Unfalsified Control; Application to automatic flight control system design

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Abstract: *Unfalsified Control Theory has been developed to provide a way for avoiding modeling uncertainties in controller design. It belongs to the class of control methods called Adaptive Supervisory Switching Control, which work by introducing in the control scheme a supervisory unit which chooses, from a set of candidate controllers the one most suited for the current plant. Unfalsified Control works by using a switching logic that dispenses with the need for a-priori knowledge of the dynamic model. At discrete moments of time, using the input/output data recorded up to that point, the supervisory calculates for each candidate controller a performance index, and compares it to a given threshold. Controllers surpassing that threshold are removed from the candidate controller set. This process is called falsification. If the controller in the loop is one such falsified controller it is replaced. In this paper we investigate the suitability of this method for aeronautical control applications. We review the theory behind this control scheme and adapt it to the case of controlling a fighter aircraft. We also provide a case study, where we test this control scheme on a simulated fighter aircraft.*

Key Words: *Unfalsified Control Theory, aeronautical control, simulated fighter aircraft.*

1 INTRODUCTION

Adaptive Control was first introduced in the 1950's in an attempt to alleviate some of the problems in controlling high speed fighter aircraft which often find themselves in conditions which are very hard to model due to their highly non-linear nature, or high number of uncertainties.

Despite initial success, Adaptive Control soon showed many deficiencies which have confined it solely to research studies.

However, new methods have been recently devised that might make Adaptive Control a reality. Over the last two decades a lot of research has been put in Adaptive Switching Supervisory Control (ASSC) (see [1], [2], [4], and [9]).

ASSC is in fact an adaptive variant of classical gain scheduling, turned, from an open loop switching mechanism to a closed loop one, by the use of a supervisory logic based on plant input/output recorded data.

A typical ASSC is depicted in Figure 1. Where a data driven "high-level unit" S, called *supervisor*, which controls each plant G belonging to the given set G of plant models by connecting an appropriate controller K from the set $\mathcal{K}$ of candidate controllers.

The supervisor decides if the currently switched-on controller works properly, and, in the negative case, it replaces it by another candidate controller.

The scheduling task (when to substitute the acting controller) and the routing task (which controller to switch on) are carried out in real time by monitoring purely data-driven test functional [1]. The main current approaches to ASSC can be subdivided into two different groups: the first consists of the so called *Multi-Model* ASSC (MASSC), wherein a dynamic nominal model is associated with every candidate controller, the second called *Unfalsified* ASSC (UASSC) [1], [9] wherein a switching logic that dispenses with the need for a-priori knowledge of the dynamic model is used. Both these methods have their advantages and disadvantages.

MASSC schemes work by comparing norms of sequences of estimation errors based on the various nominal models, as the candidate controller associated to the nominal model yielding the prediction norm of minimum magnitude is believed to be the most suitable one.

The main advantage is the fact that transient times before finding a stabilizing controller tend to be small. However this can be achieved only by using a very dense model distribution. If this condition is not enforced neither convergence to a final controller, nor boundness can be guaranteed.

In contrast, UASSC schemes described by [9], can select in finite time a final controller yielding a finite affine gain from the reference to the data, under the minimal conceivable requirement regarding the existence of a stabilizing candidate controller.

This along with the fact that the plant need not be linear makes this schemes from the robustness point of view much better suited to aerospace applications then MASSC, the asymptotic stability properties of the latter being typically only guaranteed if the unknown plant is tightly approximated by at least one nominal model.

However the main disadvantage of UASSC schemes used so far, as noted in [1] and [4], stems from the fact that they do not provide protection against the temporary insertion in the loop of destabilizing controllers, which might lead to long transient times and temporary trends to divergence before the final stabilizing controller is switched on.

In the examples provided in [9], the supervisor needs about 70 seconds before finding the stabilizing controller, which wouldn't be convenient when trying to stabilize the short period longitudinal dynamics of an aircraft.

For this reasons in [1] a scheme called Multi Model Unfalsified Adaptive Switching Supervisory Control, that combines the advantages of both methods (low transient times for MASSC and asymptotic stability for UASSC), was proposed. The purpose of the current paper is to present the Unfasified Control Theory and also present a way to adapt it for use in aerospace control namely the stabilization of the short period dynamics of a fighter aircraft.

The paper is organized as follows: Section 2 summarizes the UASSC method as well as the modifications required for its implementation in aviation are summarized, Section 3 outlines a simulation example run using the ADMIRE model, Section 4 discusses the results of the previous sections and draws a number of conclusions and future research areas are highlighted.

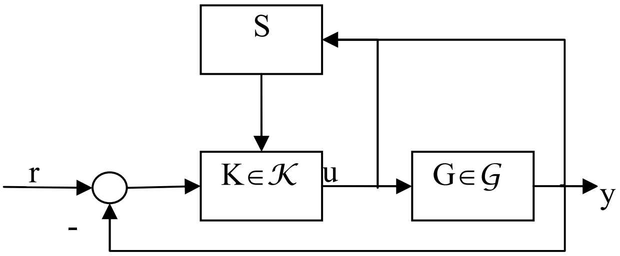

Fig. 1.Adaptive Supervisory Switching Control scheme

2 UNFALSIFIED ADAPTIVE SWITCHING CONTROL

Consider the following closed loop control system:

$$\begin{aligned} y(s) &= G(s)u(s) \\ u(s) &= K\big(r(s)-y(s)\big) \end{aligned} \tag{1}$$

where *G(s)* denotes the transfer function of the controlled system, *K(s)* stands for the controller, *r* is the reference signal, *u* and *y* are the control variable and the system output respectively.

Though UASSC methods can be used on non-linear plants, linearity will be assumed in throughout the paper for simplicity and also the case study in section 3 is conducted using robust controllers designed based on linearization of aircraft dynamic.

It is assumed *G* belongs to a plant uncertainty set $\mathcal{G}$. The controller *K* belongs to a finite family $\mathcal{K}$ of *linear time invariant* (LTI) controllers.

Definition 1 *Given a signal x(t), t≥0 is said that*

$$x_\tau(t) = \begin{cases} x(t),\ t\in[0,\tau] \\ 0,\ otherwise \end{cases}$$

represents a truncation of x(t) with the truncated norm

$$\|x_\tau\| = \left(\int_0^\tau x^2(t)\,dt\right)^{\frac{1}{2}}.$$

With the above definition the following slight generalization of input-output stability will be adopted throughout the paper [1], [4], [11]

Definition 2 *A dynamic system with the input r and the output y is called stable, or the stability is unfalsified by the data* (u, y)*, if there exist α, β ≥ 0 such that:*

$$\|y\|_\tau \le \alpha \|r\|_\tau + \beta,$$

∀ τ≥0 and for all $r \in L_{2e}$*,* L_{2e} *denoting the space of all functions with finite energy on any finite interval.*

Otherwise if $\sup\limits_{\tau\ge 0 \|[r]\|_\tau \ne 0} \dfrac{\|y\|_\tau}{\|r\|_\tau} \to \infty$*, it is said that the stability of the system is falsified by the data* (u, y)

The presence of the term $\beta \ge 0$ in the above definition is related to the situation where non-zero initial conditions of the system are taken into account. The next definition will be used in the following developments (see also [1], [4])

Definition 3 *The adaptive control problem is feasible if, for every* $G \in \mathrm{G}$*, there exists at least one controller* $K \in \mathcal{K}$ *such that the resulting system obtained by coupling K to G is stable and it accomplishes the performance objectives.*

The unfalsified adaptive control techniques are essentially based on the so-called *fictitious reference signal* and on an associated *performance index* which allows choosing appropriate controllers, $K \in \mathcal{K}$, for which the problem is feasible [9]

Definition 4 *Let the data* (u, y) *be the input and output measurements of a plant G over the time interval* $[0,\tau]$. *Then the fictitious reference signal* $\tilde{r}_K$ *associated to a controller* $K \in \mathcal{K}$ *is the signal defined over* $[0,\tau]$ *that produces the same set of data* (u, y) *if* $\mathcal{K}$ *would be connected to G.*

Note that the above definition requires the *invertibility* of *K* in which case the fictitious reference signal is given by $\tilde{r}_K = K^{-1}u + y$.

This expression of $\tilde{r}_K$ reveals another major constraint for *K,* namely it must be *minimum phase* since otherwise the fictitious reference $\tilde{r}_K$ can be unbounded for $t\to\infty$. Some aspects concerning these constraints will be discussed bellow.

The performance index $J(K,u, y,\tau)$ is a positive defined function defined on $\mathrm{K}\times\mathrm{U}\times\mathrm{Y}\times\mathrm{R}_+$ where *u* and *y* are truncated on the interval $[0,\tau]$ .It is defined according to the design specification of the controller *K* and it represents a measure of the performance provided by *K* on the time interval $[0,\tau]$.

Definition 5 *A controller* $K\in\mathcal{K}$ *is called falsified at the time τ with respect to a given cost level γ>0 by the data* (u, y) *measured on the time interval* $[0,\tau]$ if $J(K,u, y,\tau) > \gamma$. *Otherwise the controller K is called unfalsified by the measurements* (u, y) *on* $[0,\tau]$

According to the terminology used in [11] the set of all unfalsified controllers with the unfalsified cost level $\gamma > 0$ at time t stands for the *unfalsified controller set.*

The unfalsified adaptive controllers are not always safe, in the sense that some unfalsified destabilizing controllers inserted in the closed-loop can produce large signals for long intervals of time.

Definition 6 *Consider the reference signal r and the measured set of data* (u, y) *obtained by a finite number of switches of controllers* $K\in\mathcal{K}$ *mapping* $\begin{bmatrix} r(t) \\ y(t) \end{bmatrix}$ *to* $u(t)$ *and denote by* t_f *the final switching time and by* K_f *the final controller. Then the pair* (J,K) *is called cost-detectable if the following assertions are equivalent:*

a) $J(K_f,u, y,\tau)$ *is monotone increasing and bounded for* $\tau\to\infty$*;*

b) The closed loop system in Figure 1 with K_f *is unfalsified by the data* (u, y) *when* $\tau\to\infty$.

In [4], [9], [11] the following performance index is considered:

$$J(K,u,y,\tau) = \frac{\left\| w_1 * (y - \tilde{r}) \right\|^2_{L_2[0,t]} + \left\| w_2 * u \right\|^2_{L_2[0,t]}}{\left\| \tilde{r} \right\|^2_{L_2[0,t]}} \tag{2}$$

where * denotes convolution and $w_1(t)$ and $w_2(t)$ denote dynamic weighting functions used for determining controllers *K* as solutions of the mixed sensitivity problem

$$\left\| \begin{bmatrix} W_1 S \\ W_2 KS \end{bmatrix} \right\|_\infty \le \gamma$$

$S := (I + GK)^{-1}$ denoting the sensitivity function and $\|\cdot\|_\infty$ representing the H_∞ norm of the system $(\cdot)$ (for more details see [3])

A typical Unfalsified Adaptive Control Scheme is presented in Figure 2, while Figure 3 presents the algorithm by which controllers are chosen. An example is given in [9] that shows the strength of Unfalsified Control when used correctly even with the constraints mentioned above concerning the invertibility and minimum phase properties of $K \in \mathcal{K}$. However such limitations would make Unfalsified Control unusable for aerospace application where often the controllers are so complex that even if they are invertible they might not be minimum phase. Fortunately, as shown in [4], these constraints can be removed considering the coprime factorization of the controllers $K \in \mathcal{K}$. A similar idea is used in a discrete-time version in [1].

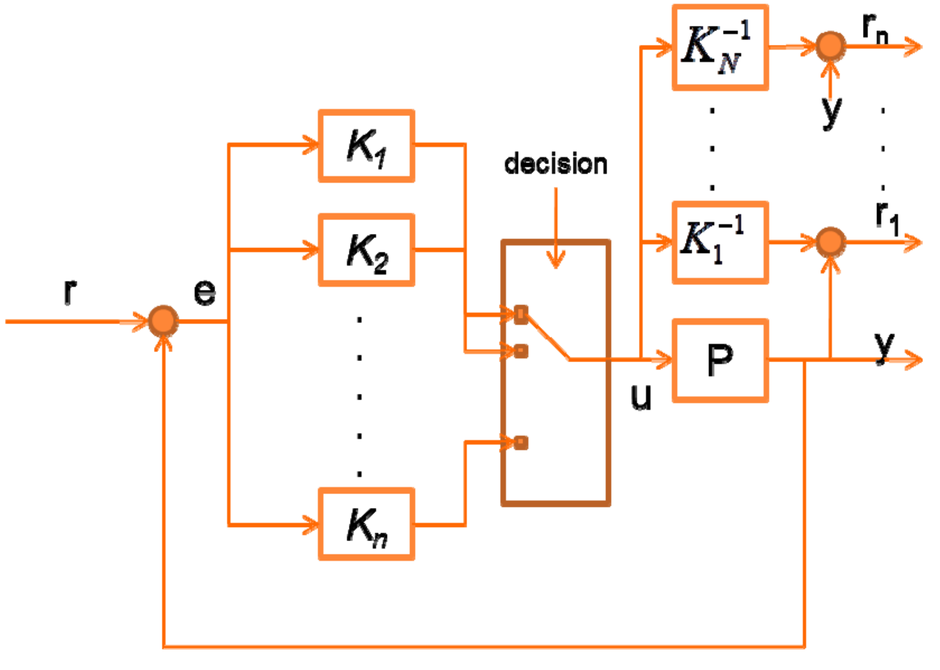

Fig. 2 Unfalsified Adaptive Supervisory Switching Control Scheme

Definition 7 *The ordered pair* (U, V) *, with* $U, V \in RH_\infty$*, where* RH_∞ *denotes the space of all asymptotically stable transfer function matrices, is called a left-coprime factorization of the transfer function G if:*

1) *V is square and invertible;*
2) $G = V^{-1}U$.

Moreover if $VV^* + UU^* = I$ *where* $V^*(s) = V^T(-s)$ *and* $U^*(s) = U^T(-s)$ *denote the adjuncts of V and U respectively, then the pair* (U, V) *is said to be a normalized left-comprime factorization of G(s).*

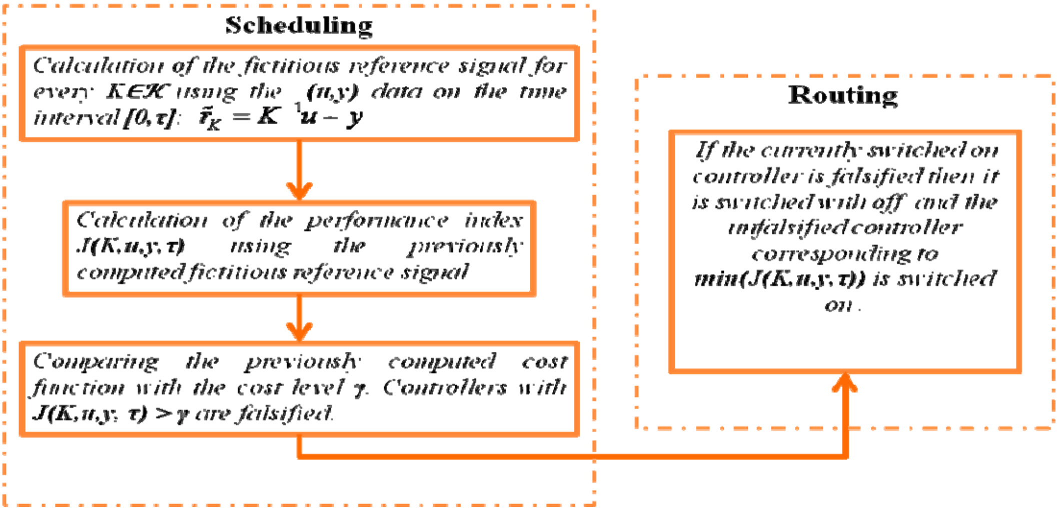

Fig. 3 Falsification algorithm

A computation method to determine left comprime factorization of a given *G(s)* may be found for instance in [6].

The closed loop system with $K=V^{-1}U$ is shown in Figure 4.

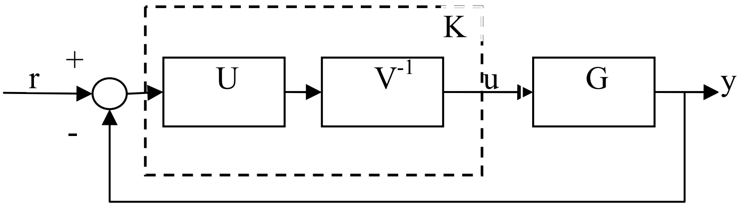

Fig. 4 Closed loop control system with $K=V^{-1}U$

The above configuration can be alternatively implemented using the so-called “observer-form” configuration (see also [4] and their references) in Figure 5, where direct algebraic computations show that the new reference signal $\tilde{z}$, which generates the data set (u, y), is given by:

$$\tilde{z} = Vu + Uy \tag{3}$$

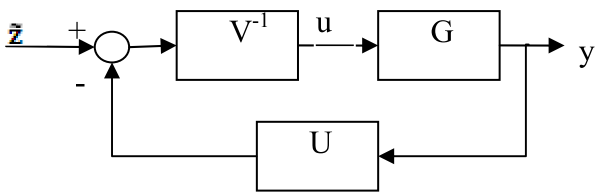

Fig. 5 *“Observer form” configuration*

Therefore the computation of the new fictitious reference $\tilde{z}$ does not require an invertibility condition; moreover when *u* and *y* are bounded, $\tilde{z}$ is bounded to, since *U* and *V* are stable.

The performance index will be determined as in (2) replacing the fictitious reference signal $\tilde{r}$ by $\tilde{z}$.

Most literature on the subject of Unfalsified Control recommends using some form of parameterization to represent the candidate controller set.

However the parameterizations given in [9] and [11], for example, are of simple controllers suitable for plants less complex then aircraft or for a limited envelope, as is the case in controlling a missile.

To be representative for aerospace applications the candidate controller set would have to include some form robust controllers, covering a large envelope, which have much higher complexity.

The authors, therefore, chose, for the application considered in the next section, the following polytopic representation of the plant family G:

$$\mathrm{G} := \left\{ G(s) \middle| G(s) = \lambda_1 G_1(s) + \ldots + \lambda_n G_n(s),\ \sum_{i=1}^{n} \lambda_i = 1;\ \lambda_i \geq 0,\ i = 1 \ldots n \right\}$$

where $G_i(s), i = 1, 2 \ldots n$ are known transfer matrices corresponding to "*n*" nominal flight conditions.

For each $G_i(s)$ one determines via the mixed-sensitivity design method mentioned in the previous section, the controller $K_1(s) = V^{-1}(s)U(s)$ where the coprime factors $V_i(s)$ and $U_i(s)$ are stable.

Then the following parameterization of the controller set is defined:

$$\mathrm{K} := \left\{ K(s) \middle| K(s) = \sum_{i=1}^{n} \lambda_i K_i, \sum_{i=1}^{n} \lambda_i = 1; \lambda_i \geq 0 \right\}$$

Based on the left coprime factorization of $K_i(s)$ $, i = 1, \ldots n$ one obtains:

$$K_\lambda(s) = \sum_{i=1}^{n} \lambda_i V_i^{-1}(s) U_i(s) = V_1^{-1} \left(\lambda_1 U_1 + \sum_{i=2}^{n} \lambda_i V_1 V_i^{-1} U_i \right) = V_1^{-1} U_\lambda \tag{4}$$

were the following notation has been introduced:

$$V_\lambda \square \lambda_1 U_1 + \sum_{i=2}^{n} \lambda_i V_1 V_i^{-1} U_i$$

From (3) it follows that the fictitious reference $\tilde{z}$ is in fact a function of λ and so is the performance index (3).

Using the collected data set (u, y) on the interval $[0, \tau]$ one can determine the optimal unfalsified controller $K_{\lambda^*}(s)$ of form (4) with

$$\lambda^* = argmin\, J(K, u, y, \tau)$$

subjected to $\sum_{i=1}^{n} \lambda_i = 1; \lambda_i \geq 0$.

The stability of the coprime factors V_i and U_i, $i = 1 \ldots n$ ensures the cost-detectability property of the pair (J, K).

3. CASE STUDY

For the case study, the ADMIRE model, provided as freeware by the Swedish Research Administration, has been used. The aim of the case study is to investigate the feasibility of unfalsified control for the control of airplane short-period dynamics. Therefore only the equations containing the angle of attack and pitch rate were used. Also to simplify the study just one of the control inputs was considered out of the three available for maneuvers in the longitudinal plane.

First a series of controllers were designed as vertices of the polytope. The design was carried out on linearizations of the short period dynamics of the ADMIRE model, at four flight conditions:

- Flight Condition 1: Mach 0.4, altitude 4500m
- Flight Condition 2: Mach 0.6, altitude 1500m
- Flight Condition 3: Mach 0.85, altitude 5500m
- Flight Condition 4: Mach 0.9, altitude 2000m

The controllers for these flight conditions were designed in the Matlab software package using the Weighted Mixed Sensitivity Criteria. As weighting functions we have used

$$W_1(s)=\frac{5(0.3s+3)}{s+0.15}\ \text{and}\ W_2(s)=\frac{1}{0.01s+1}$$

The mixed sensitivity problem was slightly altered to require the following minimization

$$\begin{bmatrix} W_1 S \\ W_2 GKS \end{bmatrix}_\infty \le \gamma$$

as such, and also because of the implementation of the modification from [4], the performance specification was changed from (2) to the following:

$$J\left(K,u,y,\tau\right)=\frac{\left\|w_1*(y-\tilde{z})\right\|^2_{L_2[0,t]}+\left\|w_2*y\right\|^2_{L_2[0,t]}}{\left\|\tilde{z}\right\|^2_{L_2[0,t]}} \tag{5}$$

where w_1 and w_2 are the impulse responses of the weighting transfer functions $W_1(s)$ and $W_2(s)$ used in the design of the four controllers $(K_1\ldots K_4)$, corresponding to the four flight conditions.

Thus we impose on the falsification algorithm that only those controllers meeting the same design criterions as the pre-designed controllers be unfalsified. The "*" symbol means convolution, $\tilde{z}$ represents the fictitious reference signal as defined by (3), y is the plant output signal, and u is the control signal. This translates into the following cost function.

$$J=\frac{\left\|\tilde{w}_1\right\|^2_{L_2[(j-1)k,jk]}+\left\|\tilde{w}_2\right\|^2_{[(j-1)k,jk]]}}{\left\|z\right\|^2_{[(j-1)k,jk]}}$$

where k is the *dwell interval* (the minimum time for which a controller is in the loop, and during which measurements of u and y are performed), j represents the index of the current dwell interval, $\tilde{w}_1$ is a vector containing the responses of the W_1 transfer function to the inputs contained in the vector $(y-z)$, $\tilde{w}_2$ is a vector containing the responses of the W_2 transfer function to the inputs contained in the vector y and z is the vector containing the fictitious reference signals calculated over the $[0,(j-1)k]$ using (5).

The cost function J is updated at the end of each interval equal to the dwell interval. The algorithm then switches-on the controller with the lowest value of J. For the polytopic representation, a precision of one digit was considered for the λ coefficients. This yielded 286 candidate controllers.

The possible values for the coefficients were stored in a vector which represented the set of candidate controllers, and were calculated in the initialization phase of the study. The case study was conducted in Matlab: a plant obtained by liniarizing the Admire model was connected in a feedback-loop with a random candidate controller. The unfalsification algorithm was run for 40 seconds.

Two different values for the dwell interval where considered: first 1 second, then 2 seconds. The reference signal used to generate the $[u, y]$ data was a compound command equal to 1 for the first 2 seconds, -1 for the next 4 seconds, 1 again for the next 8 seconds, -1 for the next 16 seconds and, finally, 1 for the last 10 seconds.

The results of the study are presented in figures 6, 7, 8, 9. Figures 6, 7 show the measured output of the plant, during the simulation with the plant linearized at Mach 0.75, altitude 5500 and an angle of attack of 12 degrees, with the dwell interval of 2 seconds in figure 6 respectively 1 second in figure 7. Figures 8 and 9 show the measured output of the plant when the plant was linearized at flight condition 4, again the dwell interval was 2 seconds in figure 8 and 1 second in figure 9.

The vertical black lines in the four figures represent the points at which the controller was switched by the algorithm. As can be seen with both plants the algorithm discarded the initial controller, which was unfit, after the first dwell interval. In the simulation that was ran at flight condition 4 the algorithm ultimately convergent to the controller that was pre-designed for it. Also it is to be noted that while a final controller was selected after at least 10 seconds, the output signal of the plant never achieved high values.

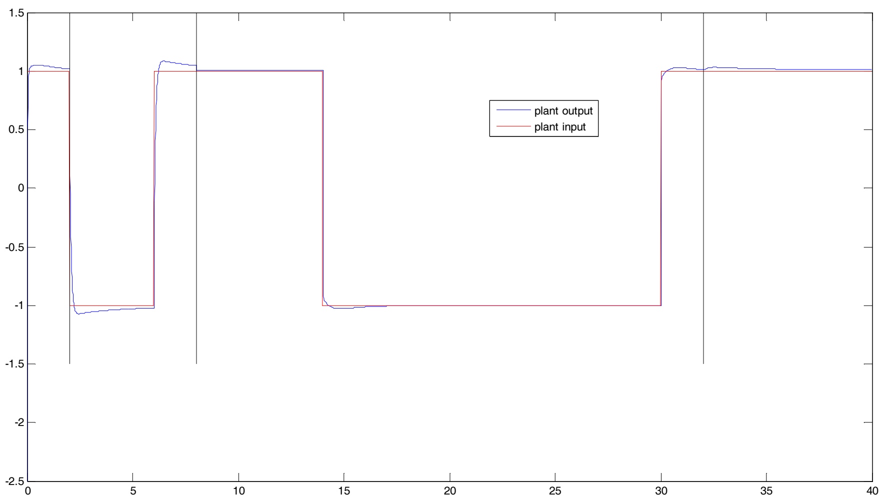

Fig. 6 Response of the closed-loop system with the plant liniarized at Mach 0.75, altitude 5500 and an angle of attack of 12 degrees. Dwell interval 2 seconds.

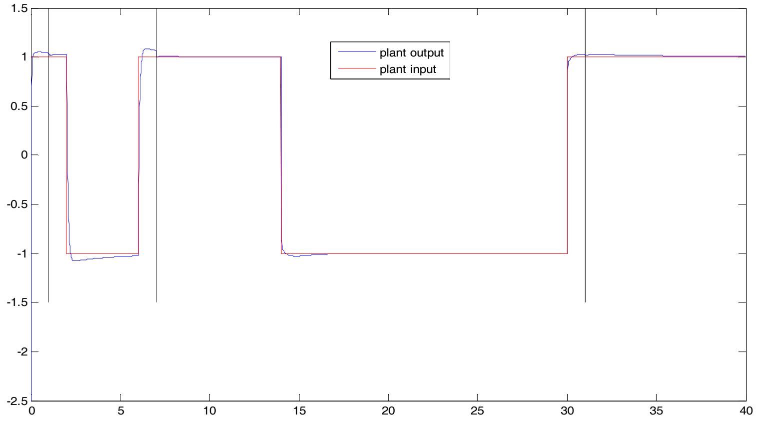

Fig. 7 Response of the closed-loop system with the plant liniarized at Mach 0.75, altitude 5500 and an angle of attack of 12 degrees. Dwell interval 1 second.

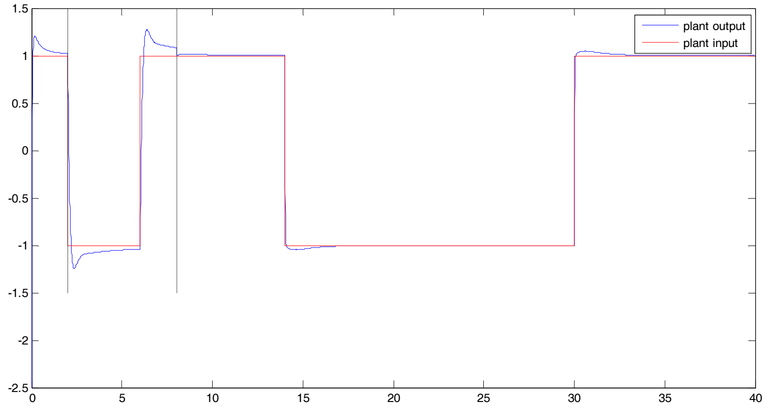

Fig. 8 Response of the closed-loop system with the plant liniarized at flight condition 4 (Mach 0.9, altitude 2000 and angle of attack of 0 degrees). Dwell interval 2 seconds.

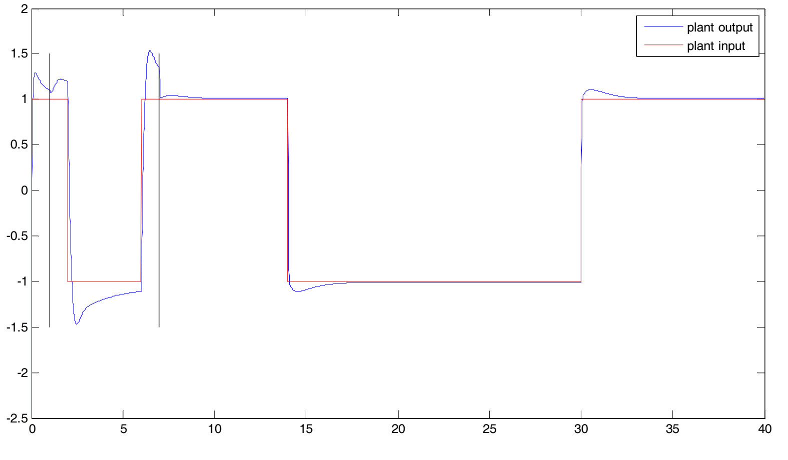

Fig. 9 Response of the closed-loop system with the plant liniarized at flight condition 4 (Mach 0.9, altitude 2000 and angle of attack of 0 degrees). Dwell interval 1 second.

4. CONCLUSIONS

The case study illustrated above confirmed the predictions of [9]. Before using the left-coprime factorization of the controllers, the authors tried using the original algorithm from [9] by calculating invertible controllers. Despite meeting this requirement the computed virtual reference signal was unbounded and thus the algorithm did not provide conclusive results.

Using the left-coprime factorization, as one can see above in section 4, the algorithm performed as specified, further more it never produced unbounded virtual references.

Still despite the promising results several problems were noticed:

- In some instances when run at the flight conditions for which the vertex controllers where calculated, the algorithm tended to choose another vertex controller. This indicates a necessity to modify the cost function to emphasis the qualities used in the design of the vertex controllers.
- The values of the cost functions calculated for the different controllers tended to be very close together which may cause the algorithm to choose controllers which are not to appropriate for the considered flight condition.

Still even with the problems specified above the Unfalsified Adaptive Switching Supervisory Control approach shows a lot of promise for the control of Unmanned Aerial Vehicles, offering a capacity to learn and adapt to quickly changing flying conditions.

Further research will be conducted by the authors to address the problems illustrated above. Namely modifications of the cost function by implementing signal analysis tools will be considered so that the algorithm can better discriminate between the validity of different controllers.

Also a generalized method of minimizing the cost function will be considered, so that the falsification algorithm becomes faster.

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