# An analytical representation of airfoils

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***Abstract***: *An important part of the construction of an aircraft is the shape of the wings. Their aerodynamics involve numerous evaluations and simulations through fluid dynamics. The reduction of the evaluation times is achieved by a parametric function. This is represented by the ratio of two polynomials. This representation will be important in order to reduce computation time for artificial network applications. The number of parameters is reduced. The analytical shape of the wing section implies analytical expression for wing parameters: maximum thickness, maximum camber, maximum camber position, minimum thickness position, and so on. These characteristics have algebraic expressions that involve arithmetic operations. We have constructed a simple mathematical wing model. We replace the discrete wing shape with a continuous form, which is described by lacunary polynomials.*

***Key Words***: *Wing geometric characteristics, lacunary polynomials, lift force, drag force*

## 1. INTRODUCTION

In this work, we present a method for airfoil parametrization using a function of the following form:

$$f(x) = \frac{\sum_{i=0}^{i=n_1} a_i x^i}{\sum_{i=0}^{i=n_2} a_i x^i}$$

We analyze the effectiveness of our method on airfoil parametrization and compare it with Class Shape Transformation (CST) [1]. We conclude that our method is comparable to CST in terms of number of parameters used and precision, but it has an advantage on heavily cambered airfoils, where CST might struggle to achieve high levels of precision.

Multi-objective optimization problems are a common occurrence in all fields of modern science and engineering, and aircraft design is no different. The shape of the aircraft wing is almost always complex, non-linear, and highly optimized for specific conditions, and computer simulations of fluid flows are a fundamental step in the process.

Aircraft wing sections are shaped like airfoils, so as to maximize lift and minimize drag. These airfoils often need to be parametrized, meaning having an analytical shape ascribed to them, in order to be properly optimized.

Niculescu et. al. [2] first came up with the idea of using a ratio of 2 polynomials in order to parametrize airfoils, using a formula of the following form:

$$f(x) = \frac{gx}{x^2 + h^2}$$

In this work we expand upon his idea, by using higher degree polynomials for both the numerator and the denominator, up to the 6th degree, and we will analyze its performance against other well-known parametrization methods.

## 2. THEORETICAL METHOD

We start with a given airfoil from publicly available databases [3], and take its coordinates at different points. The points are then divided into an upper surface, and a lower surface.

Each surface is analyzed separately. The data is then fitted with a function of our form, for different degrees for both numerator and denominator (2/4, 3/4, 6/4, etc.), as well as CST using Bernstein basis. Note that for the CST fit, since it would be a non-linear system, we have used an evolutionary algorithm (CMA-ES [5], [6], with population size 128 and 3000 iterations) to get the best solution available (the lowest RMS error). Note that for the CST fit, we have used 11 polynomial indices, as well as variable N1 and N2. We have also introduced a trailing edge thickness and chord angle for the relevant airfoils.

For the rational fit, we obtain it in 2 ways: first, through the same CMA-ES algorithm as for the CST, and second, through the curve_fit function available in python. That way, we hope to circumvent the limitations of our stochastic algorithm, and see how good our fit would be for an optimized version of the algorithm.

Two metrics are used to evaluate our method: the max. error as a % of chord, and the max. error as a % of chord.

A fit is considered acceptable if it satisfies the following 2 criteria:

1. max.error as a % of chord < 0.25% (most CFD simulations only go up to a similar accuracy)
2. average. error as a % of chord < 0.15%

After both fits are calculated we then compare them on accuracy (if both are acceptable) and *number of parameters used*.

The less parameters used, the better. Thus if our fit uses less parameters than the other methods, with a similar or better precision, then our fit is better.

We have analyzed 50 different airfoils, from a different range of engineering applications [7]. We chose the lower surface and chose a rational fit with the numerator a $6^{th}$ degree polynomial, and a $4^{th}$ degree polynomial as the denominator.

# 3. RESULTS

We then tabulated the data for both the maximum error and the root-mean-square error.

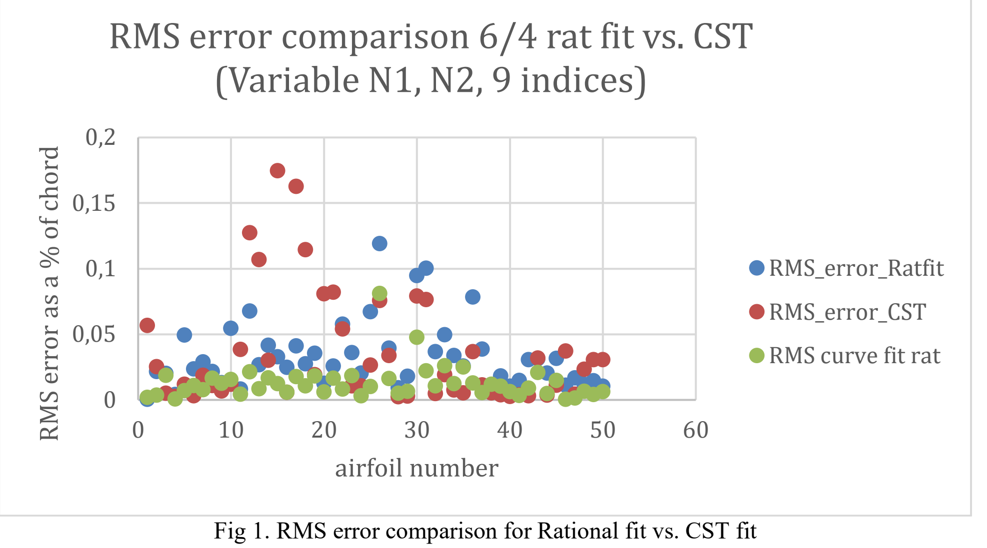


Fig 1. RMS error comparison for Rational fit vs. CST fit

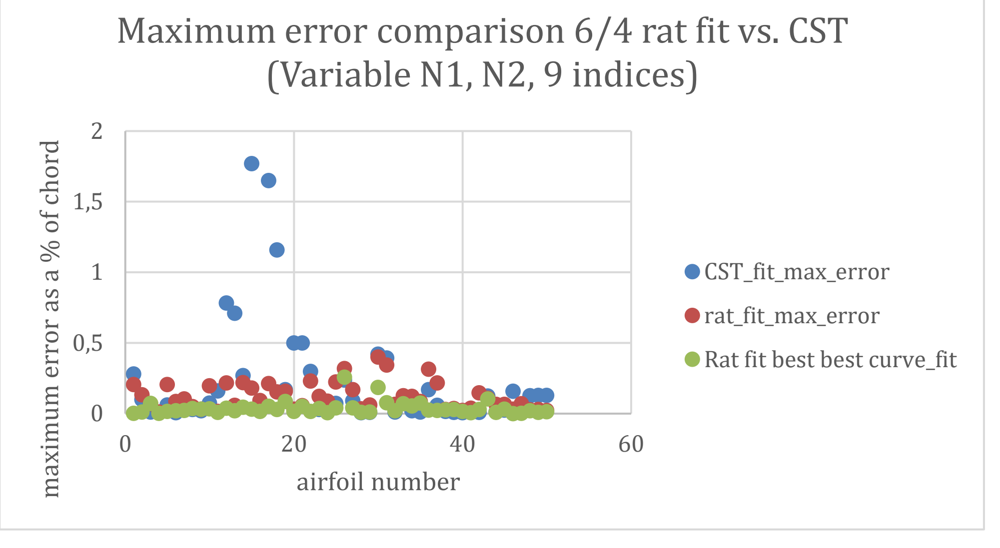


Fig 2. Maximum error comparison for Rational fit vs. CST fit

It is worth taking a further look at the outliers for the CST fit, in both RMS error and maximum error.

As stated above, these will be the airfoils that require the introduction of a trailing edge thickness, as well as a chord angle for the CST evolutionary algorithm. Since it is clear that the rational fit obtained with the curve_fit function in Python is much better than the one obtained through the CMA-ES evolutionary algorithm, we shall compare the improved CST fit with it, and not the other rational fit.

After modifying the algorithm to account for these variables, we get the following results:

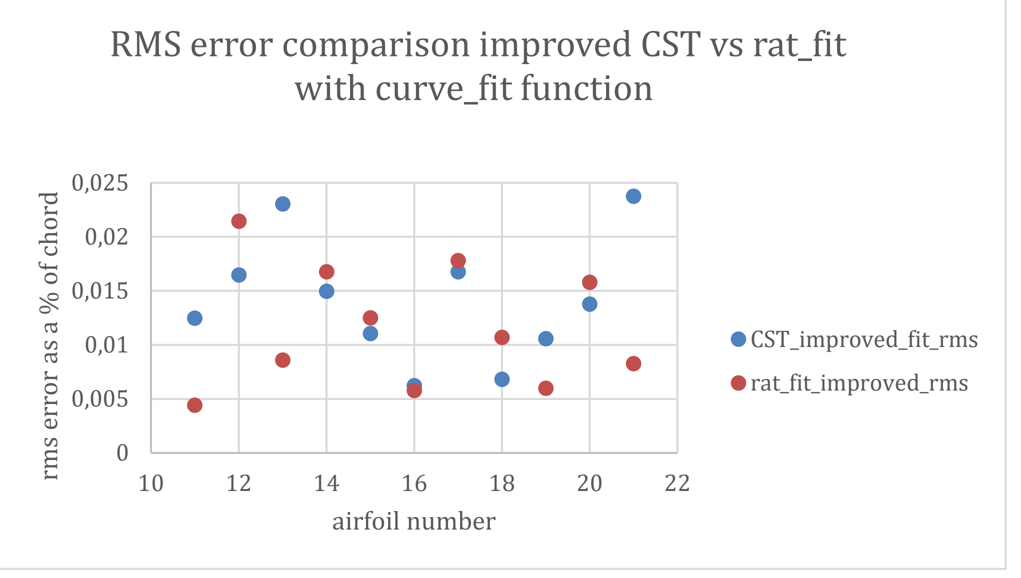


Fig 3. RMS error comparison for improved CST fit vs. Rational fit obtained through the curve_fit function

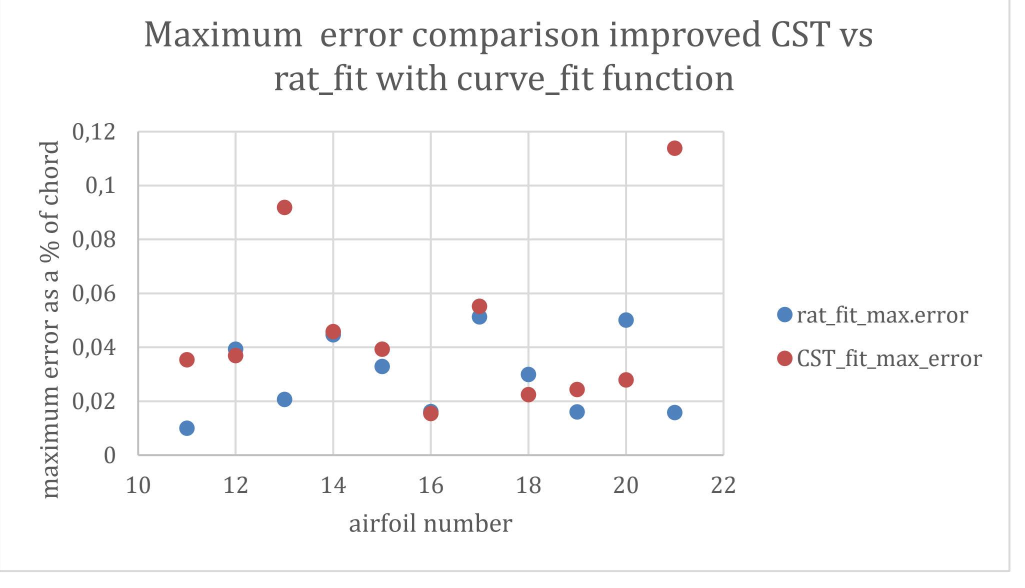


Fig 4. Maximum error comparison for improved CST fit vs. Rational fit obtained through the curve_fit function

We present here an example, as a proof-of concept (for sd7037), for the lower surface: Rational fit (6/4), for a total of 11 parameters:

$$f(x) = \frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4}$$

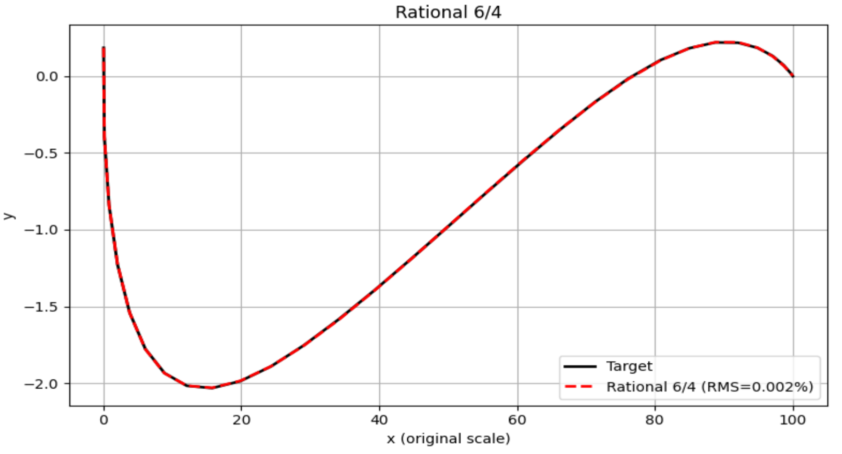


Fig. 5. Rational fit. The numerator of the fraction is a polynomial of degree 6, the denominator is a polynomial of degree 4. Rms_ error = 0.00185% of chord; Max_error = 0.00314% of chord

The values of the 11 parameters are:

a0=0.0170418694880304
a1=-49.6688070077132
a2=-10158.5833249589
a3=-157200.433447991
a4=389201.112168681
a5=-204092.114481554
a6=-17669.7386052767
b1=10968.789053471
b2=647270.779409408
b3=4308287.49187513
b4=-3860183.54224129

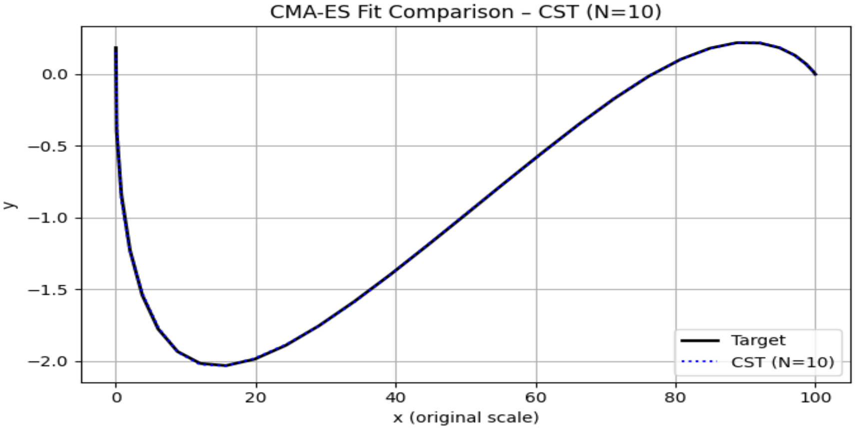


Fig. 6 CST fit (11 indexes, plus variable N1, N2 and trailing edge thickness and chord angle variables). Rms_ error = 0.01547%; Max_ error = 0.0645%

## 4. DISCUSSIONS

From the results, we can clearly see that our fit has the potential to be much better than the CST fit, if the evolutionary algorithm is properly optimized. This comes from the fact that the rational fit obtained through the curve_fit function gives lower rms and maximum errors than both fits obtained through the evolutionary algorithm. We can also see that even the rational fit obtained through the evolutionary algorithm performs comparable to the CST fit.

These observations also hold up when considering the improved CST fit that takes into account the trailing edge thickness and chord angle. From fig. 3 and fig. 4, we can see that while our fit has similar performance to the CST in terms of the rms errors, it performs consistently better in terms of the maximum error.

## 5. CONCLUSIONS

Following our data analysis, we can make the following statements:

Our fit can be used to characterise airfoils, especially airfoils where CST might struggle (such as heavily cambered ones) to a very high degree of accuracy

Thus, it can be concluded that our rational fit would be superior for airfoil parametrization in applications where heavily cambered airfoils are generally used and would be worth using over other methods.

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[7] * * * List of all airfoils used, and their corresponding numbers.


**List of all airfoils used, and their corresponding numbers:**

1. sd7037
2. ag35
3. fx60126
4. s6063
5. mh114
6. rg15
7. e205
8. naca643618
9. naca633618
10. naca4415
11. naca23015
12. ls417

13. ls413
14. sc20414
15. sc20712
16. s4053
17. sc20714
18. sc20610
19. sc20410
20. sg6051
21. sg6050
22. sg6042
23. sg6040
24. mh120
25. mh112
26. du861372
27. du84132v
28. s2055
29. s2048
30. s1210
31. s1223
32. goe703
33. fx66s196
34. fx79w151a
35. e210
36. e423
37. clark-y
38. naca632615
39. naca65210
40. naca65206
41. n64800a
42. naca64a010
43. naca6412
44. naca6409
45. naca662415
46. naca0015
47. naca0009
48. naca23012
49. naca4412
50. naca2412