L1 adaptive control design for the rigid body launch vehicle

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DOI: 10.13111/2066-8201.2023.15.4.16

Received: 23 October 2023/ Accepted: 17 November 2023/ Published: December 2023
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Abstract: *This paper investigates the use of an L1-adaptive controller to improve the performance of the Vega launch vehicle because the classical controller does not guarantee stability and tracking of the system in the transient. The L1-AC ensures uniformly bounded transient and steady-state tracking for both systems' signals, input, and output. In this paper, we used the equations of the adaptation and the L1-norm with two filters, the first one is first-order order and the second filter is third-order, we used the large adaptive gain with the first filter, also used the low adaptive gain with the second filter, and after the analysis the result numerically we found the lambda with the first filter less than 1 and the lambda with second filter larger than lambda with the first filter. The L1 adaptive controller can generate a stable system response to track the control input and the system output, both in transient and steady-state because we selected the adaptive gain large with minimize lambda. It is noted that the system response for the L1 adaptive control configuration with the first filter, as compared with the system response with the second filter, has much better performances, both from the point of view of the overshoot and rise time.*

Key Words: *L1 adaptive controller, Vega launch Vehicle, Performance and Stability, smooth transient tracking, PD controller*

1. INTRODUCTION

A launching vehicle is aerodynamically unstable and problems in designing the control system may also appear due to the parametric uncertainties and unbounded disturbances [1]. The adaptive controller ensures uniformly bounded transient and asymptotic tracking for both the system's input and output signals, simultaneously [2]. The rigid body launcher stage configuration [3]. The adaptation bounds can be improved by increasing the rate of adaptation, Fig. 1 explains that the increase in the adaptive gain (Γ) improves the tracking performance for all ($t \geq 0$), including the transient phase according to the relationship of $L_1(s) = \frac{\Gamma}{s(s+1)}$. The L1 adaptive controller ensures uniformly bounded transient and steady-state tracking for both system's signals, input, and output, compared to the same signals of a bounded reference LTI system, which assumes partial cancelation of uncertainties within the bandwidth of the control channel. In this paper, we chose the Vega launch vehicle because this launcher was calculated its performance and stability by using the classical controller in [4], but the stability and the performance of this launcher unguaranteed with this classical controller. In this study, we used the Vega launcher as a rigid body and in the next studies, we will find the performance analysis of the L1 adaptive controller in the presence of flexible modes.

By using the L1-norm condition we will find the value of $\lambda \triangleq \|G(s)\|_{L1} L < 1$, with a number of frequencies and to get the frequency that makes $\lambda < 1$, with the highest adaptive rate, and according to the Theorem 2.1.1 and Lemma 2.1.4 [2] imply that the L1 adaptive controller can generate a system response to track the system state reference and the control input reference both in transient and steady-state if we select the adaptive gain large and minimize λ [5].

In this study, we used two filters, the first one was in the first order and the second was in the third order, we will compare the results of the two filters, and we will use different adaptive gains, the first adaptive gain is higher, but the other is lower. In this study, we will submit an adaptive control solution, which ensures that the system output follows a given piecewise continuous bounded reference signal with quantifiable transient and steady-state performance bounds.

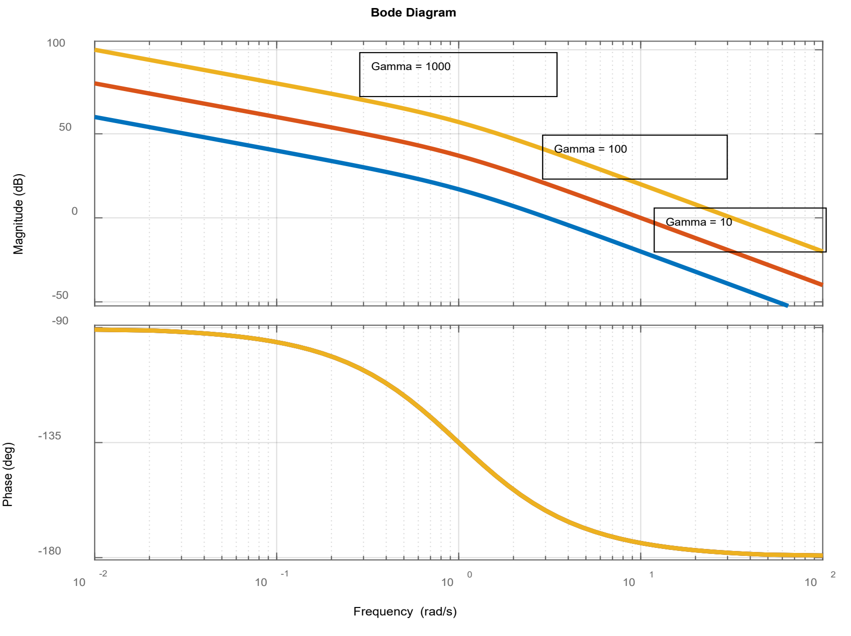


Fig. 1 - Bode plot with Gama=10,100,1000, Increasing the adaptive gain (Γ), improves the tracking performance for all t ≥ 0, including the transient phase. $L_1(s) = \frac{\Gamma}{s(s+1)}$.

2. CLOSED LOOP L1-ADAPTIVE CONTROL STRUCTURE

To study the performance of this system in the case of the two filters, we need to explain the dynamic equations and the block diagrams of this system of the Vega launch vehicle:

2.1 The class of systems as a problem formulation from [1]

$$\dot{x}(t) = A_m x(t) + b\big(u(t) + \theta^T x(t)\big), x(0) = x_0$$
$$y(t) = c^{\mathsf{T}} x(t), \tag{1}$$

where:
$x(t) \in R^n$ *: is the system state vector (measured).*
$u(t) \in R$ *: is the control signal.*
$b, c \in R^n$ *: are known constant vectors.*
A *: is the known n×n matrix, with (A, b) controllable.*
θ *: is the unknown parameter, which belongs to a given compact convex set* $\theta \in \Theta \subset R^n$.
$y(t) \in R$ *: is the regulated output.*

2.2 The control structure of the L1 Adaptive Control Architecture

$$u(t) = u_m(t) + u_{ad}(t) \\ \mathrm{u_m(t) = -\,k_m^T x(t)} \tag{2}$$

The Laplace transform of the adaptive control signal is defined as:

$$u_{ad}(s) = -C(s)\big(\hat{\eta}(s) - k_r(s)\big) \tag{3}$$

where:
k_m *: The static feedback gains.*
$k_m \in R^n$*: renders* $A_m \triangleq A - bk_m^T$ *renders.*
$u_{ad}(t)$ *: is the adaptive component.*
R(s), $\hat{\eta}(s)$ *are the Laplace transforms of r(s) and* $\hat{\eta}(s) \triangleq \hat{\theta}^T(t)x(t)$, *respectively.* $k_g \triangleq$ *-1/(*$c^\top k_m^{-1} b$*), and C(s) is a BIBO-stable and strictly proper transfer function with DC gain C(0)= 1, and its state-space realization assumes zero initialization.*
k_m *leads to the following partially closed-loop System (the system state after adding the adaptive component* $u_{ad}(t)$*):*

$$\dot{x}(t) = A_m x(t) + b\big(\theta^T x(t) + u_{ad}(t)\big), x(0) = x_0, \\ y(t) = c^\top x(t), \tag{4}$$

2.3 The state predictor of this system

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + b\left(\hat{\theta}^T(t)x(t) + u_{ad}(t)\right),\ \hat{x}(0) = x_0 \\ \hat{y}(t) = c^\top \hat{x}(t) \tag{5}$$

where:
$\hat{x}(t) \in R^n$ *:is the state of the predictor and* $\hat{\theta}(t) \in R^n$*: is the estimate of the parameter* θ.

2.4 The adaptive law of the system

$$\dot{\hat{\theta}}(t) = \Gamma \mathrm{Proj}\,(\hat{\theta}(t), -\tilde{x}^T(t)Pbx(t)), \qquad \hat{\theta}(0) = \hat{\theta}_0 \in \Theta \tag{6}$$

where:
$\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$*: the prediction errors.*
$\Gamma \in R^+$*: is the adaptation gain.*
P =$P^T > 0$*: solves the algebraic Lyapunov equation* $A_m{}^\top P + PA_m = -Q$ *for arbitrary symmetric* $Q = Q^\top > 0$.

3. L1-NORM CONDITION

The L1 adaptive controller is defined from the prior equations 2, 3 and 5 with km and C(s) verifying the following.

$$\lambda \triangleq \|G(s)\|_{L1} L < 1 \tag{7}$$

where:

$$G(s) \triangleq H(s)\big(1 - C(s)\big), \quad H(s) \triangleq (sI - A_m)^{-1}\, b\,, L \triangleq \max_{\theta \in \Theta} \|\theta\|_1 \tag{8}$$

From equations 1, 2, 3, 4 and the L1-norm condition we have:

$$\left\|x_{ref} - x\right\|_{L_\infty} \leq \frac{\gamma_1}{\sqrt{\Gamma}}, \qquad \left\|u_{ref} - u\right\|_{L_\infty} \leq \frac{\gamma_2}{\sqrt{\Gamma}} \tag{9}$$

$$\lim_{t\to\infty}\left\|x_{ref}(t) - x(t)\right\| = 0\,, \qquad \lim_{t\to\infty}\left\|u_{ref}(t) - u(t)\right\| = 0 \tag{10}$$

where:

$$\gamma_1 \triangleq \frac{\|C(s)\|_{L_1}}{1 - \|G(s)\|_{L_1} L}\sqrt{\frac{\theta_{max}}{\lambda_{min}(P)}}\,, \gamma_2 \triangleq \|H_1(s)\|_{L_1}\sqrt{\frac{\theta_{max}}{\lambda_{min}(P)}} + \left\|C(s)\theta^T + k_m^T\right\|_{L_1}\gamma_1 \tag{11}$$

The response of the closed-loop system in equation 4 with the L1 adaptive controller in equation 3 can be written (in the frequency domain) as:

$$x(s) = H(s)\, k_g C(s) r(s) + G(s)\theta^T x(s) - H(s)C(s)\tilde{\eta}(s) + x_{in}(s) \tag{12}$$

From [1] the definition of the closed-loop reference system it follows that

$$x_{ref}(s) = H(s)\, k_g C(s) r(s) + G(s)\theta^T x_{ref}(s) - x_{in}(s) \tag{13}$$

The two expressions above and the prediction error dynamics in equation 5 lead to

$$x_{ref}(s) - x(s) = G(s)\theta^T \left(x_{ref}(s) - x(s)\right) + C(s)\tilde{x}(s) \tag{14}$$

which

$$\left\|(x_{ref} - x)_\tau\right\|_{L_\infty} \leq \frac{\|C(s)\|_{L_1}}{1 - \|G(s)\|_{L_1} L}\|\tilde{x}_\tau\|_{L_\infty} \leq \frac{\|C(s)\|_{L_1}}{1 - \|G(s)\|_{L_1} L}\sqrt{\frac{\theta_{max}}{\lambda_{min}(P)}} \tag{15}$$

The closed-loop reference system in equation 13 depends upon the vector θ of unknown parameters, and hence it cannot be used for introducing the transient specifications and we consider the following LTI system, which will be referred to as a design system, with its output free of uncertainties:

$$x_{des}(s) = C(s) k_g H(s) r(s) + x_{in}(s) \tag{16}$$

$$\begin{gathered} u_{des}(s) = k_g C(s) r(s) - C(s)\theta^T x_{des}(s) - k_m^T x_{des}(s) \\ y_{des} = c^T x_{des}(s) \end{gathered} \tag{17}$$

The following upper bounds hold [2]

$$\begin{gathered} \|y_{des} - y_{ref}\|_{L_\infty} \leq \frac{\lambda}{1-\lambda}\left\|c^T\right\|_1 \left(\left\|k_g H(s) C(s)\right\|_{L_1} \|r\|_{L_\infty} + \|x_{in}\|_{L_\infty}\right) \\ \|x_{des} - x_{ref}\|_{L_\infty} \leq \frac{\lambda}{1-\lambda}\left(\left\|k_g H(s) C(s)\right\|_{L_1} \|r\|_{L_\infty} + \|x_{in}\|_{L_\infty}\right) \end{gathered} \tag{18}$$

$$\left\|u_{des} - u_{ref}\right\|_{L_\infty} \leq \frac{\lambda}{1-\lambda}\left(\|C(s)\theta^T - k_m^T\|_{L_1}\right)$$
$$\left(\left\|k_g H(s) C(s)\right\|_{L_1} \|r\|_{L_\infty} + \|x_{in}\|_{L_\infty}\right) \quad (19)$$

4. CALCULATION OF λ_1 AND λ_2 WITH RESPECT TO ω_c

The values that will be used in the time simulations and in the calculations to find the responses with L1-AC for the Vega TVC model**:**

Table 1 explain the numerical values are taken from the maximum dynamic pressure moment from [3]:

a_1	37.87
a_2	0.02737
a_3	25.54
a_6	3.2297
k_1	7.0738
k_p	- 0.9132
k_d	-0.2541
V	554

$$A = \begin{bmatrix} 0 & 1 & 0 \\ a_6 & 0 & \frac{a_6}{V} \\ -a_1 & 0 & -a_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -k_1 \\ -a_3 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0], \; D = [0], \quad k_m = [k_p \quad k_d \quad 0]$$

$$theta = \begin{bmatrix} -0.2 \\ 0 \\ -0.3 \end{bmatrix}, \qquad Gamma1 = 10, \qquad Gamma2 = 100, \qquad Omega = 4:100$$

The L1-norm $\|G_1(s)\|_{L_1} = \left\|\left(1 - C_1(s)\right)H(s)\right\|_{L_1}$ can be calculated numerically[4] by the below Fig. 2 shows $\lambda_1 = \|G_1(s)\|_{L_1} * L$ with respect to the bandwidth of the low-pass filter ω_c. Notice that for $\omega_c > 120$, we have $\lambda_1 < 1$. Choosing $\omega_{c1} = 150\ s^{-1}$ leads to $\lambda_1 = \|G_1(s)\|_{L_1} * L$=0.25< 1, and $\lambda_2 = \|G_2(s)\|_{L_1} * L = \left\|\left(1 - C_2(s)\right)H(s)\right\|_{L_1} * L$ as a function of ω_c . Notice that for $\omega_c > 50$, we have $\lambda_2 < 1$. Setting $\omega_{c2} = 55\ s^{-1}$ leads to $\lambda_2 = 0.65$

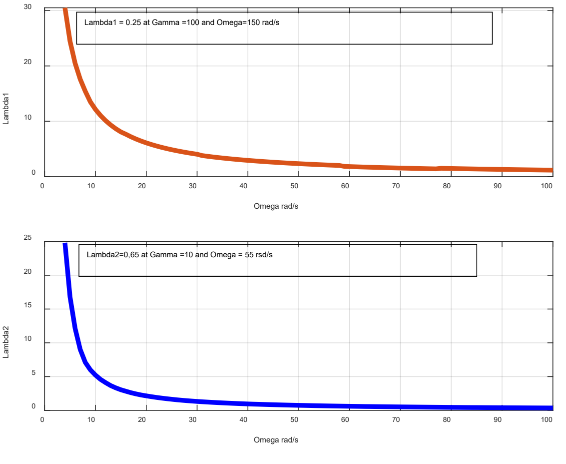


Fig. 2 - $\lambda_1$and λ_2 with respect to ω_c (rad/s)

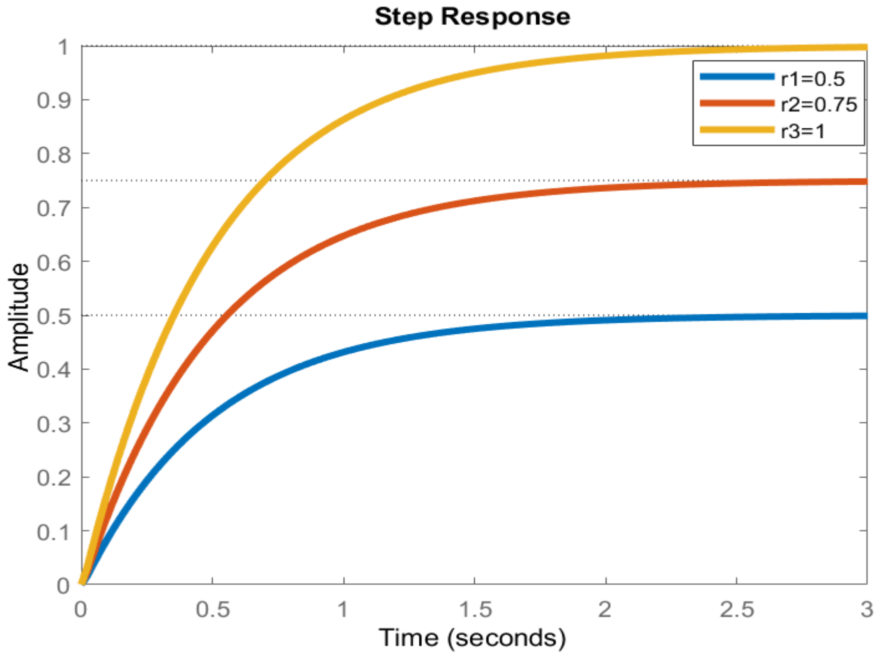
5. PERFORMANCE OF L1 ADAPTIVE CONTROLLER FOR VEGA LAUNCH VEHICLE IN CASE OF THE FILTERs $C_1(s)$ AND $C_2(s)$

By using the value of $\omega_{c1} = 150\ s^{-1}$ and the $\omega_{c2} = 55\ s^{-1}$ to [5] find the Performance of L1 adaptive controller with high-order filter C(s) for step-reference inputs with the two of the low-pass filter ($C_1(s) = \frac{150}{s+150}$) with $\Gamma = 100$, and with the high-order filters ($C_2(s) = \frac{3*55\text{^}2s+55^3}{(s+55)^3}$) with $\Gamma = 10$, for the time history for u(t) and y(t).
The responses a, b, c, d in [4] show the simulation results for the L1 adaptive controller for step reference input with the constant reference inputs r1=0.5, r2=0.75 and r3=1. By applying the two of the next relationships:

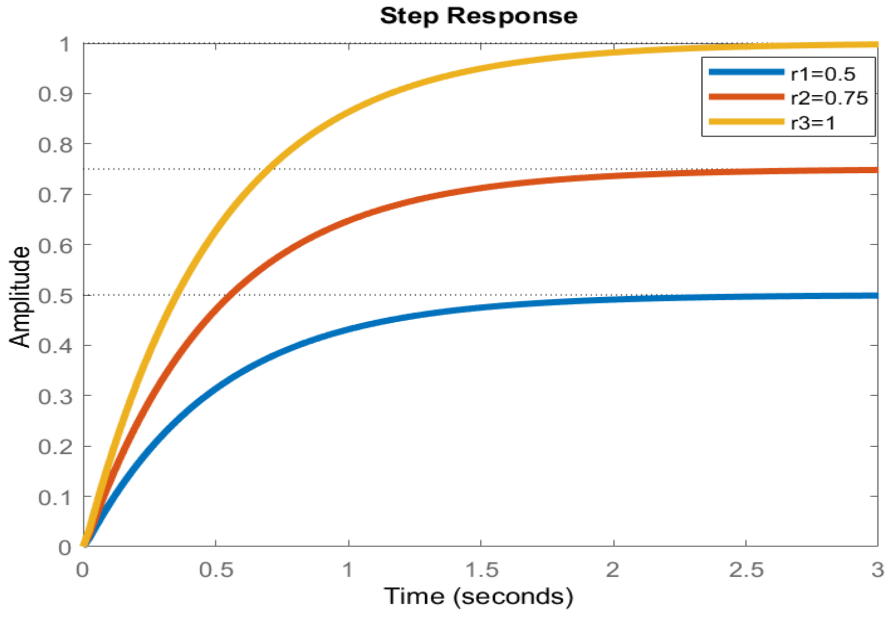
$$x(s) \approx x_{ref}(s) \approx x_{des}(s) = C(s)k_g H(s)r(s) = \frac{\omega_c}{s+\omega_c}\frac{2}{s+2}r(s) \tag{20}$$

$$u(s) \approx u_{ref}(s) = (-2 - C(s)\theta)x_{ref}(s) + C(s)2r(s) \tag{21}$$

We note that the first of these relationships implies that the control objective is met, while the second states that the L1 adaptive controller approximates $u_{ref}(t)$, which partially cancels θ and the L1 adaptive controller leads to scaled control inputs and scaled system outputs for scaled reference inputs [6]

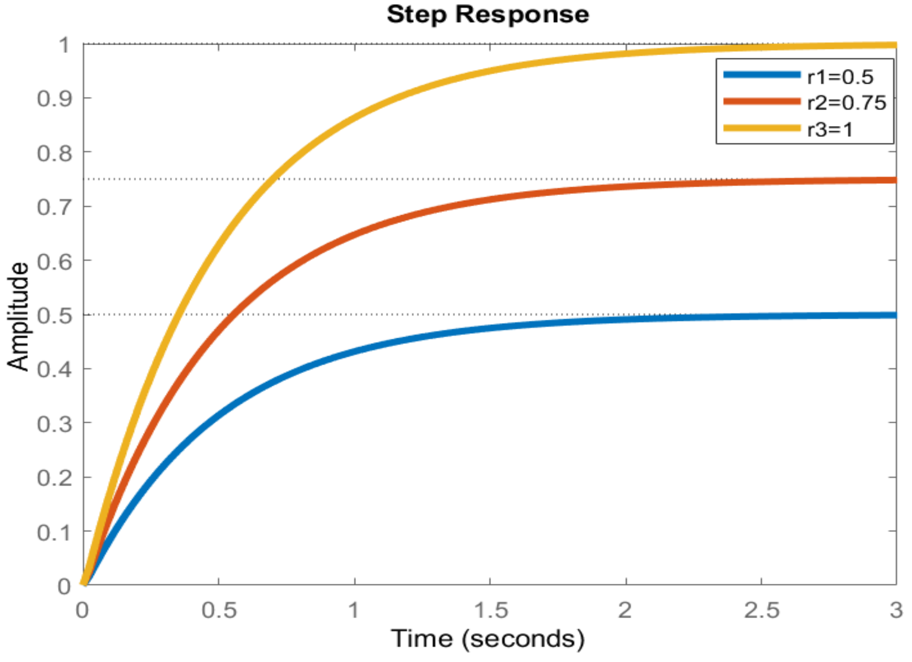


(a)Time history of $y(s)$ with($C_1(s) = \frac{150}{s+150}$)

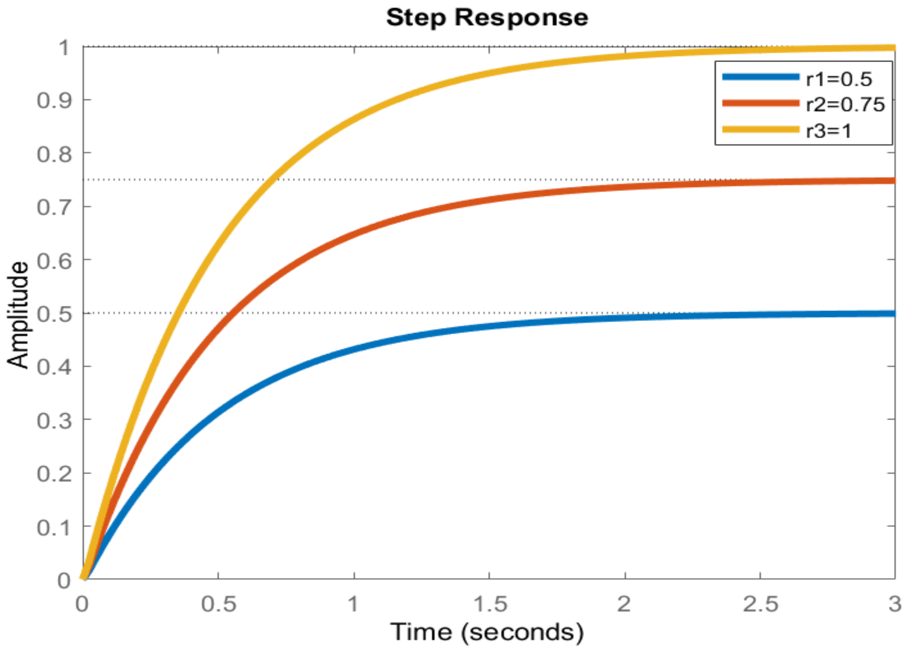


(b) Time history of $u(s)$ with $(C_1(s) = \frac{150}{s+150})$

Fig. 3 - Performance of L1 adaptive controller with $C_1(s) = \frac{150}{s+150}$,



(a) Time history of $y(s)$ with $(C_2(s) = \frac{3*55\text{^}2s+55^3}{(s+55)^3})$



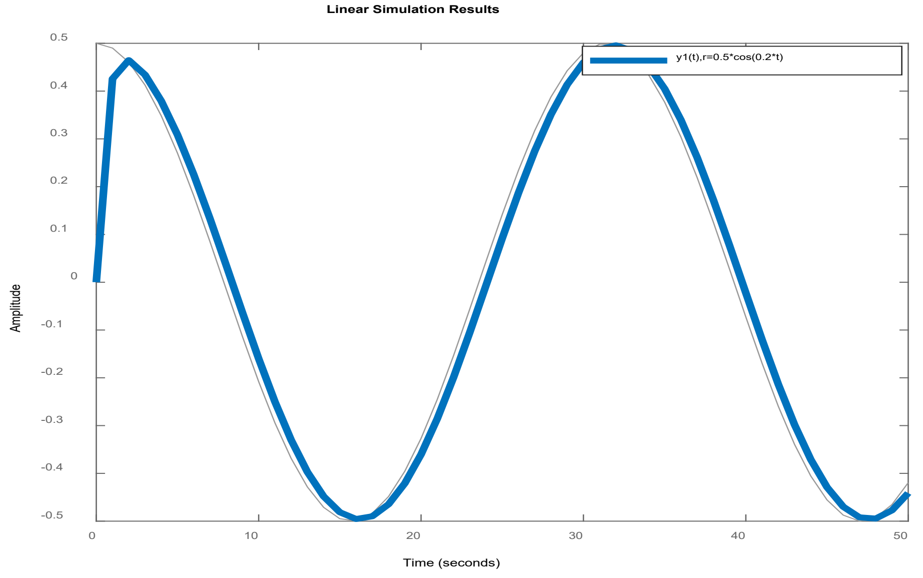
(b) Time history of $u(s)$ with ($C_2(s) = \frac{3*55^2s+55^3}{(s+55)^3}$)

Fig. 4 - Performance of L1 adaptive controller with $C_2(s) = \frac{3*55^2s+55^3}{(s+55)^3}$ for step-reference inputs.

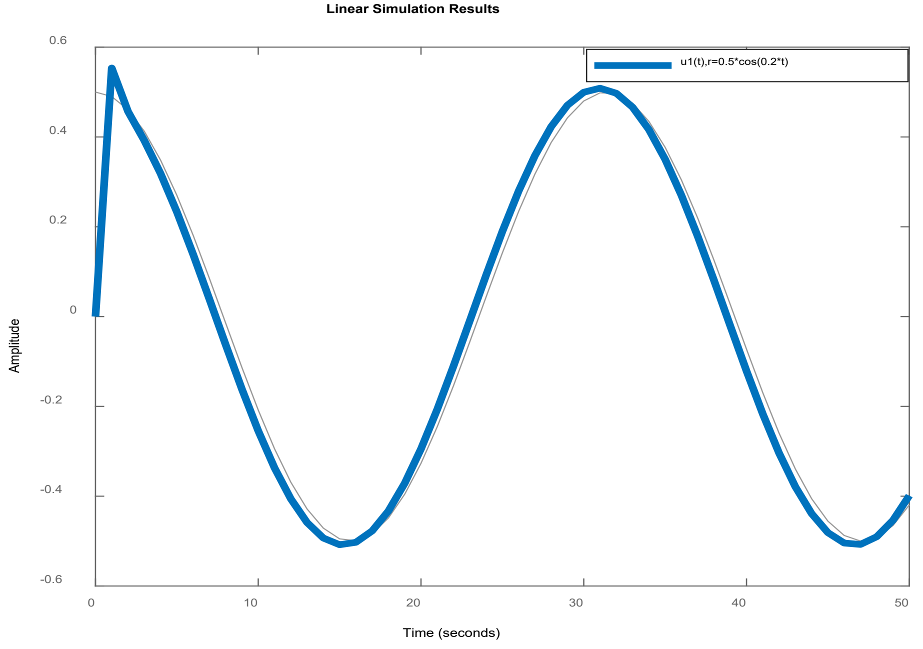
Fig. 3 (a and b) [6], the L1 adaptive controller and the system state approximate $u_{ref}(t)$ in (equation 21)and $x_{ref}(t)$ in (equation 20) respectively. Therefore, $y(t)$. Fig. 3.a approximates the output response of the LTI system to the input signal $r(t)$. And hence [7], its transient performance specifications, such as overshoot and settling time, can be derived for every value of θ, but if we further minimize λ (λ_1=0.25), the L1 adaptive controller leads to uniform transient performance of $\boldsymbol{y(t)}$ independent of the value of the unknown parameter θ. For the resulting L1 adaptive control signal one can characterize the transient specifications such as its amplitude and rate of change for every θ ∈ Θ, using $u_{des}(t)$. Fig. 4.a, $y(t)$approximates the output response of the LTI system to the input signal $r(t)$, and we see the uniformly and bounded tracking between y(t) and u(t) in steady state response, but we see the overshoot in transient of the Fig. 4.b because the adaptive rate of the filter $\boldsymbol{C_2}$ is lower than the filter $\boldsymbol{C_1}$ and $\lambda_2 = 0.65$ is larger than $\lambda_1 = 0.25$,therefore the performance with filter $\boldsymbol{C_1}$ is better than the performance with filter $\boldsymbol{C_2}$ because the large of adaptive rate with minimized λ. From Fig. 4 we can illustrate that high-order filters $\boldsymbol{C_2(s)}$ may give the opportunity to use relatively small adaptive gains keeping relatively with same performance.

6. PERFORMANCE OF L1 ADAPTIVE CONTROLLER WITH C_1 (s) WITH REFERENCE SIGNAL *r*=0.5cos (0.2*t*)

In Fig. 5 we show the performance for the time-varying reference signal [8], r=0.5cos (0.2t) without any retuning of the controller and $y(t)$ Fig. 5.a approximates the output response of the LTI system to the input $r(t)$. plot the system response and the control signal for a different uncertainty r(t) without any retuning of the controller [9], and in Figures 5a and 5.b we see the tracking between $y(t)$, $u(t)$ and $r(t)$ with the L1-adaptive controller.



(a) $y(t)$ (Blue) and $r(t)$ (grey)



(b) $u(t)$ (Blue) and $r(t)$ (grey)

Fig. 5 - Performance of L1 adaptive controller with $C_1(s) = \frac{150}{s+150}$ for r=0.5cos (0.2t)

7. CONCLUSIONS

The adaptation bounds can be improved by increasing the rate of adaptation Fig. 1 and the performance bounds can be systematically improved by increasing the adaptation rate. In contrast, the robustness bounds can be appropriately addressed via known methods from linear systems theory. The L1 adaptive controller in (figures 3, 4, and 5) ensures uniformly bounded transient and steady-state tracking for both system's signals, input, and output, which has guaranteed transient response for the system's signals, input, and output, simultaneously, in addition to stable tracking as compared to the same signals of a bounded reference LTI system, which assumes partial cancelation of uncertainties within the bandwidth of the control channel. To valuable the Performance of the L1 adaptive controller of Vega TVC, we used two filters, the first low-pass filter C1 with the first order and the second high-order filter C2 with the third order and we used the high adaptive gain (100) with C1 and the low adaptive gain (10) with C2, the results according to Fig. 2 were (ω_c1=150 s^(-1), Lambda1=0.25 with filter C1, and ω_c2=55 s^(-1), Lambda2=0.65), therefore according to (figures 3, 4 and 5) the L1 adaptive controller with C1, leads to the uniform transient performance of y(t) because we select the large adaptive gain and minimize λ1. In (figures 3 and 4), the L1 adaptive controller can generate a system response to track u(t) and y(t), both in transient and steady-state if we select the adaptive gain large and minimize λ. In Fig. 5 the performance for the time-varying reference signal r=0.5cos (0.2t) is without any retuning of the controller, and y(t) approximates the output response of the LTI system to the input r(t), and the L1 adaptive controller and the system state approximate u_{ref} (t) and x_{ref} (t), respectively [10], because using the adaptive gain large and minimize λ.

ACKNOWLEDGEMENTS

I would like to thank the National University of Science and Technology Politehnica of Bucharest, Faculty of Aerospace Engineering and the Ph.D School, for their very helpful suggestions which led to the improvement of my abilities and for these invitations.

This paper is an extended version of the presentation given at the *40th edition of the Conference "Caius Iacob" on Fluid Mechanics and its Technical Applications*, 19 - 20 October 2023, session Dynamical Systems, at the headquarters of the INCAS, Blvd. Iuliu Maniu 220, sector 6, Bucharest, Romania.

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