The Multipoint Global Shape Optimization of Flying Configuration with Movable Leading Edges Flaps

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***Abstract**: The aerodynamical global optimized (GO) shape of flying configuration (FC), at two cruising Mach numbers, can be realized by morphing. Movable leading edge flaps are used for this purpose. The equations of the surfaces of the wing, of the fuselage and of the flaps in stretched position are approximated in form of superpositions of homogeneous polynomes in two variables with free coefficients. These coefficients together with the similarity parameters of the planform of the FC are the free parameters of the global optimization. Two enlarged variational problems with free boundaries occur. The first one consists in the determination of the GO shape of the wing-fuselage FC, with the flaps in retracted position, which must be of minimum drag, at higher cruising Mach number. The second enlarged variational problem consists in the determination of the GO shape of the flaps in stretched position in such a manner that the entire FC shall be of minimum drag at the second lower Mach number. The iterative optimum-optimorum (OO) theory of the author is used for the solving of these both enlarged variational problems. The inviscid GO shape of the FC is used only in the first step of iteration and the own developed hybrid solutions for the compressible Navier-Stokes partial-differential equations (PDEs) are used for the determination of the friction drag coefficient and up the second step of iteration of OO theory.*

***Key Words:** Aerodynamical global shape optimization, Multipoint design by morphing, Supersonic flow, Hybrid and meshless solutions for the three-dimensional compressible Navier-Stokes PDEs*

1. INTRODUCTION

Let us consider an FC, which shall optimal fly at two different supersonic cruising Mach numbers over sea and land. It can be realised by morphing, using movable leading edge flaps, like the birds in gliding flight. The determination of the GO shape of FC with flaps in retracted and in stretched positions lead to two enlarged variational problems with free boundaries. The OO theory developed by the author is a strategy for the determination of the GO shape of FC inside of a class of elitary FCs defined by their chosen common properties.

Two elitary FCs belong to the same class, if their surfaces are expressed in form of superpositions of polynomes with the same maximal degree, their planforms can be related through affine transformation, they fulfill the same constraints and their surfaces are optimized in classical way by considering the similarity parameters of planform constant.

A lower-limit hypersurface of the inviscid drag functional $C_d^{(i)}$ as function of the similarity parameters ν_i of the planform is defined, namely:

$$(C_d^{(i)})_{opt} = f(\nu_1, \nu_2, ..., \nu_n). \tag{1}$$

Each point of this hypersurface is obtained by solving a classical variational problem with given boundaries (i.e. a given set of similarity parameters). The position of the minimum of this hypersurface, which is numerically determined, gives us the best set of the similarity parameters and the FC's optimal shape, which corresponds to this set, is at the same time the global optimized FC's shape of the class.

This OO theory was used by the author for the inviscid global optimization of the shapes of three models, with respect to minimum drag, at the cruising Mach numbers $M_\infty = 2,\ 2.2,\ 3.0$, respectively, it is: Adela (a delta wing alone) and two fully-integrated delta wing fuselage FCs, Fadet I and Fadet II.

More recently, an iterative OO theory was developed in order to compute the friction drag coefficient, to perform the viscous global optimal design and to take care of the requests of the structure in its early steps of iteration, via weak interaction.

2. DETERMINATION OF THE INVISCID GLOBAL OPTIMIZED SHAPE OF THE WING-FUSELAGE CONFIGURATION, WITH FLAPS IN RETRACTED POSITION

Firstly, the enlarged variational problem of the determination of the inviscid GO shape of the integrated wing-fuselage FC with the flaps in retracted position is considered. It shall have a minimum inviscid drag at higher cruising Mach number, M_∞. This FC is treated like an equivalent integrated delta wing, fitted with two artificial ridges, which are located along the junction lines between the wing and the fuselage. The downwash w on the thin component of this integrated FC is supposed to be continuous and expressed in the form of a superposition of homogeneous polynomes in two variables, namely:

$$w = \tilde{w} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k} \left|\tilde{y}\right|^k , \tag{2}$$

The downwashes w^* and $\overline{w}'^*$ on the wing and on the fuselage zone are expressed in form of two different superpositions of homogeneous polynomes :

$$w^* = \tilde{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}^*_{m-k-1,k} \left|\tilde{y}\right|^k , \qquad \overline{w}'^* = \overline{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \overline{w}^*_{m-k-1,k} \left|\tilde{y}\right|^k . \tag{3a}$$

$$\left(\tilde{x}_1 = \frac{x_1}{h_1} ,\ \tilde{x}_2 = \frac{x_2}{\ell_1} ,\ \tilde{x}_3 = \frac{x_3}{h_1} ,\ \tilde{y} = \frac{y}{\ell} ,\ \ell = \frac{\ell_1}{h_1} ,\ \nu = B\ell ,\ B = \sqrt{M_\infty^2 - 1} \right) \tag{3b}$$

The coefficients $\tilde{w}_{m-k-1,k}$, $\tilde{w}^*_{m-k-1,k}$ and $\overline{w}^*_{m-k-1,k}$, together with the similarity parameters ν and $\overline{\nu} = Bc$ of the planforms of entire FC and of the fuselage of the integrated FC, are the free parameters of optimization and ℓ_1 and h_1 are the half-span and the depth of this FC. The quotient $k = \overline{\nu} / \nu$ between the similarity parameters of the planforms of the wing and of the fuselage of this FC depends on the purpose of FC and is here considered constant.

The corresponding axial disturbance velocities u and u^* on the thin and thick-symmetrical components of the FC, obtained by the author, by using the hydrodynamic analogy of Carafoli and the principle of minimal singularities (which fulfill the jumps along the singular lines) are, as in [1]:

$$u=\ell\sum_{n=1}^{N}\tilde{x}_1^{n-1}\left\{\sum_{q=1}^{E\left(\frac{n-1}{2}\right)}\tilde{C}_{n,2q}\tilde{y}^{2q}\cosh^{-1}\sqrt{\frac{1}{\tilde{y}^2}}+\sum_{q=0}^{E\left(\frac{n}{2}\right)}\frac{\tilde{A}_{n,2q}\tilde{y}^{2q}}{\sqrt{1-\tilde{y}^2}}\right\}, \quad (4)$$

$$u^*=\ell\sum_{n=1}^{N}\tilde{x}_1^{n-1}\left\{\sum_{q=1}^{E\left(\frac{n-1}{2}\right)}\tilde{C}^*_{n,2q}\tilde{y}^{2q}\cosh^{-1}\sqrt{\frac{1}{\nu^2\tilde{y}^2}}+\sum_{q=0}^{E\left(\frac{n-2}{2}\right)}\tilde{D}^*_{n,2q}\tilde{y}^{2q}\sqrt{1-\nu^2\tilde{y}^2}\right.$$

$$+\sum_{q=0}^{n-1}\tilde{H}^*_{nq}\tilde{y}^q\left[\cosh^{-1}M_1+(-1)^q\cosh^{-1}M_2\right] \quad (5)$$

$$\left.+\sum_{q=0}^{n-1}\tilde{G}^*_{nq}\tilde{y}^q\left[\cosh^{-1}N_1+(-1)^q\cosh^{-1}N_2\right]\right\}.$$

$$\left(M_{1,2}=\sqrt{\frac{(1+\nu)(1\mp\nu\,\tilde{y})}{2\nu(1\mp\tilde{y})}}\ ,\quad N_{1,2}=\sqrt{\frac{(1+\overline{\nu})(1\mp\nu\,\tilde{y})}{2(\overline{\nu}\mp\nu\,\tilde{y})}}\right)$$

For a given value of ν, the optimization of the shapes of its thin and thick-symmetrical components can be separately treated. The constraints of the determination of the inviscid GO shape are: the given lift, pitching moment and the Kutta condition along the subsonic leading edges of the thin FC component (in order to cancel the induced drag at cruise and to suppress the transversal conturnement of the flow around the leading edges, in order to increase the lift) and the given relative volumes of the wing and of the fuselage zone, the cancellation of thickness along the leading edges and the new introduced integration conditions along the junction lines between the wing and fuselage zone of the thick-symmetrical FC component (in order to avoid the detachment of the flow along these lines).

The similarity parameter ν of the planform of FC is sequentially varied and the inviscid drag functional of optimal FCs, for each corresponding value of ν, is obtained by solving a classical variational problem with fixed boundaries and a lower limit-line of the inviscid drag functional of optimal FCs, as function of this similarity parameter ν, is determined (for FCs with subsonic leading edges, it is: $0<\nu<1$). The position of the minimum of this limit-line gives the optimal value of the similarity parameter $\nu=\nu_{opt}$ and the corresponding optimal FC is, at the same time, the global optimized FC of the class. The inviscid GO shape of the integrated model Fadet II is presented in the (Fig. 1), as exemplification. The lift, the pitching moment and the pressure coefficients C_ℓ , C_m and C_p of this model were measured in the trisonic wind tunnel of the DLR-Cologne, in the frame of some research contracts of the author, sponsored by the DFG. The comparisons of theoretical and experimental-correlated values of lift and pitching moment coefficients of model Fadet II are presented in the (Fig. 2a,b) and the comparison between the theoretical and the linear interpolated experimental determined values of pressure coefficients, along the central section on the upper side of the model Fadet II, are presented in the (Fig. 3a-c) for the angles of attack $\alpha=-8^\circ,\ 0^\circ,\ 8^\circ$.

The very good agreement between experimental and theoretical determined inviscid analytical hyperbolical solutions of the lift, pitching moment and pressure coefficients of FCs with subsonic leading edges, at moderate angles of attack (deduced by the author in closed forms), leads to the following remarks: the validity of the three-dimensional hyperbolical analytical potential solutions for the axial disturbance velocity with the chosen balanced singularities and the corresponding developed software for the computation of aerodynamical coefficients given above are confirmed; the influence of friction upon these coefficients is neglectable; the flow is laminar, as supposed here and it remains attached in supersonic flow, for larger angles of attack than by subsonic flow. If the FC has sharp subsonic leading edges, is flattened enough and flies at moderate angles of attack, the flight with characteristic surface, instead of the flight with shock wave surface, is confirmed.

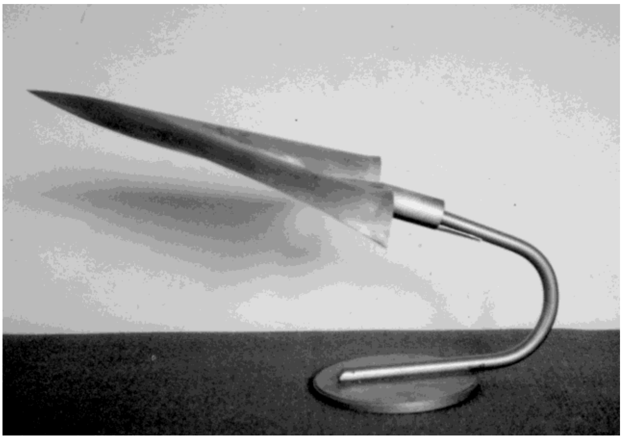

Fig. 1 The Inviscid, Integrated and Global Optimized Shape of the Model Fadet II

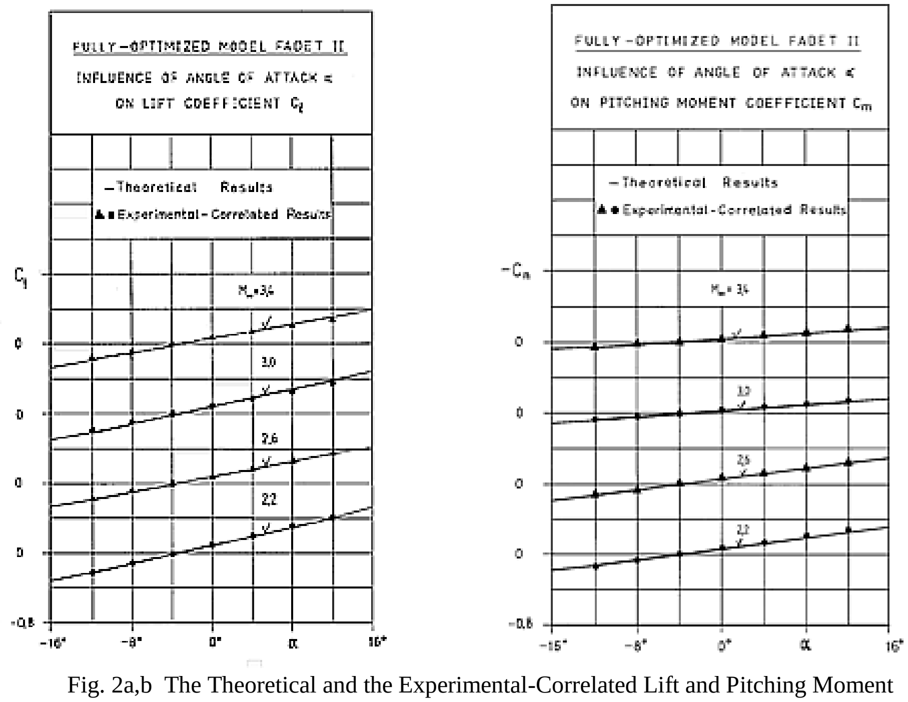

Fig. 2a,b The Theoretical and the Experimental-Correlated Lift and Pitching Moment Coefficients of Integrated, Global Optimized Model Fadet II.

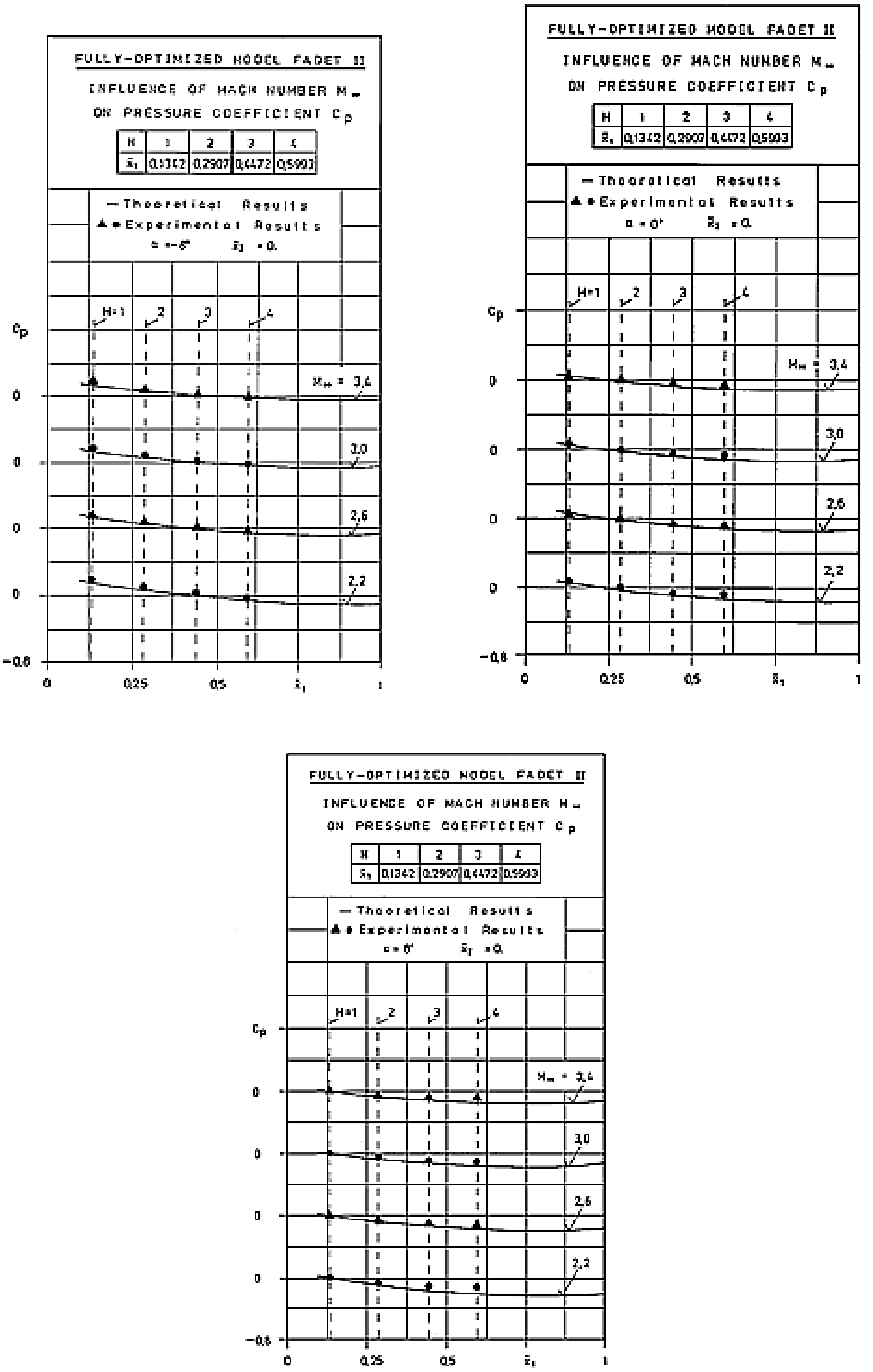

Fig. 3a-c The Theoretical and the Linear Interpolated Values of the Pressure Coefficient on the Longitudinal Central Section of the Upper Side of the Inviscid Global Optimized Wing-Fuselage Model Fadet II for the Angles of Attack $\alpha = -8^{\circ},\ 0^{\circ},\ 8^{\circ}$

3. DETERMINATION OF THE INVISCID GLOBAL OPTIMIZED SHAPE OF THE WING-FUSELAGE CONFIGURATION, WITH FLAPS IN STRETCHED POSITION

Now the second enlarged variational problem of the determination of the inviscid GO shape of the integrated wing-fuselage FC with the flaps in stretched position is considered. It shall have a minimum inviscid drag at lower cruising Mach number, M_{∞}^{*}. This FC is treated like an equivalent integrated delta wing, fitted with two artificial ridges, which are located along the junction lines between the wing and the fuselage and other two artificial ridges along the junction lines between the wing and the stretched flaps.

The downwash w on the thin component of the integrated wing-fuselage of the FC and the downwashes w^{*} and $\overline{w}'^{*}$ on its thick-symmetrical component remain unchanged and are given, as in (2) and, respectively, in (3a,b).

The downwash $w^{''}$ of the thin component of the stretched flaps and $w^{''*}$ of their thick-symmetrical components are also supposed to be expressed in form of superpositions of homogeneous polynomes in two variables, with free coefficients, namely:

$$w^{''} \equiv \tilde{\tilde{w}} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{\tilde{w}}_{m-k-1,k} \left|\tilde{y}\right|^k \qquad w^{''*} \equiv \tilde{\tilde{w}}^{*} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{\tilde{w}}^{*}_{m-k-1,k} \left|\tilde{y}\right|^k \tag{6a,b}$$

The corresponding axial disturbance velocities on the thin and on the thick-symmetrical components of the entire FC (i.e. integrated wing-fuselage with stretched flaps) are, as in [1], of the form:

$$\begin{aligned} u = \ell \sum_{n=1}^{N} \tilde{x}_1^{n-1} \Bigg[& \sum_{q=0}^{n-1} \tilde{\tilde{A}}_{nq} \tilde{y}^q \left(\cosh^{-1} N_1^{'} + (-1)^q \cosh^{-1} N_2^{'} \right) \\ & + \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\tilde{\tilde{F}}_{n,2q} \tilde{y}^{2q}}{\sqrt{\bar{k}^2 - \tilde{y}^2}} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \tilde{\tilde{C}}_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{\tilde{k}^2}{\tilde{y}^2}} \Bigg] \end{aligned} \tag{7}$$

$$\left(N_{1,2}^{''} = \sqrt{\frac{\left(1+\tilde{k}\right)\left(\tilde{k} \mp \tilde{y}\right)}{2\tilde{k}\left(1 \mp \tilde{y}\right)}} \;, \quad \tilde{k} = \frac{L}{\ell} \right)$$

and

$$\begin{aligned} u^{*} = \ell \sum_{n=1}^{N} \tilde{x}_1^{n-1} \Bigg[& \sum_{q=0}^{n-1} \tilde{\tilde{H}}^{*}_{nq} \tilde{y}^q \left(\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right) \\ & + \sum_{q=0}^{E\left(\frac{n}{2}\right)} \tilde{\tilde{D}}^{*}_{n,2q} \sqrt{1 - \tilde{v}^2 \tilde{y}^2} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \tilde{\tilde{C}}^{*}_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\tilde{v}^2 \tilde{y}^2}} \\ & + \sum_{q=0}^{n-1} \tilde{\tilde{G}}^{*}_{nq} \tilde{y}^q \left(\cosh^{-1} S_1 + (-1)^q \cosh^{-1} S_2 \right) \Bigg]. \end{aligned} \tag{8}$$

$$(M'_{1,2} = \sqrt{\frac{(1+\tilde{\nu})\,(1\mp\nu\ \tilde{y})}{2(\tilde{\nu}\mp\nu\ \tilde{y})}} \ ,\ \ S_{1,2} = \sqrt{\frac{(1+\tilde{\nu})\,(1\mp\nu\ \tilde{y})}{2(\bar{\nu}\mp\nu\ \tilde{y})}})$$

The free parameters of this second enlarged variational problem are the free coefficients $\tilde{\tilde{w}}_{m-k-1,k}$ and $\tilde{\tilde{w}}^*_{m-k-1,k}$ of the downwashes $\tilde{\tilde{w}}$ and $\tilde{\tilde{w}}^*$ on the thin and thick-symmetrical components of the flaps and the similarity parameter $\tilde{\nu} = B'L$ of the planform of the stretched flaps. Here is L the dimensionless half-span of the integrated wing-fuselage FC with the flaps in stretched position and $B' = \sqrt{M_\infty^{*2} - 1}$.

The constraints for the thin component are: the lift and the pitching moment coefficients are given and the Kutta condition on leading edges is fulfilled and for the thick-symmetrical component the relative volume of the flaps is given, the thickness along the leading edges is null and the integration conditions along the junction lines wing-flaps are satisfied.

4. HYBRID SOLUTIONS FOR THE COMPRESSIBLE NAVIER-STOKES PDEs

The author proposes hybrid numerical solutions for the NSL's PDEs, which use the hyperbolical analytical solutions given before, as in [1,2], twice, namely: as outer flow, at the NSL's edge (instead of the undisturbed parallel flow used by Prandtl in his boundary layer theory) and, secondly, the velocity's components are products between the corresponding potential velocities and polynomial expansions with arbitrary coefficients, which are used to satisfy the NSL's PDEs. Let us now introduce a spectral variable η, it is:

$$\eta = \frac{x_3 - Z(x_1, x_2)}{\delta(x_1, x_2)} . \qquad (0 < \eta < 1) \tag{9}$$

Hereby $Z(x_1, x_2)$ is the equation of the surface of the flattened FC and $\delta(x_1, x_2)$ is the NSL’s thickness distribution. The spectral forms of the axial, lateral and vertical velocity’s components u_δ, v_δ and w_δ, the density function $R = \ln\rho$ and the absolute temperature T are here proposed, and the physical equation of ideal gas for the pressure p and an exponential law of the viscosity μ versus T are used:

$$u_\delta = u_e \sum_{i=1}^{N} u_i \eta^i , \quad v_\delta = v_e \sum_{i=1}^{N} v_i \eta^i , \quad w_\delta = w_e \sum_{i=1}^{N} w_i \eta^i , \quad R = R_w + (R_e - R_w) \sum_{i=1}^{N} r_i \eta^i ,$$

$$T = T_w + (T_e - T_w) \sum_{i=1}^{N} t_i \eta^i , \qquad p \equiv R_g \rho T = R_g e^R T , \qquad \mu = \mu_\infty \left[\frac{T}{T_\infty} \right]^{n_1} \tag{10a,g}$$

Here R_w and T_w are the given values of R and T at the wall, R_g and T_∞ the universal gas constant and the absolute temperature of the undisturbed flow and n_1 the viscosity exponent, u_e, v_e, w_e, R_e and T_e are the values of u, v, w, R and T at the NSL’s edge, obtained from the outer inviscid hyperbolical potential flow, and u_i, v_i, w_i, r_i and t_i are their free spectral coefficients, which are used to fulfill the NSL’s PDEs. The use of the density function $R = \ln\rho$ (instead of the density ρ), proposed here, combined with the formulas for R, T, u, v, w, p and μ, and with the collocation method allows the determination of the spectral coefficients of all the physical entities, namely: R, T, p and μ only as functions of the spectral coefficients u_i, v_i and w_i. The spectral forms (10a-e) automatically satisfy the boundary conditions, at wall $(\eta = 0)$. If the forms (10a-g) are introduced in the NSL's PDEs of impulse and the collocations method is used, the spectral coefficients u_i, v_i and w_i of the velocity's components are obtained by the iterative solving of a linear algebraic system with slightly variable coefficients. The analytical properties of these hybrid numerical NSL's solutions are: they have correct last behaviors, correct and balanced jumps along the singular lines (like subsonic leading edges, junction lines wing/fuselage and wing/ leading edge flaps in stretched position), according to the minimum singularities principle, no inviscid/viscous interface is needed and, for the supersonic flow, the condition on the characteristic surface is fulfilled.

5. THE VISCOUS, ITERATIVE OPTIMUM-OPTIMORUM THEORY

The viscous iterative OO theory uses the inviscid hyperbolical potential solutions as start solutions and the inviscid GO shape of the FC, **only** in its first step of iteration. An intermediate computational checking of this inviscid GO shape of the FC is made with own hybrid solvers, for the three-dimensional NSL.

The friction drag coefficient $C_d^{(f)}$ of the FC is computed and the inviscid GO shape is checked also for the structural point of view. A weak interaction aerodynamics/structure is proposed. In the second step of optimization, the predicted inviscid GO shape of the FC is corrected by including these additional constraints in the variational problem and of the friction drag coefficient in the drag functional. The chart flow of the iterative OO theory is given in the (Fig. 4).

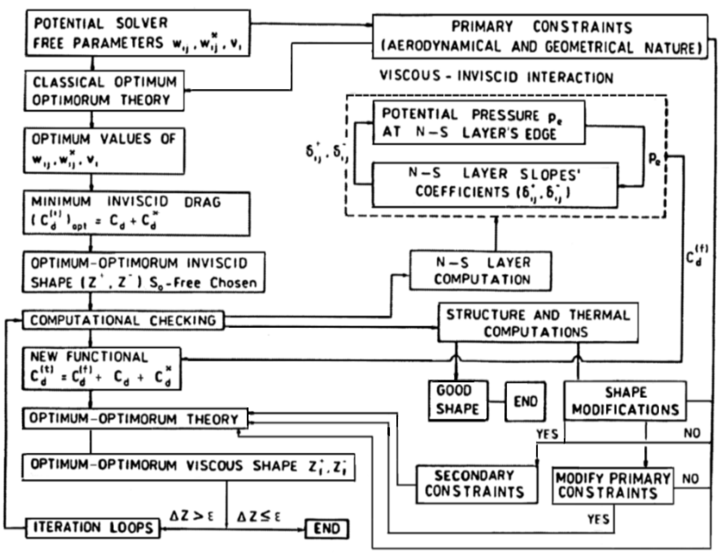

Fig. 4 The Iterative Optimum-Optimorum Theory

6. CONCLUSIONS

- The hybrid solutions for the Navier-Stokes layer and the iterative OO theory can be applied for the both extended variational problems of the determination of the GO shape of FC with retracted and stretched flaps;

- the analytical potential solutions of the same FC are used twice, namely, at the edge of NSL and to reinforce the numerical solutions of the NSL;

- the inviscid GO shape of FC is an adequate surrogate shape of FC and is used in the first step of iteration of the iterative OO theory;

- the morphing by using movable leading edge flaps allows the FC with retracted flaps to fly with subsonic leading edges at higher cruising Mach number, which flight is optimal by a smaller aspect ratio;

- the use of FC with retracted flaps augments the lift by take off and landing.

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