shown in Eq (3), where P_x is the weight equation for the point of interest.

$$P_x(i,j) = \frac{1}{\sqrt{(x-i)^2 + (y-j)^2}} \tag{3}$$

The graphic significance of Eq (2) and Eq (3) is shown in Figure 2.

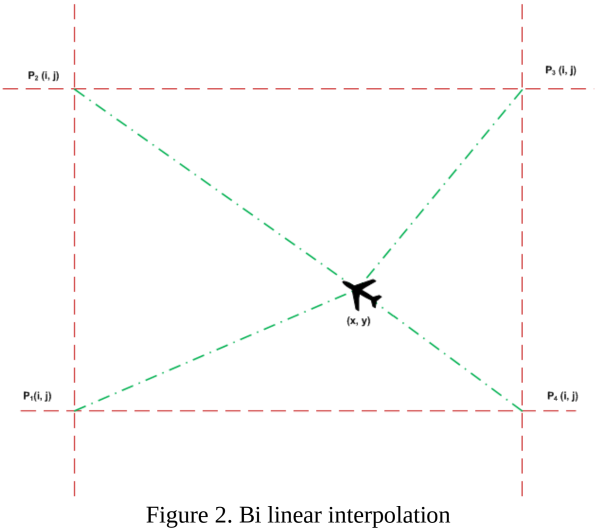

Figure 2. Bi linear interpolation

The aircraft speed relative to the ground (ground speed - GS) is influenced by the wind; tailwind increases the GS while headwind decreases it. The GS is computed from the IAS as follows. When an aircraft is flying in the atmosphere, the IAS (in knots or in Mach) being measured by the instruments is always the same regardless of wind speed. Compensation calculations for the pressure and density of this speed are performed afterwards, to find the True Air Speed (TAS). The GS is computed from the TAS and the wind speed (WD) with the wind triangle as shown in Figure 3.

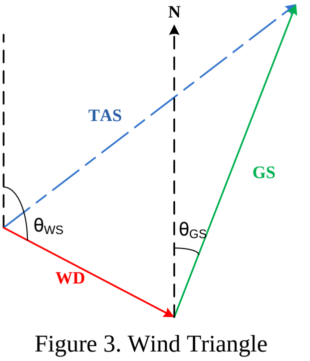

Figure 3. Wind Triangle

Figure 3 indicates that the wind speed vector and the GS vector form the TAS, as shown in equations (4) to (6):

$$TAS^2 = TAS_x^2 + TAS_y^2 \tag{4}$$

$$TAS_x = GScos\theta_{GS} + WScos\theta_{WS} \tag{5}$$

$$TAS_y = GSsin\theta_{GS} + WScos\theta_{WS} \tag{6}$$

where WS is the wind speed, θ_{GS} is the destination angle, and θ_{WS} is the wind angle. By substituting equations (4) and (5) into (6), a second-degree equation can be obtained. The GS is computed by solving Eq (7)

$$GS^2 - 2(GS)(WS)(cos\theta_{GS}cos\theta_{WS} + sin\theta_{GS}sin\theta_{WS}) + WS^2 - TAS^2 = 0 \tag{7}$$

<u>Cost Function</u>

Fuel consumption represents only a part of the flight cost. In order to optimize the total flight cost, both fuel and flight time have to be considered. Flight time represents the operating costs of the aircraft, such as crew salary, the need for the aircraft to be at a certain waypoint at a given hour, etc. A variable called the Cost Index (CI) is used to assign a cost to time in terms of fuel. The fundamental rationale of the cost index concept is to achieve minimum overall trip cost by means of a trade-off between operating costs per hour and the fuel needed to perform the flight, as the final cost of a flight depends on both time and fuel consumption. Eq (8) is the objective function to be minimized:

$$Total\ Cost = Fuel\ Consumption + CI * Flight\ Time \tag{8}$$

Where Fuel Consumption is given in Kg, CI is given in Kg/hr and Flight Time is in hours. The higher the CI, the greater the priority given to flight time over fuel consumption. A CI of zero would give 100% of the priority to fuel consumption with zero consideration to flight time.

III. OPTIMIZATION ALGORITHM

Aircraft flight phases have different degrees of importance, depending on the type of flight (short or long haul). For short flights, it is more important to find the most economical speeds and altitude for the climb and descent phases, because the cruise phase is too short to have muich impact on the flight cost. For long haul flights, however, the descent becomes the least important phase of the flight to be optimized, and it is the cruise phase that is the most important. Cruise is influenced by the climb phase, as a plane arriving too heavy at the TOC would demand more fuel for the cruise phase. Therefore, it is important to consider all three flight phases to find the optimal flight plan, which is why the algorithm proposed in this paper takes into account all three flight phases.

<u>Algorithm Overview</u>

The algorithm is composed of the following steps:
Step 1: Data Input and validation
Step 2: Fuel verification and final weight estimation
Step 3: Climb and TOC location calculations
Step 4: Descent cost estimation and TOD location estimation
Step 5: Cruise and step climb cost calculations
Step 6: Optimal finder
Step 7: Descent Calculation and cruise distance correction
Step 8: Results display
These steps are described in detail below.

Step 1: Data Input and its validation

The algorithm-required inputs are the aircraft take-off weight, the take-off time, the CI, the initial and final coordinates, the desired step climb, and the cruise-range segments. Most of these inputs are included in the standard information that any FMS would need to begin its calculations. However, the altitude of the desired step climbs and the cruise range are extra information required for the algorithm. The cruise range is the distance at which the weight is revised, and where a step climb evaluation is performed, i.e. the interpolation range between two waypoints. The altitude of the step climb is the height the aircraft would climb if a step climb is to be executed.

Step 2: Fuel requirements' verification and final weight estimation

The algorithm verifies that the take-off is coherent with the flight distance. If the maximum take-off weight is exceeded, or if too much fuel remains at the end of cruise the algorithm sends a message requiring the user to modify the initial take-off weight. For this paper, the final weight calculation is performed at 0.805 Mach at 36,000 ft. These values of speed and altitude are the average values of those available for this aircraft model. The weight at the end of cruise will help to estimate the descent cost in step 4.

Step 3: Climb and TOC location calculations

In the aircraft model, 169 possible climb combinations (IAS/Mach number) can be calculated. For a constant IAS and varying Mach number for climb, it was observed that the crossover altitude was located at different altitudes: the higher the Mach number required for a constant IAS, the higher the crossover altitude will be located. This relationship is used to reduce the number of computations for climb; instead of computing the climb costs for each IAS/Mach Number combination, a given IAS climb is only computed once for the highest possible altitude. The IAS climb costs are then taken at the desired altitude for the desired Mach. For example, for IAS 250, the maximal Mach number available is 0.84, so that for the 250/0.84 pair, the crossover altitude is 41,581 ft. The algorithm computes the climb at an IAS of 250 from the current altitude until the altitude of 42,000 ft. Next, the algorithm has to compute the climb for the same IAS of 250 at a lower Mach number that is 250/0.835 with a crossover altitude of 39,885. It is obvious that because the climb was performed at the same IAS, the values can be safely transmitted from the 250/0.84 climb to the 250/0.835 climb, because the climb in IAS is the same as the one calculated up to the crossover altitude of 39,885. The values above the crossover altitude must be calculated using the Mach Climb database. Figure 5 shows how the climbs in IAS are overlapped and how the climb in Mach number begins to appear as their crossover altitudes are found.

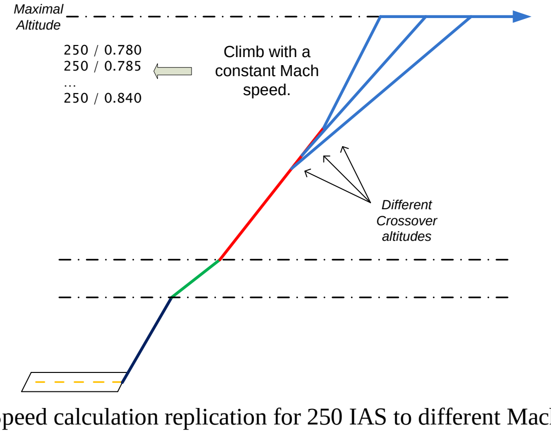

Figure 4. Speed calculation replication for 250 IAS to different Mach numbers

The result of this stage is a climb cost matrix and a TOC location matrix. The first helps with the final cost calculation. The second is essential in the cruise cost calculation, since the cruise begins at the end of the TOC.

Step 4: Descent cost estimation and TOD location estimation.

With the aircraft weight at the TOD calculated in step 2, an estimate of the descent will be calculated for all the IAS/Mach number pairs for each altitude available for the cruise. This estimation is calculated in order to define an estimated TOD location for each pair. This information is important for the cruise calculation, as it indicates where the cruise calculation procedure should stop. To reduce the time it takes to compute the descent estimation, it can be assumed that the weight of the aircraft has a negligible effect on the descent, as shown below. Figure 5 is the plot of the descent cost from 43,000 ft at the maximal and minimal allowable weights and speeds. It can be seen that at low speeds (240 IAS) the curves overlapped, so that there is virtually the same fuel consumption for the maximal and minimal weights (120,000 and 170,000 kg). The difference between the fuel burn variations with altitude increases gradually and achieves its highest value at the maximum speed of 365 IAS, where the fuel consumption difference is 104 kg at 43,000 ft for the maximal weight of 170,000 kg.

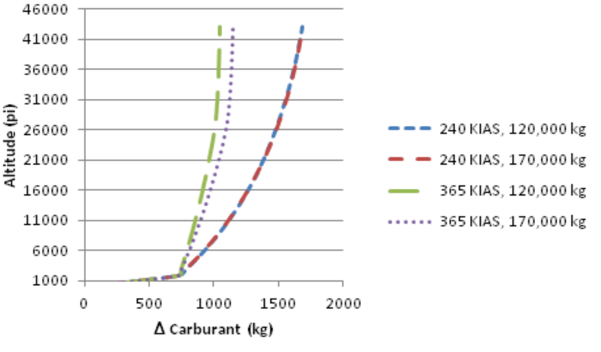

Figure 5. Variation of fuel burned with altitude in the descent phase for different speeds and weights

For the TOD estimation, the variable of interest is the distance traveled. Figure 6 is the plot of distance traveled at the maximal and minimal allowable descent weights and the maximal and minimal descent speeds. The distance traveled variation at the lowest speed is practically zero, as the curves are overlapped. The distance difference gradually increases with the increase of speed, up to the maximum speed of 365 IAS for the maximal weight as shown in Figure 6. It can be observed that this variable has a low sensitivity.

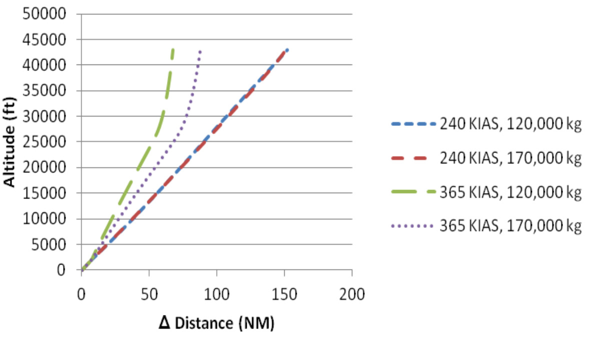

Figure 6. Variation of distance traveled with altitude in the descent for different speeds and weights

The final descent estimation is calculated for a single weight at the maximum altitude. The distance of a descent from a lower altitude is obtained by replicating the results of the calculated descent. This assumption of taking just the maximal altitude is only valid for the estimation, and not for the final descent calculation.

When descending in Mach, the number of computations can be reduced, similar to the process explained in Step 3 for climb. However, during descent, the replicated values are those related to the Mach number. The fuel burn, and horizontal distance are in IAS are the ones being calculated during the descent. The IAS in the descent regime is not related to the climb IAS. The algorithm calculates the descent cost from all the available altitudes. For each altitude, the Mach/IAS combination with the lowest cost is the one selected, and distance traveled and the cost are saved to a matrix table for further reference.

Step 5: Cruise and step climb calculations

This phase will normally be the longest to calculate because of the large number of interpolations required. The distance traveled has a strong effect on the calculation time due to the number of computations. The minimal cruise distance that the algorithm can calculate is equal to the distance segment defined in the inputs. From the TOC at the desired altitude and speed (calculated in Step 3), the algorithm calculates the cruise cost per distance segments of the length defined in the cruise range while tracking the estimated TOD location (calculated in step 4). The last segment of the cruise may not be exactly the same as the other segments of the nm. For example, a cruise distance of 860 nautical miles is divided into 34 distances of 25 nm and the last 35th distance of 10 nm. The weight of the aircraft is updated for each of these and then the influence of weather on the flight is calculated.

Search and calculation of step climbs during cruise

During this phase, the algorithm determines if a step climb can help to reduce the cruise cost. The algorithm verifies if the fuel consumption would be lower at higher altitudes than the current cruise altitude. The algorithm will perform this verification until there is no further possible climb. The algorithm will not do this verification at the TOC; since it is the first waypoint of the cruise, a climb at this point signifies that the airplane is not yet at its optimal altitude. Figure 7 shows the step climb in the cruise phase evaluation.

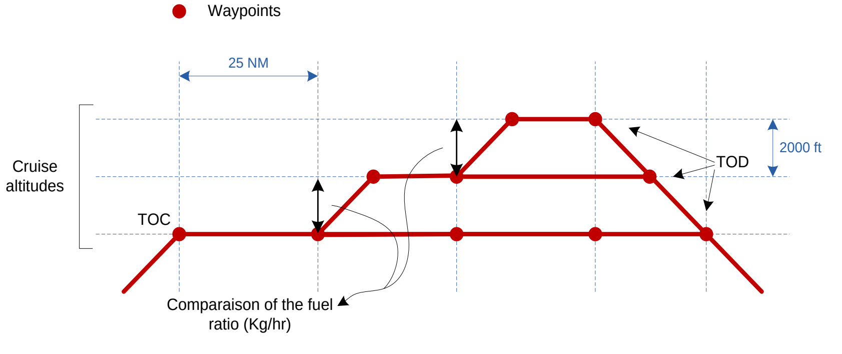

Figure 7. Step Climb altitudes comparison

In this figure, the distance between waypoints is 25 nm and the separation of altitudes is 2,000 ft. When the aircraft reaches the first waypoint of cruise, the fuel flow (kg/hr) of the current flight level trajectory is calculated, and then the fuel flow after a pre-defined step climb is calculated.

If the fuel flow would be reduced by climbing to a new flight altitude, the algorithm suggests climbing; otherwise the aircraft maintains the original flight altitude.

Whether maintaining the original flight level or climbing to a new one, when the aircraft reaches the next waypoint, a similar type of comparison of fuel ratios between two flight altitudes is performed until the maximal altitude or the TOD is reached. Step-climbs of 2,000 ft or 4,000 ft are performed depending on the user, since it is the altitude change recommended by the literature[60].

Altitude search range selection

Computing all the available altitudes with all the possible climbs would require significant amount of computation resources which would lead to an algorithm that could not be implemented in an FMS. To reduce the computation time, a procedure to select a range of altitudes to calculate is executed.

It was observed that the optimal altitudes particular for each Mach number were condensed in a limited range of altitudes that never exceeded 6,000 ft. Evaluating more than 6,000 ft would increase the space search and make the algorithm too heavy, but a search space of less than 6,000 ft would be too small and there would not be enough confidence about finding the optimal altitude.

To define where this 6.000 ft layer is to be located, using the first Mach number available in the PDB, the algorithm calculates all the cruise altitudes costs starting at the minimum altitude of 20,000 ft. The algorithm then searches for an optimal candidate altitude from among those results.

Therefore, for the next Mach number, the algorithm does not begin to calculate from the first available cruise altitude in the PDB, but instead calculates from 6,000 ft lower than the pre-optimal altitude mentioned above.

For example, for a given pre-optimal altitude of 36,000 ft, which can be translated into a flight level of FL360, calculated at the first Mach number of 0.75; the calculation of the next Mach number (0.785) will not begin at 20,000 ft, but from 30,000 ft because of the 6,000 ft that will be used to search for the final optimal solution. The algorithm will thus save time by calculating the cruises beginning from 30,000 ft instead of from the minimal altitude available in the performance model.

Step 6: Optimal Finder

To find the optimal trajectory, the costs of all the IAS/Mach climb pairs with their altitudes and descents are calculated using Equation (8). The IAS/Mach climb with the cruise and descent that produces the least expensive trajectory is defined as the optimal.

Step 7: Descent final calculation

Because the descent used in the above calculations is estimated, the real descent is needed to provide the aircraft with the real TOD, the real flight cost and the expected time of arrival (ETA). Based on the knowledge of the optimal flight parameters, such as IAS climb speed, Mach cruise speed, flight altitude and the estimated TOD, the final descent will be re-calculated. If the airplane is not found exactly at the final coordinates at the end of descent, the length of the cruise will be modified accordingly by adding or removing distance in the cruise phase, and the descent phase will be recalculated to attempt to end the flight at the exact coordinates.

Thus, the position of the TOD will change as well as the total time and fuel required for the flight. However, the descent will not be recalculated for a third time; the effect of weight and the changes in the location of the TOD after the second recalculation are negligible in the calculation of the descent as explained above. Figure 9 is a roadmap of the steps followed in this algorithm.

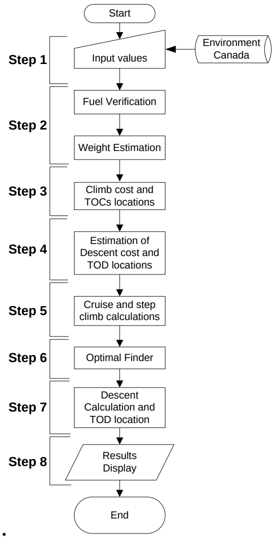

Figure 8. Algorithm steps roadmaps

Step 8: Results Display

The computed trajectory is provided with the fuel requirements, the Estimated Time of Arrival (ETA), and the computation time.

IV. RESULTS

This section describes the simulations performed to validate the optimized trajectories. The optimized flight plan was compared against those provided by the FMS of reference, which uses the same performance model as the algorithm developed in this paper. The FMS of reference was coupled with FlightSim® developed by Presagis®.

Reference trajectories' cost comparison

For the search of the optimal vertical reference trajectory, the initial weight used was the maximum take-off weight for testing aircraft, while the maximum weight at descent would always be below 170 tons. The Edmonton Airport was chosen for the departure of most of these flights because of its geographical position, allowing many different flight distances to be traveled. The weather conditions were considered to be ISA for these tests, and thus wind was not taken into account. This elimination of the weather influence was done because the algorithm uses weather in a way that is not comparable to how the weather is used by the reference FMS. A total of 10 flights were optimized with the reference FMS. The flight profiles proposed by the FMS can be seen in Table 2. The same 10 flights under the same flight conditions where optimized with the algorithm proposed in this paper. The proposed flights are shown in Table 3. The flight parameters observed were the cruise altitudes and flight speeds. In these tables, FL390 is equivalent to a Flight Level of 39,000 ft. The speeds V are given in knots, and the Mach number is non-dimensional. The CI value selected was 0 in order to observe the maximum amount of fuel that can be reduced.

Table 2. FMS optimal output flight parameters

Fight #	Cruise altitude start (ft)	Cruise altitude end (ft)	V_{Climb} (knots/knots/ND)	V_{Cruise} (ND)	$V_{Descent}$ (ND /knots/knots)
1	FL376	FL396	250/316/0.824	0.824	0.800/300/240
2	FL376	FL376	250/300/0.824	0.824	0.800/300/240

3	FL370	FL390	250/319/0.824	0.824	0.800/300/240
4	FL381	FL381	250/313/0.824	0.824	0.800/300/240
5	FL388	FL388	250/310/0.824	0.824	0.800/300/240
6	FL387	FL387	250/311/0.824	0.824	0.800/300/240
7	FL372	FL392	250/320/0.824	0.824	0.800/300/240
8	FL385	FL385	250/312/0.824	0.824	0.800/300/240
9	FL391	FL391	250/308/0.824	0.824	0.800/300/240
10	FL390	FL390	250/290/0.810	0.824	0.800/300/240

Table 3. Algorithm optimal output flight parameters

Flight #	Cruise altitude Start (ft)	Cruise altitude end (ft)	V_{Climb} (knots/knots/ND)	V_{Cruise} (ND)	$V_{Descent}$ (ND /knots/knots)
1	FL340	FL360	250/310/0.82	0.82	0.78/260/250
2	FL360	FL360	250/310/0.82	0.82	0.78/260/240
3	FL340	FL360	250/310/0.82	0.82	0.78/260/240
4	FL360	FL360	250/300/0.82	0.82	0.78/260/240
5	FL360	FL360	250/300/0.82	0.82	0.78/260/240
6	FL360	FL360	250/300/0.82	0.82	0.78/260/240
7	FL340	FL360	250/310/0.82	0.82	0.78/260/240
8	FL360	FL360	250/300/0.82	0.82	0.78/250/240
9	FL300	FL300	250/270/0.79	0.79	0.79/300/240
10	FL360	FL360	250/300/0.81	0.81	0.81/280/240

It can be seen that the parameters of Table 2 and Table 3 are different; this is because the new algorithm optimizes the route as a complete trajectory and not by phases as with the current FMS. This can be seen especially by comparing flights 9 and 10 where the cruise speeds and the altitudes are completely different.

The flights cost of each flight profile in Table 2 and Table 3 was evaluated using FlightSim® and compared; for example, flight one of Table 2 was compared with flight one of Table 3 and so on. The results of these comparisons are presented in Table 4.

Table 4. Comparison of fuel consumption between the FMS of reference and the proposed algorithm

Flight	Departure Airport	Arrival Airport	Distance (nm)	Fuel Savings Program vs. FMS (%)	Fuel Savings Program vs. FMS (kg)	Computation Time (s)
1	Edmonton	Toronto	1456	2.17	500	41
2	Edmonton	Chicago	1233	1.63	319	35
3	Edmonton	Houston	1611	1.57	407	45
4	Edmonton	San Francisco	1010	2.17	353	32
5	Edmonton	Vancouver	438	2.37	178	15
6	Edmonton	Yellowknife	550	2.25	203	16
7	Edmonton	Ottawa	1537	1.55	379	43
8	Edmonton	Winnipeg	640	2.49	275	17
9	Edmonton	Fort McMurray	217	5.59	235	9
10	Montreal	Thunder Bay	663	2.40	259	20

The fuel required to execute the flight plan proposed by the new algorithm was always less than the amount of fuel required to execute flight plans proposed by the reference FMS. An average savings of 2.4% in fuel consumption can be computed from Table 4. For flight test #9, the saving (%) obtained by using the algorithm's parameters is more than double the average fuel savings achieved by this algorithm. This significant fuel economy is due to the

FMS' inability to choose the flight parameters (IAS/Mach climb speed, Mach/altitude of cruise and Mach/IAS descent speed) due to the short flight distance. The source of this inability comes from the FMS optimization methodology; optimizing the parameters for each phase and not for the whole flight. In this case, the FMS chose a cruise altitude that was too high for the corresponding distance, and thus a high quantity of fuel was required for the climb phase.

Weather calculations' validation

The proposed algorithm and the reference FMS handle meteorological information differently, in ways that are not directly compatible. The FMS accepts information for up to 4 altitudes along the whole trajectory for a large number of waypoints along the route. This information is introduced manually by the pilot. Linear interpolations between those points are then performed to estimate the weather at the location of the aircraft. The weather for the proposed algorithm is a dynamic grid with the weather at different altitudes. However, how the weather is used by the FMS and by the developed algorithm is similar. The FMS and the algorithm as well as FlightSim® use the triangle of wind to compute the effect of wind on the the ground speed.

Comparing their values to those of FlightSim® would help to validate the weather effect computations. FlightSim® is limited to 16 meteorological columns per waypoint. In other words, 16 points at different altitudes from the environment Canada weather model. These points would help to cover only small distances. The maximum distance that can be covered is 400 nm. The limitation of FlightSim® against the algorithm is that the meteorological information is fixed in time, which means that the exact information for the time when the aircraft will pass through of all the points has to be exactly known, instead to computed in in real time as the algorithm does. This information can be recovered while running the algorithm.

Since the exact meteorological information can be introduced in the algorithm and FlightSim®. The meteorological conditions were only tested for the algorithm and FlightSim® in a cruise phase of 253 nm at FL320. The parameters from the grid obtained from Weather Canada and introduced in FlightSim® were the temperature, wind angle and wind speed. Three slices of the grid were introduced in FlightSim® to perform the interpolation for the required FL, one below and one above. This test was not performed for the FMS of reference since it is not possible to introduce the identical information used for the algorithm because of how this device calculates meteorological influence. Table 5 shows that the differences between the algorithm and FlightSim® predictions are low; the fuel error can be attributed to the interpolations effect. This test does not intend to show that our method is better than the one used in FlightSim®; in order to prove that, a flight with the exact experimental data would need to be tested in both platforms. However, these tests serve to validate that the developed algorithm is able to correctly compute the effects of weather.

Table 5. Flights with meteorological conditions.

Flight	**Parameters**	**Flightsim**	**Algorithm**	**Difference**	**Difference (%)**
1	**Fuel (kg)**	3575.69	3563.79	11.90	-0.33%
	Time (s)	1936.60	1932.57	4.03	-0.21%
2	**Fuel (kg)**	3524.00	3551.65	-27.65	0.78%
	Time (s)	1939.00	1941.15	-2.15	0.11%

V. CONCLUSION

In this paper an algorithm able to calculate the VNAV profile was developed and implemented using a numerical performance model. This algorithm was developed with the aim of being implemented in a current generation of FMSs. By comparing the flight costs of the optimized vertical reference profiles delivered by the algorithm with those delivered by the FMS of reference, it was observed that the algorithm provided trajectories that were more economical in terms of fuel burn than those from the reference FMS. An average savings of 2.4% were reported, with a maximal savings of 5.59% for a particular flight. An analysis of the descent fuel burn and required distance sensitivity due to total weight revealed practical considerations to be taken into account to reduce the computation time. It was also observed that the computation time in climb can be reduced by replicating the fuel burn computed for similar climbs. Along with more economical vertical trajectories of reference, the proposed algorithm successfully integrated a precise and efficient meteorological model. Instead of linear interpolations between fixed meteorological variables captured by the pilot, this new algorithm downloads the latest meteorological data from Environment Canada.

The algorithm's computation time is relatively low. This time was on the order of minutes on the computer on which it was tested. The computation capacity of FMSs is low; the complete algorithm execution can be separated into different cycles of the FMS.

VI. FUTURE WORK

Although the calculation times are acceptable, it is desirable to implement techniques to reduce the number of computations required to find the optimal trajectory. For example, a pre-selection method based on the flight time, the distance to travel and the initial total weight of the aircraft can be implemented to better define the optimal candidate altitudes and speeds of cruise and climb before beginning a flight. To more easily incorporate the weather influence in a flight, the bilinear interpolation can be reduced to a simple interpolation by which the calculus time per point can be reduced. A further study should be conducted to observe the weather influence using this proposed method. This algorithm follows the trajectory and the free flight concept. It would be worthwhile investigating the addition of restrictions such as altitude and speed as well as traffic along the route to be considered, to define a more realistic route. The present algorithm calculates the trajectory once; the airplane continues to fly in that trajectory throughout the whole flight. As future improvements, the optimal path could be continuously recalculated during flight, considering the current conditions of the airplane and any possible updates of the meteorological information, thus providing better routes using the real aircraft data. A means to visualize the weather forecast used in this algorithm could be further developed and added to the FMS, allowing the pilot to gain immediate knowledge of this information. This could help to avoid hazardous weather. Weather information is used here to find altitudes for the VNAV where the wind may be favorable during the cruise phase; always calculated by the great circle (geodesic). However, this algorithm could also consider weather information to optimize the lateral trajectory of reference to follow. The total cost of the flight could thus be reduced even more if a lateral reference trajectory is coupled with the vertical trajectory of reference.

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