Physico-mathematical model of the contact between the sealing O-ring and the sealing surfaces

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Abstract: *The most common type of sealing ring used in the hydraulic system of an aircraft and its equipment is the O-ring sealing system used in radial sealing, static or dynamic. In order to better understand the sealing phenomenon itself and how wear of O-ring sealing rings occurs, it is essential to define a physico-mathematical model of the contact between the sealing O-ring and the sealing surfaces.*

Key Words: *O-ring, radial sealing, hydraulic system, aircraft*

1. GENERAL CONSIDERATIONS AND WORKING HYPOTHESES

Regardless of the type of radial seal (shaft type or bore type), after the final assembly, the sealing ring will be subjected to radial compression between two cylindrical surfaces: a bore surface on the outside of the sealing ring and a spindle-like surface, on the inside of the sealing ring, because the radial dimension of the sealing ring before deformation is d_1, but after deformation gets $h_c = (d_z - d_b) / 2 < d_1$, as can be seen in Fig. 1.

Therefore, between the sealing rings and the sealing surfaces, a contact pressure p_{cz} and p_{cb} will be generated, spread over the width of the contact strip after a certain law that will be subsequently established, the result of which is q_z and q_b, respectively on the unit of the length of the contact strip, as in Fig. 2.

The lower indexes z and b refer to the contact of the sealing ring with the bore type surface, and the shaft type surface, respectively.

In Fig. 1, *1* - is the undeformed sealing ring; *1'* - the sealing ring after final assembly; *2* - the shaft type part; *3* - the bore type part.

In the case of shaft seals, d_b is the diameter of the shaft part and d_z - the bottom diameter of the recess in the bore type part. Also, it is necessary to observe the condition:

$$(d_2 + 2d_1) >= d_z \tag{1}$$

In the case of boreholes, d_z is the diameter of the bore type part, and d_b is the bottom diameter of the bore in the shaft-type part. Also, it is necessary to observe the condition:

$$d_2 <= d_b \tag{2}$$

Geometric conditions (1) and (2) ensure that the sealing ring is correctly positioned in the recess after the partial assembly. In case of non-compliance, the sealing is compromised either due to over-stressing of the sealing ring or due to its inadequate layout in relation to the plane of symmetry of the groove.

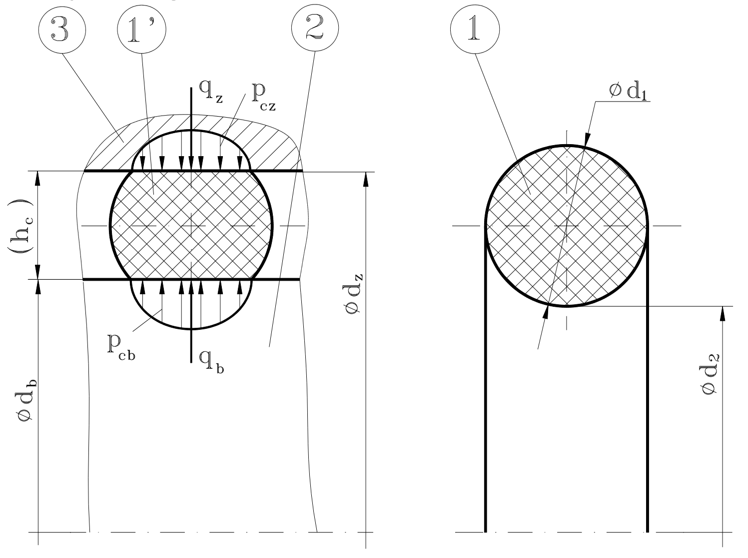

Fig. 1 - Radial compression of the sealing O-ring after the final assembly

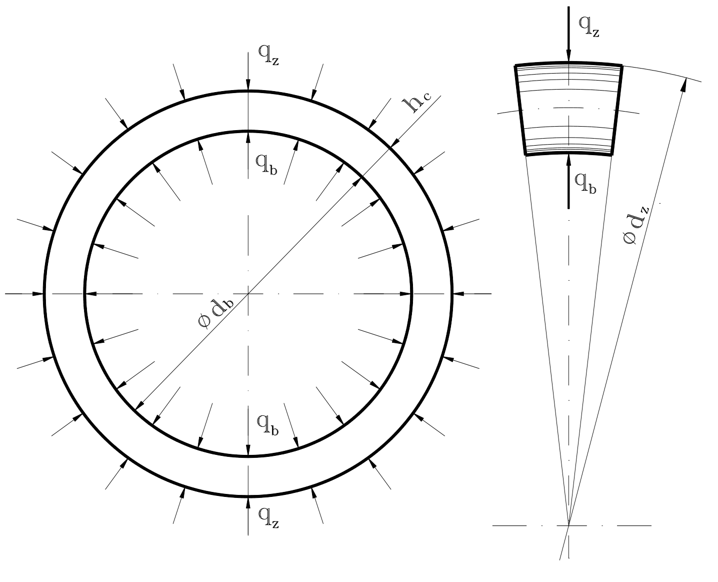

Fig. 2 - Distribution of the resultants of the contact pressures on the sealing ring

In other terms, it is made clear that shaft and bore type parts are made of special steels characterized by a longitudinal elastic modulus (Young's module) - $E \approx 210000$ MPa and a cross-contractive coefficient (Poisson coefficient) $\boldsymbol{\nu} \approx 0.3$.

The highly elastic material of the sealing ring (e.g., NBR nitrate butadiene medium) is characterized by the following: E = 8.5-10 MPa and $\boldsymbol{\nu} \approx 0.5$. This means that at the same application, the deformation of the steel piece is approximately 20000 times less than that of the sealing ring, thus being practically negligible in this case. Therefore, the steel parts will still be considered undeformable. The value of Poisson's coefficient $\nu \approx 0.5$ in the case of elastomers, has the meaning that they deform without changing their volume.

2. LINEAR CONTACT BETWEEN THE SEALING RING AND THE CYLINDRICAL CONTACT SURFACES

As shown in the previous point, between the sealing ring and the sealing surfaces there are contact pressures p_{cz} or p_{cb} distributed over the width of the contact strips $2b_z$ and $2b_b$ respectively, the resultants of which are q_z and q_b for the ring sector with the unitary length, as shown in Fig. 2.

The deformations suffered by the sealing ring (see Fig. 3) δ_b and δ_z, respectively, must comply with the following condition:

$$\delta_b + \delta_z = d_1 - h_c \tag{3}$$

where $h_c = (d_z - d_b) / 2$ represents the height of the sealing ring groove.

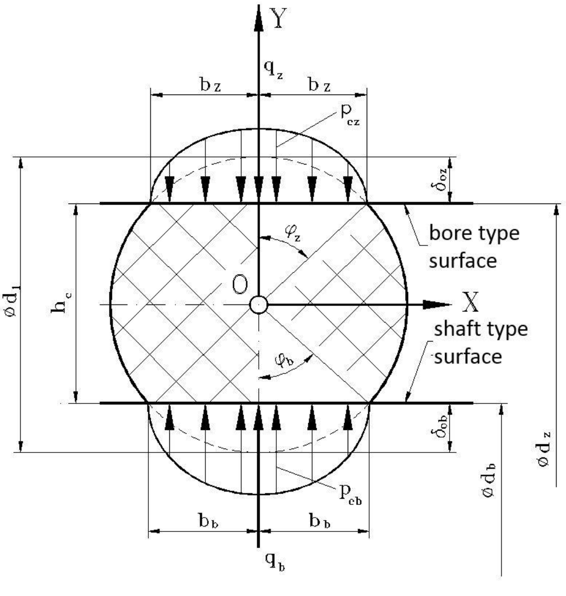

Fig. 3 - Contact between O-ring and sealing surfaces

In the first instance, it will be assumed that the surfaces in contact are smooth and there is no friction between them. Due to the symmetry of the contact with the cylindrical surfaces and the lack of discontinuities, it is enough to study the stress and deformation state in a single cross-section, whatever it is.

The calculation of the distribution of contact pressures for their maximum value for the half-width of the contact strip for the deformations suffered by the sealing ring will be further deduced, considering the sealing ring, a Winkler elastic medium [1].

In point O (the center of cross section of the O-ring) an Oxy reference system is attached, as shown in Fig. 3, and φ_z and φ_b are the semi angles of the contact chords $2b_z$ and $2b_b$, respectively. The deformations of the sealing ring δ_b or δ_z represent no more than 15% of d_1, but the relative variation in the mean diameter of the sealing ring, after the final assembly, is within the range between: -2% ... + 4%.

In accordance with the foregoing, the pressure distribution law for $p_{cb}(x)$ and $p_{cz}(x)$, respectively is of the form:

$$p_{\text{cb}}(x) = \frac{k}{d_1} \cdot \delta_b(x) \tag{4}$$

$$p_{\text{cz}}(x) = \frac{k}{d_1} \cdot \delta_z(x) \tag{5}$$

where k/d_1 is a constant dependent on the nature of the elastomeric material and the geometric characteristics of the contact, determined on the basis of the recommendations in [1] so as to verify the experimental distributions of contact pressures published in [2, 3, 4, 5, 6]:

$$\frac{k}{d_1} = C \cdot \frac{E}{h_c \cdot (1-\nu^2)} \tag{6}$$

where C is a numerical constant equal to the value of d_1; ν and E are the Poisson coefficient, and the longitudinal elastic modulus of the sealing ring material, respectively.

To simplify the writing, the ratio $E/(1-\nu^2)$, called the elastic characteristic of the elastomer, is denoted by E', and the previous relationship takes the form:

$$\frac{k}{d_1} = C \cdot \frac{E'}{h_c} \tag{7}$$

The radial deformations $\delta_b(x)$ and $\delta_z(x)$ (see Fig. 3) are calculated with the relations:

$$\delta_b(x) = \sqrt{r_1^2 - x^2} - (r_1 - \delta_{ob}) \tag{8}$$

$$\delta_z(x) = \sqrt{r_1^2 - x^2} - (r_1 - \delta_{oz}) \tag{9}$$

in which δ_{ob} and δ_{oz} are the maximum radial deformations at the initial contact points, and the laws of variation of the contact pressures become:

$$p_{\text{cb}}(x) = \frac{k}{d_1}\left(\sqrt{r_1^2 - x^2} - (r_1 - \delta_{ob})\right) \tag{10}$$

$$p_{\text{cz}}(x) = \frac{k}{d_1}\left(\sqrt{r_1^2 - x^2} - (r_1 - \delta_{oz})\right) \tag{11}$$

where $r_1 = d_1/2$ is the radius of the sealing ring cross section.

For the unit length ring sector, the results of these pressures distributed on the contact strip of width $2b$ are q_b and q_z, respectively, as shown in Fig. 3:

$$q_b = \int_{-b}^{+b} p_{\text{cb}}(x)\,\text{dx} \tag{12}$$

$$q_z = \int_{-b}^{+b} p_{\text{cz}}(x)\,\text{dx} \tag{13}$$

and they must respect the equilibrium condition:

$$\pi \cdot d_b \cdot q_b = \pi \cdot d_z \cdot q_z \tag{14}$$

After integration, the expressions of the resultants q_b and q_z become:

$$q_b = 2\frac{k}{d_1}\left[b_b(\delta_{ob} - r_1) + 0{,}5 \cdot r_1^2\left(\varphi_b + 0{,}5 \cdot \sin(2\varphi_b)\right)\right] \quad (15)$$

$$q_z = 2\frac{k}{d_1}\left[b_z(\delta_{oz} - r_1) + 0{,}5 \cdot r_1^2\left(\varphi_z + 0{,}5 \cdot \sin(2\varphi_z)\right)\right] \quad (16)$$

The maximum radial deformations δ_{ob} and δ_{oz} are determined from condition (3) and from the equilibrium condition (14) which, after integration, is written as:

$$\frac{d_b}{d_z} = \frac{\left[b_z(\delta_{oz} - r_1) + 0{,}5 \cdot r_1^2\left(\varphi_z + 0{,}5 \cdot \sin(2\varphi_z)\right)\right]}{\left[b_b(\delta_{ob} - r_1) + 0{,}5 \cdot r_1^2\left(\varphi_b + 0{,}5 \cdot \sin(2\varphi_b)\right)\right]} \quad (17)$$

where φ_z and φ_b and the half-widths of the contact strip b_z and b_b can be expressed according to δ_{ob} and δ_{oz}, which allows them to be determined by numerical solving:

$$\varphi_b = \text{arctg}\left(\frac{b_b}{r_1 - \delta_{ob}}\right) \quad (18)$$

$$\varphi_z = \text{arctg}\left(\frac{b_z}{r_1 - \delta_{oz}}\right) \quad (19)$$

$$b_b = \sqrt{\delta_{ob}(d_1 - \delta_{ob})} \quad (20)$$

$$b_z = \sqrt{\delta_{oz}(d_1 - \delta_{oz})} \quad (21)$$

In conclusion, the contact pressure $p_{cb}(x)$ or $p_{cz}(x)$ is parabolically distributed over the width of the contact strip having a maximum in the initial contact point ($x = 0$):

$$p_{cb}(0) = p_{ob} = p_{b\max} = \frac{k}{d_1} \cdot \delta_{ob} \quad (22)$$

$$p_{cz}(0) = p_{oz} = p_{z\max} = \frac{k}{d_1} \cdot \delta_{oz} \quad (23)$$

For some subsequent calculations, it is useful to establish the calculation relations for the average contact pressure p_{cbm} or p_{czm} by reporting their resultants q_b and q_z to the contact area:

$$p_{cbm} = \frac{k}{d_1}\left[\delta_{ob} - r_1 + 0{,}5\frac{r_1^2}{b_b}\left(\varphi_b + 0{,}5 \cdot \sin(2\varphi_b)\right)\right] \quad (24)$$

$$p_{czm} = \frac{k}{d_1}\left[\delta_{oz} - r_1 + 0{,}5\frac{r_1^2}{b_z}\left(\varphi_z + 0{,}5 \cdot \sin(2\varphi_z)\right)\right] \quad (25)$$

3. THE INFLUENCE OF PRESSURE FROM THE SEALED ENVIRONMENT

The pressure in the sealed medium acts on the surface of the sealing ring which is in contact with it, as in Fig. 4.

Experimentally, it was found that, due to the viscous component of the sealing ring material, the pressure of the medium is not fully transmitted, but with a subunitary transmission coefficient, denoted s, which depends on the Poisson coefficient of the sealing ring material $\nu = 0.48\ldots0.496$ [2, 3]:

$$s = \frac{\nu}{1-\nu} = 0.90\ldots0.98 \tag{26}$$

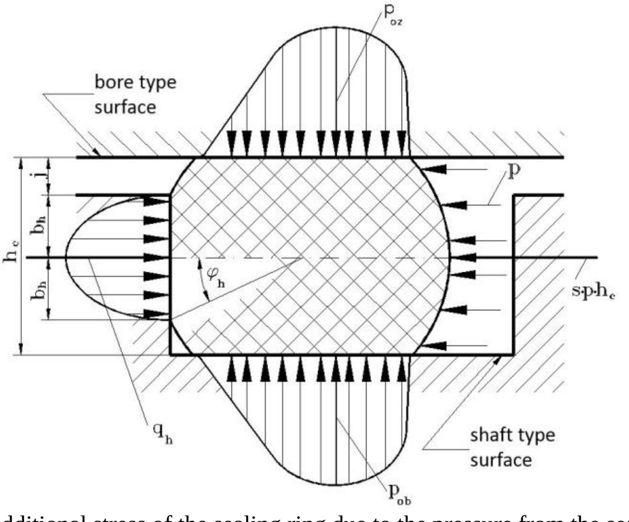

Fig. 4 - Additional stress of the sealing ring due to the pressure from the sealed environment

In Fig. 4 all previous notations remain valid and the new notations have the following meanings: b_h is the half-width of the contact strip between the sealing ring and the shoulder of the groove; q_h - resultant of the contact pressure between the sealing ring and the side surface of the groove; p - the pressure from the sealed environment; $s \cdot p \cdot h_c$ - the resultant of the pressure in the sealed medium per unit of length of the median diameter of the sealing ring; j - maximum radial clearance of the hole shaft fit.

The equilibrium condition requires that the result of $s \cdot p \cdot h_c$ and q_h are equal:

$$s \cdot p \cdot h_c = q_h \tag{27}$$

and q_h is calculated in a similar relation to (15) and (16):

$$q_h = 2\frac{k}{d_1}\left[b_h(\delta_h - r_1) + 0{,}5 \cdot r_1^2\left(\varphi_h + 0{,}5 \cdot \sin(2\varphi_h)\right)\right] \tag{28}$$

in which φ_h and δ_h, can be expressed according to the half-width of the contact strip b_h:

$$\delta_h = r_1 - \sqrt{r_1^2 - b_h^2} \tag{29}$$

$$\varphi_h = \text{arctg}\left(\frac{b_h}{\sqrt{r_1^2 - b_h^2}}\right) \tag{30}$$

In Fig. 4 it is noted that if the width of the contact strip $2b_h$ exceeds the upper edge of the shoulder of the groove, then the sealing ring material will be expelled into the sealed gap j under pressure p from the sealed medium. This phenomenon is called extrusion and it occurs when the following condition is met:

$$2 \cdot {b_h}^3(h_c - 2 \cdot j) \tag{31}$$

since it is very likely that the side contact strip is distributed symmetrically in relation to the height h_c of the groove.

On the other hand, the pressure in the sealed medium is also transmitted to the other sealing surfaces and changes the size and distribution of contact pressures between the sealing ring and the cylindrical sealing surfaces, as shown in Fig. 5.

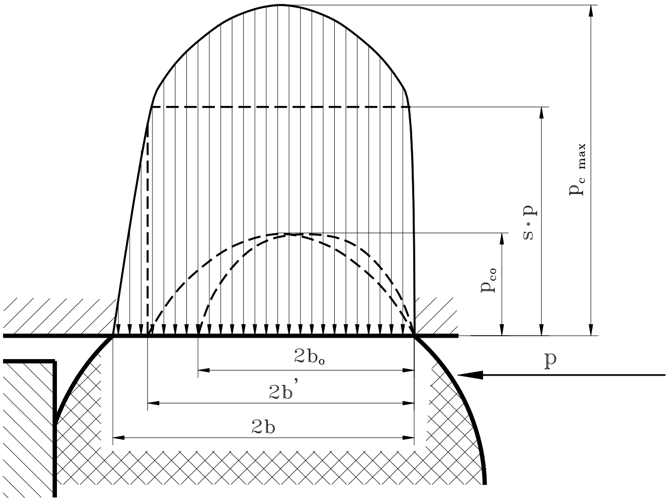

Fig. 5 - Diagram of the contact pressure distribution under pressure from the sealed environment

Therefore, the maximum values of the contact pressures between the sealing ring and the cylindrical sealing surfaces become:

$$p_{\mathrm{cb}_{\max}} = p_{\mathrm{ob}} + s \cdot p \tag{32}$$

$$p_{\mathrm{cz}_{\max}} = p_{\mathrm{oz}} + s \cdot p \tag{33}$$

Based on the experimental results [2, 3, 5, 6], regarding the experimental variation of the ratio $2b/d_1$, depending on the environmental pressure p and the hardness of the elastomeric material, we could establish an analytical calculation ratio for the ratio $2b/d_1$, with very good precision, as shown in Fig. 6:

$$2\,b/d_1 = 1 - (\mathrm{IRHD}/90) \cdot p^{-0,5} \tag{34}$$

On the basis of the experimental results [2, 3, 5, 6] regarding the pressure distribution, we compiled the calculation scheme of Fig. 6, where the width of the contact strip $2b$ ($2b_b$ or $2b_z$) is determined by relation (34) and the width of strip $2b'$ can be determined with the following relationship, which is also determined on the basis of the analysis of the experimental results mentioned above:

$$2b' = 2b \cdot (3p)^{-0,075} \tag{35}$$

The contact pressure distributed on strip $2b$ (see Fig. 5) is calculated with the following relation, for $x \in [-b', b']$:

$$p_c(x) = s \cdot p + \frac{k}{d_1}\left(\sqrt{r_*^2 - x^2} - (r_* - \delta_o)\right) \tag{36}$$

in which:

$$r_* = \frac{b'^2 + \delta_o^2}{2\delta_o} \tag{37}$$

$$\delta_o = \frac{\delta_{\text{ob}} + \delta_{\text{oz}}}{2} \tag{38}$$

and with the below relationship for $x \in (2b', 2b]$:

$$p_c(x) = s \cdot p \cdot \left(\frac{2b - x}{2b - 2b'}\right) \tag{39}$$

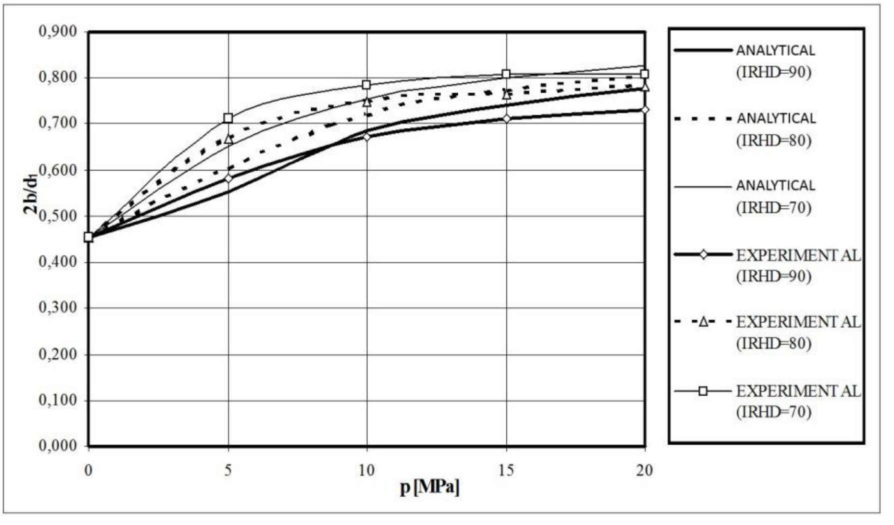

Fig. 6 - Variation of ratio 2b/d1 with variation of elastomer pressure and hardness

The correctness of the calculation relations (36) and (39) was verified by the comparative analysis of the pressure distributions obtained experimentally [2, 3, 5, 6] and analytically, as shown in Fig. 7.

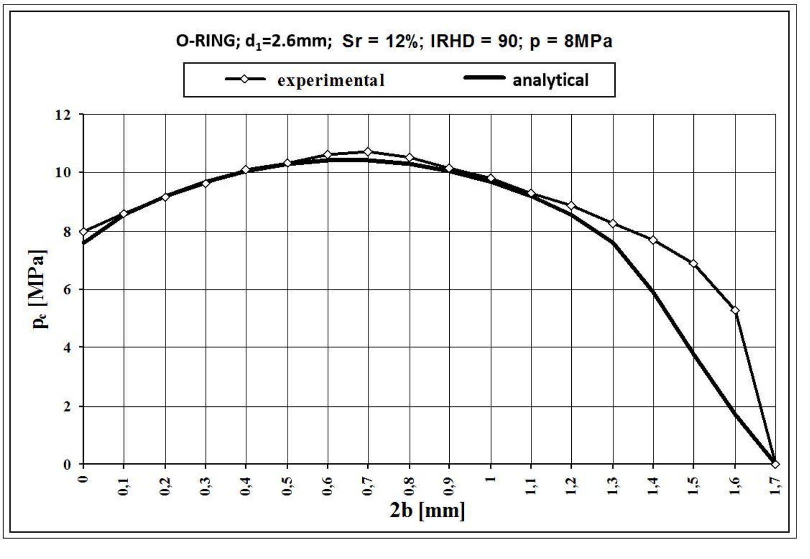

Fig. 7 - Distribution of contact pressures on strip $2b$ (experimental and analytical)

4. CONCLUSIONS

The maximum values of the contact pressures between the sealing ring and the cylindrical sealing surfaces is calculated with (33).

Based on the experimental results, regarding the experimental variation of the ratio $2b/d_1$, depending on the environmental pressure p and the hardness of the elastomeric material, we have established an analytical calculation ratio for ratio $2b/d_1$ (34), with very good precision, as shown in Fig. 6

The contact pressure distributed on strip $2b$ (see Fig. 5) is calculated with (36) or (39), for $x \in [-b', b']$.

The correctness of the relations (36) and (39) was verified by comparative analysis of the pressure distributions obtained experimentally and analytically, as shown in Fig. 7.

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ABBREVIATION

IRHD: International Rubber Hardness Degrees.